# Demand Uncertainty, Selection, and Trade *


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**Abstract**

In a theoretical framework that encompasses complete and incomplete information environments, this paper shows that trade is more elastic when exporters are uncertain about the foreign demand for their goods. Under complete information, firms condition their export decisions on both productivity and demand. Under incomplete information, firms select into exporting based on productivity alone, which encourages exporting and therefore makes trade more elastic at the extensive margin. We show that because of these differences in selection, the identification of trade elasticities under incomplete information requires quantity data, and under complete information requires sales data. Using Brazilian exporter-level data on sales and quantities, we quantify trade elasticities and the welfare gains from trade under both information environments. We find that welfare gains from trade are smaller under uncertainty and that the difference in welfare increases when demand shocks are more dispersed. Finally, we find evidence for this paper's mechanism from estimating an otherwise standard gravity equation that accounts for information availability and demand dispersion.



**Keywords**: Demand uncertainty, firm size distribution, extensive margin, selection, trade elasticities, welfare.

**JEL**: F12, F13.


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# 1 Introduction

Welfare implications for standard trade theory, most recently developed in Arkolakis, Costinot, and Rodríguez-Clare (2012a) and Melitz and Redding (2015), show that partial elasticities of trade with respect to variable trade costs are key parameters for evaluating the welfare gains from trade. While these implications are derived from a broad class of models in which firms have complete information about their economic environment, a growing branch of the trade literature has demonstrated that models with uncertainty along the lines of Jovanovic (1982) are well suited to match salient patterns of empirically observed firm behavior.[1] However, normative implications of such an alternative information structure, particularly for measurements of trade elasticities and the welfare gains from trade, are not yet well understood.[2]

In this paper, we vary the information available to potential exporters in a standard trade model and study the impact of uncertainty about demand in foreign markets on the welfare gains from trade. We find that under uncertainty trade flows are more elastic to changes in variable trade costs and the welfare gains from trade are lower compared to an environment with complete information. We show that these results arise because selection into export activity is more stringent when potential exporters have complete information, as potential exporters would not knowingly enter an unprofitable market. When potential exporters face uncertainty about demand in foreign markets, a larger number of firms engage in risky export activity but only few firms become large exporters relative to economies with complete information about demand.

This paper highlights the selection mechanism theoretically and quantitatively. To do so, we study two stylized economic environments that represent opposing assumptions about the information firms possess when making export decisions. The first economic environment is a stylized version of Timoshenko's (2015b) trade model, which embeds firm-level learning about demand along the lines of Jovanovic (1982) in a standard trade model. In this model, firms make export decisions after observing their firm-specific productivity but before observing their firm-specific demand in foreign markets. Hence, firms make export quantity and participation decisions based on *productivity* alone. Export sales thereafter depend on productivity, through export quantity decisions, and the realization of demand shocks in foreign markets. The second economic environment we consider is a standard trade model with

[1]These papers incorporate Jovanovic (1982) learning mechanism into the Melitz (2003) model, which features monopolistically competitive exporters that are heterogeneous in productivity and learn about their unobserved idiosyncratic demand in foreign markets. See Arkolakis, Papageorgiou, and Timoshenko (2018) for implications for firm growth as a function of age and size, Timoshenko (2015b) for implication for firm product switching behavior, and Bastos, Dias, and Timoshenko (2018) for implications for firm input and output pricing behavior.

[2]A notable exception is Arkolakis, Papageorgiou, and Timoshenko (2018), who characterize constrained efficiency of a model in which firms learn about demand, but do not engage in international trade.

complete information (see Melitz (2003), Bernard, Redding, and Shott (2010), Arkolakis et al. (2012a), Melitz and Redding (2015)), in which firms observe both productivity and demand shocks prior to making export decisions. Thus, firms in the standard trade model fully observe their *profitability* (defined as their combination of productivity and demand) when making export decisions, while firms in the uncertainty economy do not fully observe their profitability (they only observe the productivity component of it).

Using the structure of the two models, we prove that uncertainty lowers welfare gains from trade under a mild set of conditions. In both economic environments, the partial elasticity of trade flows with respect to variable trade costs can be expressed as a sum of intensive and extensive margin components. The intensive margin component measures changes in trade flows arising from incumbent firms, and is common across information environments. On the other hand, the extensive margin component, which measures changes in trade flows arising from the selection of exporters into trade activity, depends on the degree of information available to potential entrants. We prove that the extensive margin elasticity is larger when demand is not known prior to export activity. In particular, we show how to rewrite the extensive margin elasticity as a hazard rate – of the distribution governing *productivity shocks* in the environment with uncertainty, and of the distribution governing *profitability shocks* in the environment with complete information. We can construct profitability shocks as a mean-preserving spread of productivity, which implies that the hazard rate associated with the profitability distribution is lower than the hazard rate associated with the productivity distribution. This insight allows us to conclude that the extensive margin of the partial trade elasticity, and therefore the overall partial trade elasticity, is higher in the environment with uncertainty than with complete information. Thus, trade is more elastic when firms face greater uncertainty which lowers welfare gains from trade all else equal.

Quantitatively, we show that the distinction between firm *productivity* and *profitability* is key for identifying trade elasticities in economic environments with and without uncertainty. In an environment with complete information, profitability can be described by export sales data, which therefore identify trade elasticities in a complete information environment. However, in a world with demand uncertainty, structural estimation strategies that identify trade elasticities from export sales data are inappropriate, as export sales data contain information that does not directly influence how a firm responds to changes in trade costs. Instead, export quantity data does contain the necessary information about productivity to identify trade elasticities.

Using Brazilian firm-level data, we quantify the magnitude of the trade elasticity under different information environments and find that trade elasticities are on average 7% larger when estimated assuming demand uncertainty. In order to estimate trade elasticities, we adapt the structural elasticity estimation approach in Bas, Mayer, and Thoenig (2017) to our model with demand uncertainty and to the model with complete information. We discipline the complete information model's profitability distribution with empirical export sales data and discipline the demand uncertainty model's productivity distribution with data on quantities exported. We apply this estimation strategy to Brazilian export data and find that demand uncertainty amplifies an increase in export sales arising from new entrants in response to a decline in variable trade costs - the extensive margin response - relative to the model with complete information. We further find that the amplification effect is larger when an export destination exhibits larger demand dispersion, as measured by the variance of demand shocks.

We quantitatively measure the welfare gains from trade in the two information environments and find that uncertainty lowers the gains from trade. This occurs because uncertainty weakens the selection mechanism that underlies standard trade models. When firms have complete information, entry is based on realized profits and firms only enter into exporting if they know they will be profitable. On the other hand, when firms have incomplete information, entry is based on expected profits and a greater number of firms enter since they believe they will be profitable. Furthermore, lower variable trade costs encourage more entry, so greater uncertainty amplifies the response of entry by increasing the mass of potential entrants that believe they will be profitable – as opposed to the complete information economy, in which more firms also enter due to lower costs, but at a lower rate due to knowing their profitability. With incomplete information, given that firms with low or negative realized profit (but positive expected profit) also enter, the average exporter size is smaller than the average exporter in the complete information economy. Therefore, lower variable costs encourage more exporters to enter foreign markets but (all else equal) increase average firm size by less than in the full information environment.

Finally, we provide reduced form empirical evidence for the effect of information uncertainty on the partial trade elasticity. In particular, we use bilateral aggregate trade data and distance data from Head, Mayer, and Ries (2010) and the *CEPII GeoDist* database (Mayer and Zignago, 2011) to perform a series of cross-sectional gravity regressions that control for information uncertainty and demand dispersion. We measure the information availability with ethic, linguistic, and religious diversity scores from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003). We argue that idiosyncratic demand shocks are less observable by exporters in a demographically diverse export destination, since exporters will have less precision in predicting demand from demographics in destinations with greater demographic heterogeneity. Likewise, greater homogeneity in the underlying population would make demand more predictable. We find that the interaction term on distance and each of the diversity scores is negative and statistically significant, lending support for our quantitative results.

We next construct a theory consistent measure of demand dispersion. Our theoretical framework implies that the variation in the dispersion of sales across export destinations occurs solely due to differences in the dispersion of the destination specific demand shocks. Hence, we construct the demand dispersion measure as the standard deviation of export sales of an origin country to a destination country, normalized by the average standard deviation of export sales across all destinations for the given origin country. The triple interaction term between distance, information uncertainty and dispersion in a standard gravity regression will therefore measure the additional effect of demand dispersion on the amplification effect of uncertainty. Using export sales dispersion data from the World Bank Exporter Dynamics Database (Fernandes et al., 2016), we find that the triple interaction term is also negative and statistically significant, lending further support for our quantitative results.

This paper shows that the information structure faced by firms is crucially important for measuring the extensive margin response to a decline in trade costs. For countries or industries in which exporters face demand uncertainty, assuming away information asymmetries understates the magnitude of extensive margin adjustments to changes in variable trade costs and, therefore, understates the welfare gains from trade.

Our work contributes to the growing literature on decomposing trade elasticities. Chaney (2008) shows that the partial elasticity of trade with respect to variable trade costs can be decomposed into an intensive and an extensive margin of adjustment components. Melitz and Redding (2015) further show that the extensive margin of adjustment crucially depends on the distributional assumptions with respect to the sources of firm-level heterogeneity. Sager and Timoshenko (Forthcoming) characterize a flexible distribution that well describes firm-level heterogeneity and find the extensive margin trade elasticity to be small. This paper demonstrates that selection into exporting (and hence the extensive margin of trade elasticity), depends on the information structure faced by firms.

Our work also contributes to a literature on measuring trade elasticities. Imbs and Mejean (Forthcoming) find that there is substantial heterogeneity in bilateral trade elasticities due to heterogeneity in countries' industrial production. Furthermore, Imbs and Mejean (2015) document that elasticities computed using industry-level data are often larger than those using aggregated data. This paper demonstrates that firms' information sets affect trade elasticity measurement and documents an amplification effect on trade elasticities attributed to uncertainty faced by firms in foreign markets.

A related strand of literature estimates trade elasticities based on complete information models of trade. Eaton and Kortum (2002) and Simonovska and Waugh (2014) use the aggregate trade flows and prices data to estimate trade elasticities. In contrast, Caliendo and Parro (2015) rely on trade flows and tariffs data. We contribute to this literature by providing an alternative method based on a structural model of trade to compute trade elasticities and show how an assumption about the information structure faced by firms alters elasticity estimates.

Our paper also relates to the literature on information asymmetries in trade. Two highly related papers are by Timoshenko (2015a) and Dickstein and Morales (2016). Timoshenko (2015a) finds that past continuous export history predicts current export choice. Dickstein and Morales (2016) extends this work and demonstrates that firm-level information such as lagged sales and industry averages predict exporting, but only for large firms (e.g., in terms of productivity or domestic sales). The authors find that small firms do not seem to make decisions based on this additional information. While these papers uncover a large set of factors that predict export decisions, our paper abstracts from such possible correlations in order to highlight as clearly as possible the interaction between information and trade elasticities.[3]

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 provides a set of proofs to characterize the difference between trade elasticities in economies with and without uncertainty, describes the implications of uncertainty for identifying key model parameters when estimating trade elasticities, and contrasts welfare implications across model environments. Section 4 details our data and method for estimating trade elasticities. Section 5 presents elasticity estimation result compares welfare results from a counterfactual trade liberalization between the two information environments. Section 6 estimates a gravity equation that controls for information availability and demand dispersion. Section 7 concludes. Appendix A provides a detailed description of the demand uncertainty model and complete information model. Appendix B provides proofs to all Propositions. Appendix C demonstrates that our results are robust to the alternative firm-level choice variable under uncertainty.

[3] Furthermore, Bergin and Lin (2012) show that the entry of new varieties increases at the time of the announcement of the future implementation of the European Monetary Union, suggesting that changes in the information available to firms have immediate consequences for firms' decisions. Lewis (2014) studies the effect of exchange rate uncertainly on trade; Allen (2014) shows that information frictions help to explain price variation across locations; Fillat and Garetto (2015) show that aggregate demand fluctuations can explain variation in stock market returns between multinational and non-multinational firms; Handley and Limao (2015) show that when trade policy is uncertain, there is less entry into foreign markets.

# 2 Theoretical Framework

In this section we consider a model with heterogeneous firms that export products in markets characterized by monopolistic competition. This environment is similar to that in Melitz (2003), and we assume exogenous entry as in Chaney (2008). All derivations are relegated to Appendix A.

There are $N$ countries and $K$ sectors, such that each country is indexed by $j$ and each sector is indexed by $k$. Each country is populated by a mass of $L_j$ identical consumers.

## 2.1 Demand

Preferences of a representative consumer in country $j$ are represented by a nested constant elasticity of substitution utility function given by

$$U_j = \prod_{k=1}^{K} \left[ \left( \sum_{i=1}^{N} \int_{\omega \in \Omega_{ijk}} \left(e^{\theta_{ijk}(\omega)}\right)^{\frac{1}{\epsilon_k}} c_{ijk}(\omega)^{\frac{\epsilon_k - 1}{\epsilon_k}} d\omega \right)^{\frac{\epsilon_k}{\epsilon_k - 1}} \right]^{\mu_k}, \tag{1}$$

where $\Omega_{ijk}$ is the set of varieties in sector $k$ consumed in country $j$ originating from country $i$, $c_{ijk}(\omega)$ is the consumption of variety $\omega \in \Omega_{ijk}$, $\epsilon_k$ is the elasticity of substitution across varieties within sector $k$, $\theta_{ijk}(\omega)$ is the demand shock for variety $\omega \in \Omega_{ijk}$, and $\mu_k$ is the Cobb-Douglas utility parameter for goods in sector $k$ such that $\sum_{k=1}^{K} \mu_k = 1$.

Each consumer owns a share of domestic firms and is endowed with one unit of labor that is inelastically supplied to the market. Cost minimization yields a standard expression for the optimal demand for variety $\omega \in \Omega_{ijk}$, given by

$$c_{ijk}(\omega) = e^{\theta_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}, \tag{2}$$

where $p_{ijk}(\omega)$ is the price of variety $\omega \in \Omega_{ijk}$, $Y_{jk}$ is total expenditures in country $j$ on varieties from sector $k$, and $P_{jk}$ is the aggregate price index in country $j$ in sector $k$.[4]

## 2.2 Supply

Each variety $\omega \in \Omega_{ijk}$ is supplied by a monopolistically competitive firm. Each firm can potentially supply one variety of a product from each sector. Upon entry, a firm is endowed with an idiosyncratic labor productivity level $e^{\varphi}$ and a set of idiosyncratic product-destination specific demand shocks $\{\theta_{ijk}\}$.[5] Productivity and demand shocks are drawn from indepen-

[4]Note that $Y_{jk} = \mu_k Y_j$, where $Y_j$ is aggregate income in country $j$.

[5]Following the finding of Foster, Haltiwanger, and Syverson (2008) who document that idiosyncratic firm-level demand shocks rather than productivity account for a greater variation in sales across firms, we focus on the demand shocks that are firm specific. Each firm from country $i$ draws a separate demand shock for

dent distributions. Denote by $g^{\varphi}_{ijk}(.)$ the distribution from which firms draw productivity, $\varphi$, and by $g^{\theta}_{ijk}(.)$ the distribution from which firms draw demand shock, $\theta_{ijk}$. Firms from country $i$ face fixed costs, $f_{ijk}$, and variable costs, $\tau_{ij}$, of selling output to country $j$. Fixed and variable costs are denominated in units of labor.

## 2.3 Environment with Uncertainty

We introduce information asymmetries by considering a stylized version of the learning model in Jovanovic (1982) as was embedded into a trade model in Timoshenko (2015b) to a trade context and thereby assume that firms do not possess complete information when making export decisions. In particular, while firms always observe their productivity shock, $\varphi$, they do not observe their demand shocks, $\theta_{ijk}$, when making export decisions. Therefore, firms choose a quantity to export to each destination market before knowing destination-specific demand shocks for their product.[6] Firms choose export quantities to maximize *expected* profits, subject to consumer demand (2) and prior beliefs about demand, $E_{\theta}(\exp(\theta_{ijk}/\epsilon_k))$.[7] Henceforth, the subscript of the expectation operator indicates that the expectation is taken with respect to the distribution of the random variable indicated in the subscript.

The firm's decision problem yields an expression for the optimal quantity exported, given by

$$q_{ijk}(\varphi) \quad = \quad B^{q}_{ijk} \cdot e^{\epsilon_k \varphi}. \tag{3}$$

where $B^{q}_{ijk}$ is an origin-destination-industry fixed effect.[8] The corresponding productivity

each destination $j$ and industry $k$. The $ijk$ subscript on $\theta_{ijk}$ therefore indicates that idiosyncratic firm-level demand shocks are origin-destination-sector specific. The superscript does not refer to an aggregate origin-destination-sector level shock that is common across firms. We abstract from such potential aggregate shocks since they do not affect properties of the distribution of quantities or sales across firms, which is the focus of our analysis.

[6] Alternatively, we can assume that firms choose and commit to a price at which they will export and engage in production, once demand shocks are realized and foreign markets order products. While we conform to the standard learning model of Timoshenko (2015b) by assuming that firms choose quantities, we confirm in Appendix C that both model setups yields similar results.

[7] We assume that prior beliefs are the same across firms and equal the population mean. This assumption does not fundamentally change our results. In principle, we could expand the set of shocks on which a firm bases its quantity decision to any idiosyncratic shocks including demand expectations which could be arbitrarily correlated with firm productivity. What is important is that some idiosyncratic information is known to a firm before making decisions, and some idiosyncratic information is revealed to that firm after those decisions have been made.

[8] We refer the reader to Appendix A.1.1 for a derivation of and full expression for the origin-destination-industry fixed effects found in equations (3), (4) and (5).

entry threshold is given by

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} \quad = \quad \frac{B^{\varphi}_{ijk}}{\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}}, \tag{4}$$

where $B^{\varphi}_{ijk}$ is an origin-destination-industry fixed effect. Once all goods are supplied to markets, demand shocks are realized and prices clear the goods markets for each variety. A firm's realized export sales are given by

$$r_{ijk}(\theta_{ijk},\varphi) \quad = \quad B^r_{ijk}\cdot e^{(\epsilon_k-1)\varphi+\frac{\theta_{ijk}}{\epsilon_k}}, \tag{5}$$

where $B^r_{ijk}$ is an origin-destination-industry fixed effect.

**Trade Elasticity:** The aggregate trade flow from country $i$ to country $j$ in industry $k$ is defined as

$$X_{ijk} \quad \equiv \quad M_{ijk}\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} r_{ijk}(\theta,\varphi)g^{\theta}_{ijk}(\theta)\frac{g_{ijk}(\varphi)}{Prob_{ijk}(\varphi>\varphi^*_{ijk})}d\theta d\varphi,$$

where $M_{ijk}$ is the mass of firms exporting from country $i$ to country $j$ in industry $k$, and $\varphi^*_{ijk}$ is the productivity entry threshold. The partial elasticity of trade flows with respect to the variable trade costs is given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} \quad = \quad \underbrace{(1-\epsilon_k)}_{\substack{\text{level of the}\\ \text{partial trade elasticity}}}\left(\underbrace{1}_{\substack{\text{intensive margin}\\ \text{contribution}}} + \underbrace{\gamma_{ijk}(\phi^*_{ijk})}_{\substack{\text{extensive margin}\\ \text{contribution}}}\right), \tag{6}$$

where $\phi^*_{ijk}=(\epsilon_k-1)\varphi^*_{ijk}$ is a rescaled productivity, and the extensive margin is given by

$$\gamma_{ijk}(\phi^*_{ijk}) \quad = \quad \frac{e^{\phi^*_{ijk}}g^{\phi}_{ijk}(\phi^*_{ijk})}{\int_{\phi^*_{ijk}}^{+\infty}e^{\phi}g^{\phi}_{ijk}(\phi)d\phi}. \tag{7}$$

Equation (6) decomposes the partial trade elasticity into intensive and extensive margin components, and equation (7) shows that the extensive margin of the partial trade elasticity is governed by the rescaled productivity entry threshold, $\phi^*_{ijk}$, and the rescaled productivity distribution, $g^{\phi}_{ijk}(\cdot)$.

## 2.4 Environment with Complete Information

In a complete information environment, firms observe both the productivity and demand shocks upon entry.[9] Firms' decisions therefore depend on a single profitability parameter defined by $z_{ijk} \equiv (\epsilon_k - 1)\varphi + \theta_{ijk} = \phi_k + \theta_{ijk}$. Productivity and demand shocks therefore simultaneously determine selection into exporting through their impact on the export entry threshold, the export sales distribution, and the partial trade elasticity. In particular, under complete information, the single *profitability* entry threshold, $z^*_{ijk}$ is given by

$$e^{z^*_{ijk}} = B^{\varphi}_{ijk}. \tag{8}$$

Subsequently, a firm's export sales are given by

$$r^{CI}_{ijk}(z_{ijk}) = B_{ijk} e^{z_{ijk}}, \tag{9}$$

and the partial trade elasticity is given by

$$\eta^{CI}_{ijk} = \underbrace{(1-\epsilon_k)}_{\substack{\text{level of the} \\ \text{partial trade elasticity}}} \left( \underbrace{1}_{\substack{\text{intensive margin} \\ \text{contribution}}} + \underbrace{\gamma^{CI}_{ijk}(z^*_{ijk})}_{\substack{\text{extensive margin} \\ \text{contribution}}} \right), \tag{10}$$

where the superscript 'CI' stands for the 'Complete Information' environment, and the extensive margin is given by

$$\gamma^{CI}_{ijk}(z^*_{ijk}) = \frac{e^{z^*_{ijk}} g^z_{ijk}(z^*_{ijk})}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz}, \tag{11}$$

where $g^z_{ijk}(.)$ is the distribution of firm profitability.

# 3 Characterization of Trade Elasticities

In this section we characterize the partial elasticity of trade with respect to variable trade costs. In Section 3.1 and Section 3.2, we prove that the trade elasticity increases with the degree of uncertainty that firms face. Finally, in Section 3.3 we show how the elasticities connect to the welfare gains from trade.

## 3.1 Properties of the Partial Trade Elasticity

In this section, we characterize the partial trade elasticity and prove (under mild conditions) that the extensive margin of trade is larger under incomplete information than complete

[9] Appendix A.2 contains a formal description of the complete information economy.

information. We show how to write the extensive margin of the partial trade elasticity as a hazard rate, and use standard results on second-order stochastic dominance to characterize the response of the elasticity to increases in the variance of demand shocks.

In comparing the expressions for the partial trade elasticity between the incomplete versus complete information environments, equations (6) versus (10) clearly indicate that the two elasticities differ solely along the extensive margin dimension. Furthermore, from equations (7) and (11) observe that the extensive margin component of the partial trade elasticity admits the same functional form in both information environments and can be simply written as a function of the entry threshold (denoted by $x$) and its respective distribution $g(.)$,

$$\gamma(x) \equiv \frac{e^x g(x)}{\int_x^{+\infty} e^z g(z) dz}. \tag{12}$$

For notational compactness, we have dropped all subscripts and will focus solely on the functional form of the extensive margin elasticity. To proceed, we make standard assumptions on the distribution $g(x)$.

**Assumption 1 (A1)** *The probability density function $g(x)$ has the following properties:*

*(i) $x \in \mathbb{R}$ is the support of the distribution,*

*(ii) $E(e^x) \equiv \int_{-\infty}^{+\infty} e^z g(z) dz$ exists and is finite, and*

*(iii) the function $\log\left(\int_x^{+\infty} e^z g(z) dz\right)$ is concave in $x$.*

Assumption (iii) ensures that function $\gamma(x)$ is a monotonically increasing function of $x$. Intuitively, Assumption (iii), that the log of the conditional expectation of profitability is a concave function of the threshold value, requires that the upper tail of the distribution $g(x)$ does not have too much mass.[10] Without such a restriction, total sales of marginal firms relative to average sales could become very small as the threshold increases, and the extensive margin elasticity, $\gamma(x)$, might not be monotonically increasing in $x$. Monotonicity of $\gamma(x)$ allows us to characterize the effect of uncertainty on the extensive margin elasticity. Accordingly, Proposition 1 below establishes two novel properties of the extensive margin $\gamma(x)$.

**Proposition 1** *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

*(i) $\gamma(x) \equiv [e^x g(x)] / \int_x^{+\infty} e^z g(z) dz$ is an increasing function of $x$.*

[10]Heavy-tailed distributions, e.g. distributions that violate assumption (ii), are sometimes said to have the property of log-convexity.

(ii) *Denote the extensive margin elasticity associated with $g(x)$ as $\gamma(x)$. Let $\tilde{g}(x)$ be a mean preserving spread of $g(x)$, with extensive margin elasticity $\tilde{\gamma}(x)$. Then $\gamma(x)$ and $\tilde{\gamma}(x)$ satisfy the single crossing property. That is, there exists $x^*$ such that $\tilde{\gamma}(x) \leq \gamma(x)$ for all $x \geq x^*$, and $\tilde{\gamma}(x) \geq \gamma(x)$ for all $x \leq x^*$.*

To provide some intuition for why Proposition 1 holds, notice that the extensive margin $\gamma(x)$ can be written as

$$\gamma(x) = \frac{h(x)}{1 - H(x)}, \tag{13}$$

namely, the extensive margin of the partial trade elasticity is a hazard rate associated with a random variable $X$ distributed according to $h(x)$, where $h(x) \equiv e^x g(x) / \int_{-\infty}^{+\infty} e^z g(z) dz$. Notice that $\int_{-\infty}^{+\infty} h(x) dx = 1$ and $h(x) \geq 0$. Hence, $h(x)$ is the probability density function. The corresponding cumulative distribution function is then given by

$$H(x) = \int_{-\infty}^{x} e^z g(z) dz \Big/ \int_{-\infty}^{+\infty} e^z g(z) dz. \tag{14}$$

The corresponding survival function is given by $1 - H(x) = \int_{x}^{+\infty} e^z g(z) dz / \int_{-\infty}^{+\infty} e^z g(z) dz$.[11]

As such, $\gamma(x)$ inherits properties of the hazard rate function the first of which is being a monotonically increasing function. Notice that part (iii) of A1 ensures that distribution $h(x)$ has a log-concave survival function. Log-concavity of the survival function ensures that the corresponding hazard rate is increasing: Notice from equation (13) that $\gamma'(x) = -d^2 \log(1 - H(x))/dx^2$ which is positive if and only if $\log(1 - H(x))$ is concave.

Part (ii) of Proposition 1 shows that the extensive margin elasticity as a function of threshold values $x \in \mathbb{R}$ exhibits a single crossing property. The single crossing property establishes that the extensive margin elasticity function associated with the cumulative distribution function of the mean preserving spread, $\tilde{\gamma}(x)$, only crosses the extensive margin elasticity function associated with the less dispersed cumulative distribution function, $\gamma(x)$, once from above.

To provide further intuition for the single-crossing property of function $\gamma(.)$, recall that, by part (ii) of Proposition 1, $\tilde{g}(x)$ is a mean preserving spread of $g(x)$. Hence $G(x)$ crosses $\tilde{G}(x)$ once from below. As we show in the proof of Proposition 1 in Appendix B, this single-crossing property is also preserved when defining a distribution according to the transformation in equation (14), and is also preserved by $\gamma(x)$. Therefore, $\gamma(x)$ also crosses $\tilde{\gamma}(x)$ from below once, and for all values of $x$ sufficiently large we know $\gamma(x) > \tilde{\gamma}(x)$.

[11] Representation (13) is obtained by normalizing the numerator and the denominator of $\gamma(x)$ by $\int_{-\infty}^{+\infty} e^z g(z) dz$ as follows $\gamma(x) = \left[ e^x g(x) / \int_{-\infty}^{+\infty} e^z g(z) dz \right] / \left[ \int_{x}^{+\infty} e^z g(z) dz / \int_{-\infty}^{+\infty} e^z g(z) dz \right]$.

### 3.2 Comparison of the Partial Trade Elasticities

A corollary of part (ii) of Proposition 1 is that the single crossing property of the extensive margin holds for any affine transformation of the abscissa for either of the functions.

**Corollary 1** *Let $g(x)$ be a probability density function satisfying A1. For all $a \in \mathbb{R}$ there exists $x^*(a)$ such that $\gamma(x) \geq \tilde{\gamma}(x+a)$ if $x \geq x^*(a)$, and $\gamma(x) \leq \tilde{\gamma}(x+a)$ if $x \leq x^*(a)$.*

Together, Proposition 1 and Corollary 1 imply that the extensive margin of the partial trade elasticity is larger under incomplete information.

Observe from equations (6), (7), (10), and (11) that the difference in the partial trade elasticity between the two information environments arises from the extensive margin component $\gamma(x)$. Further, the difference in the extensive margin arises from two separate channels: the entry threshold, $\phi^*_{ijk}$ versus $z^*_{ijk}$, and the distribution of the corresponding shock, $g^{\phi}_{ijk}(.)$ versus $g^{z}_{ijk}(.)$. Assume that both distributions satisfy Assumption 1. It is worth noting that distributions that are commonly used in the trade literature, such as a Normal, and the Double EMG distribution, which we will use in Section 4.4, satisfy A1.

Recall that profitability shock $z$ is defined as $z = \phi + \theta$, where $\phi$ and $\theta$ are independent and are drawn from the probability density functions $g^{\phi}_{ijk}(.)$ and $g^{\theta}_{ijk}(.)$ respectively. Without a loss of generality we can assume that the mean of $\theta$ equals zero. In this case, $g^{z}_{ijk}(.)$ is a mean-preserving spread of $g^{\phi}_{ijk}(.)$. Hence, by part (ii) of Proposition 1 we know that $\gamma_{ijk}(x) > \gamma^{CI}_{ijk}(x)$ for a sufficiently high entry threshold $x$, and therefore trade is more elastic under incomplete information given the same threshold value.

We, next, show that $\gamma_{ijk}(x) > \gamma^{CI}_{ijk}(x)$ holds even if the entry thresholds are different. From equations (4) and (8), the entry threshold $\phi^*_{ijk}$ under incomplete information versus $z^*_{ijk}$ under complete information are related as follows: $\phi^*_{ijk} = z^*_{ijk} - \log\left(E_{\theta}\left(\exp(\theta_{ijk}/\epsilon_k)\right)\right)^{\epsilon_k}$. Therefore, the proper comparison of elasticities involves comparing $\gamma_{ijk}(x)$ and $\gamma^{CI}_{ijk}(x+a)$, where constant $a \equiv \log\left(E_{\theta}\left(\exp(\theta_{ijk}/\epsilon_k)\right)\right)^{\epsilon_k}$. By Corollary 1, for $\phi^*_{ijk}$ high enough, $\gamma_{ijk}(\phi^*_{ijk}) > \gamma^{CI}_{ijk}(\phi^*_{ijk} + a) = \gamma^{CI}_{ijk}(z^*_{ijk})$, i.e. trade is less elastic under complete information.

To summarize, trade is less elastic under complete information because the distribution of the shock that determines the partial trade elasticity is more dispersed under complete information than under uncertainty. Under uncertainty, trade elasticity is determined by the distribution of productivity, while under complete information trade elasticity is determined by the distribution of the profitability shock, a mean preserving spread of productivity. As a result, the associated hazard rate, and consequently the partial trade elasticity, is lower under complete information than under uncertainty.

We next demonstrate that differences in the magnitude of the extensive margin contribution to the partial trade elasticity between the two information environments are intimately linked to the differences in the measurements of the welfare gains from trade through their effect on total trade flows.

## 3.3 Welfare

In both information environments, the total trade flows between country $i$ and $j$ in industry $k$ can be written as

$$X_{ijk} \quad = \quad J_i \epsilon_k w_i f_{ijk} \frac{1}{\gamma_{ijk}(x^*)} g_{ijk}^x(x^*), \tag{15}$$

were $J_i$ is the exogenous mass of entrants in country $i$, $w_i$ is the wage rate in country $i$, and $x^*$ and $g_{ijk}^x(.)$ are the relevant entry threshold and the probability distribution function of the underlying variable.[12]

Equation (15) provides a crucial link between the extensive margin of the partial trade elasticity and trade flows and shows that, holding all else constant, the two variables are inversely related. As shown in Section 3.1, the value of the extensive margin elasticity is lower under complete information relative to an environment with uncertainty. Hence, holding all else constant, the corresponding value of trade flows is higher and the implied domestic trade share is lower in an environment with complete information relative to uncertainty. This leads to higher potential welfare gains from trade as suggested by the Arkolakis et al. (2012b) result extended to a multi-sector environment:

$$\ln(W_i) \propto \sum_{k=1}^{K} \frac{\mu_k}{|\eta_{ik}|} \ln \left( \frac{\pi_{ii,k}}{L_{ik}} \right)^{-1}, \tag{16}$$

were $W_i$ denotes welfare (real income) of country $i$, $L_{ik}$ denotes country $i$'s employment in sector $k$, and $\pi_{ii,k}$ denotes the share of expenditure in sector $k$ devoted to domestic goods. A lower value of the extensive margin of the partial trade elasticity, $\gamma_{ijk}$, leads to a lower absolute value of the partial trade elasticity, $|\eta_{ik}|$, and a lower value of the domestic trade share, $\pi_{ii,k}$. As per equation (16), the welfare is inversely related to both of these magnitudes, and therefore the welfare gains from a decline in variables trade costs are higher for lower values of the extensive margin of the partial trade elasticity. Hence, a model with complete information will predict higher welfare gains from trade relative to a model with uncertainty.

[12] In the environment with uncertainty, $x^*$ is the rescaled productivity entry threshold $(\epsilon_k - 1)\varphi_{ijk}^*$; in the environment with complete information, $x^*$ is the profitability entry threshold $z_{ijk}^*$.

# 4 Quantifying Trade Elasticities

In this section we quantify and compare trade elasticities between and environment with uncertainty and complete information. Using insights from Section 2 we first demonstrate that different data identify trade elasticity in the two information environments. In Section 4.1 we show that the information contained in export quantity shoudl be used to identify the partial trade elasticity in economies with incomplete information, while export sales data matters for the identification of the partial trade elasticity in economies with full information. Accordingly, we next use data on the distribution of export quantities and sales across Brazilian firms for exports by destination-industry over time to quantify and compare trade elasticities between and environment with uncertainty and complete information. In doing so we adapt the estimation approach suggested by Bas, Mayer, and Thoenig (2017) and extended by Sager and Timoshenko (Forthcoming). Relative to these papers, we extend the approach to an environment with demand uncertainty.

## 4.1 Identification of the Partial Trade Elasticity

We demonstrate that the distinction in the information contained in export quantity versus export sales data matters for the identification of the partial trade elasticity.

Comparing equations (6) and (10) makes clear that the information environment directly impacts the identification of the extensive margin of the partial trade elasticity. Intuitively, the information available to firms at the time of making export decisions determines how responsive their decisions are to changes in variable trade costs. Therefore, quantifying the partial trade elasticity in the complete and incomplete information economies requires different data for proper identification.

As can be seen from equations (10) and (11), under complete information the trade elasticity is identified by the profitability distribution, $g^z_{ijk}(.)$, and the profitability entry threshold, $z^*_{ijk}$. Profitability, is exactly the information that determines firm entry decisions by equation (8) and subsequently the equilibrium the distribution of firm sales by equation (9). Hence, export sales data contain the information about profitability that is necessary to identify the partial trade elasticity under complete information environment.

As can be seen from equations (6) and (7), under incomplete information the partial trade elasticity is identified by the rescaled productivity distribution, $g^\phi_{ijk}(.)$, and the rescaled productivity entry threshold, $\phi^*_{ijk}$. Hence, the partial trade elasticity is governed by productivity, which, under incomplete information, is the only information available to the firms at the time at which they make export decisions. As can be seen from equations (3) and (5), the model implies that productivity can be identified from data on export quantities, but not export sales. Equation (3) shows that, conditional on variables common to all firms (contained in $B^q_{ijk}$), a firm's export quantity decision is entirely governed by its firm-level productivity shock, $\varphi$. Since firms only observe their productivity, and productivity is the only idiosyncratic information upon which firms base their export decisions, productivity alone determines a firm's production response to a potential change in variable trade costs. Therefore, equation (3) shows that such productivity information is contained exactly in the export quantity data.

This is in contrast to the information embedded in the export sales data. Equation (5) shows that, conditional on aggregate variables that are common across all firms ($B^r_{ijk}$), a firm's export sales depend on both the firm's known productivity, $\varphi$, and the firm's subsequent idiosyncratic demand realization, $\theta_{ijk}$. The idiosyncratic firm-level demand shocks only affect the realized distribution of sales across firms, but play no role in a firm's decision making process and therefore do not affect the firm's response to a change in variable trade costs. As a result, the model implies that, relative to export quantity data, export sales data contain additional information that does not directly influence firms export decisions.

Note that productivity and demand are two standard interpretations of model ingredients relative to the data. Following the trade literature, the model's productivity and demand shocks stand in for any variation that allows the model to be consistent with the data on sales and quantities. What is important for our paper is that, in environments with uncertainty, productivity shocks stand in for any information that firms possess at the time of making export decisions while demand shocks stand in for any information that determines sales and that is revealed after exporting goods.

To summarize, quantity data identifies the extensive margin contribution to the partial trade elasticity by enabling inference about the productivity distribution and productivity entry threshold. In contrast, it is only appropriate to identify the partial trade elasticity using export sales data when firms possess complete information about their demand.

## 4.2 Estimation Approach

In this section we detail our approach to estimating partial trade elasticities in the presence of demand uncertainty. As shown in equation (6), in an environment with uncertainty, selection occurs based on the productivity alone. Hence the extensive margin of trade elasticity depends on the productivity entry threshold and the distribution of the productivity draws. Both can be recovered using the data on the distribution of export quantity as we now describe.

Consider the following change of notation. Let $\tilde{\varphi} \equiv \epsilon_k \varphi$ and denote by $g^{\tilde{\varphi}}(.)$ the probability distribution function of $\tilde{\varphi}$. Given the change in notation, $g^{\tilde{\varphi}}(.)$ is the distribution of

$\varphi$, $g^{\varphi}_{ijk}(.)$, scaled by the elasticity of substitution, $\epsilon_k$.

Given this change in notation, the partial trade elasticity can be expressed as

$$\eta_{ijk} \;=\; (1-\epsilon_k)\left(1+\frac{\epsilon_k}{\epsilon_k-1}\frac{e^{\frac{\epsilon_k-1}{\epsilon_k}\tilde{\varphi}^*}g^{\tilde{\varphi}}(\tilde{\varphi}^*)}{\int_{\tilde{\varphi}^*}^{+\infty}e^{\frac{\epsilon_k-1}{\epsilon_k}\varphi}g^{\tilde{\varphi}}_{ijk}(\varphi)d\varphi}\right). \tag{17}$$

The distribution $g^{\tilde{\varphi}}(.)$ can be directly recovered from the empirical distribution of the log-export quantity. Taking the logarithm of equation (3) yields

$$\log q_{ijk} \;=\; \log B^q_{ijk}+\tilde{\varphi}. \tag{18}$$

Observe that the distribution of log-export quantity is given by the distribution of $\tilde{\varphi}$ shifted by a constant. Hence, parameters of $g^{\tilde{\varphi}}_{ijk}(.)$ can potentially be recovered from fitting the distribution to the empirical distribution of log-export quantity. Note however that the model implies endogenous selection into exporting. Hence equation (3) and therefore (18) hold only when $\tilde{\varphi}>\tilde{\varphi}^*_{ijk}$ or when $\log q_{ijk}>\log q^*_{ijk}$. To account for the endogenous selection, we follow the approach by Sager and Timoshenko (Forthcoming). Namely, we proceed by fitting a truncated probability distribution function $g^{\tilde{\varphi}}_{ijk}(.)$ to the log-export quantity data and take the truncation point $\log q^*_{ijk}$ to be given by the zeroth percentile of the corresponding log-export quantity distribution.

Given the scaled productivity distribution, $g^{\tilde{\varphi}}_{ijk}(.)$, we follow Bas et al. (2017) in recovering the scaled productivity threshold, $\tilde{\varphi}^*_{ijk}$, by matching the model-implied average-to-minimum ratio to that in our quantity data. The model-implied average-to-minimum ratio of export quantities is given by

$$\text{Average-to-Minimum Ratio} \;=\; e^{-\tilde{\varphi}^*_{ijk}}\int_{\tilde{\varphi}^*_{ijk}}^{+\infty}\frac{e^{\varphi}g^{\tilde{\varphi}}_{ijk}(\varphi)}{Prob^{\tilde{\varphi}}_{ijk}(\varphi>\tilde{\varphi}^*_{ijk})}d\varphi. \tag{19}$$

In the next section we describe the data on export quantity and sales that we are going to use to quantify the partial trade elasticities.

## 4.3 Data

The data come from the Brazilian customs declarations collected by SECEX (*Secretaria de Comercio Exterior*).[13] The data record export value and weight (in kilograms) of the shipments at the firm-product-destination-year level. A product is defined at the 6-digit Harmonized Tariff System (HS) level. We use the data for the period between 1997 and 2000, when both the sales and the weight data are available.

[13] For a detailed description of the dataset see Molinaz and Muendler (2013). The data have further been used in Flach (2016) and Flach and Janeba (2017).

We proxy the theoretical notion of export quantity with an empirical measure of export weight.[14] The properties of export weight differ substantially across industries. Hence, we further conduct our analysis at the destination-year-industry level where we define an industry as a 6-digit HS code.

We define an observation to be a distribution of export quantity across firms for a given destination-year-industry triplet, and focus on observations where at least 100 firms export.[15]

The final sample consists of 190 destination-year-industry observations, and covers 14 destinations and 35 industries. Table 1 provides summary statistics of log-export quantities and log-sales distributions in our sample.

Next, we apply the described elasticity estimation approach to quantify the partial trade elasticities.

## 4.4 Parameter Estimates

**The Export Quantity Distribution:** To recover the partial trade elasticities we proceed by, first, assuming that the productivity is drawn from a Double EMG distribution, $DEMG(m, \upsilon^2, \xi_L, \xi_R)$. The resulting log-export quantity distribution, $g_{ijk}^{\tilde{\varphi}}(.)$, then also follows a Double EMG distribution, $DEMG(\mu, \sigma^2, \lambda_L, \lambda_R)$ with parameters scaled by the elasticity of substitution, $\epsilon_k$, and described by the following cumulative distribution function:[16]

$$\begin{aligned} G(\varphi) = \Phi\left(\frac{\varphi-\mu}{\sigma}\right) \quad &- \quad \frac{\lambda_L}{\lambda_L+\lambda_R} e^{-\lambda_R(\varphi-\mu)+\frac{\sigma^2}{2}\lambda_R^2}\Phi\left(\frac{\varphi-\mu}{\sigma}-\lambda_R\sigma\right) \\ &+ \quad \frac{\lambda_R}{\lambda_L+\lambda_R} e^{\lambda_L(\varphi-\mu)+\frac{\sigma^2}{2}\lambda_L^2}\Phi\left(-\frac{\varphi-\mu}{\sigma}-\lambda_L\sigma\right), \end{aligned} \tag{20}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.[17]

The Double EMG distribution provides a very flexible generalization of common distributional assumptions used in the literature. From equation (20), for example, as $\sigma \to 0$ and $\lambda_L \to 0$, the Double EMG distribution converges to an Exponential (Pareto) distribution, as assumed in Chaney (2008). As $\lambda_L \to +\infty$ and $\lambda_R \to +\infty$, the Double EMG distribution converges to a Normal distribution, as assumed in Bas et al. (2017) and Fernandes et al. (2015). As $\sigma \to 0$, the Double EMG converges to a Double Exponential (Pareto) distribution. By assuming the Double EMG distribution we, therefore, allow the data to recover

[14] Export weight is used as a measure of export quantity in a number of studies including Bastos et al. (2018).

[15] The thresholds of 100 firms ensures that an empirical distribution can be accurately described by percentiles. This threshold is also consistent with the literature. See Fernandes et al. (2015), Sager and Timoshenko (Forthcoming).

[16] The parameters of the productivity versus log-export quantity are related as follows: $\mu = \epsilon_k m$, $\sigma^2 = \epsilon_k^2 \upsilon^2$, $\lambda_L = \xi_L/\epsilon_k$, and $\lambda_R = \xi_R/\epsilon_k$.

[17] For notational compactness we drop the $ijk$ subscripts in this section.

the best fit of distribution between the Exponential, Normal, Double Exponential or the corresponding convolutions.[18]

For each destination-year-industry observation, we choose distribution parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$ so that the percentiles of the theoretical log-quantity distribution match the percentiles of the empirical log-quantity distribution. We follow Sager and Timoshenko (Forthcoming) in estimating the parameters of the Double EMG distribution using a Generalized Method of Moments (GMM) procedure that minimizes the sum of squared residuals,

$$\min_{(\mu,\sigma^2,\lambda_L,\lambda_R)} \sum_{i=1}^{N_P} \left(q_i^{data} - q_i(\mu, \sigma^2, \lambda_L, \lambda_R)\right)^2,$$

where $q_i^{data}$ is the $i$-th percentile of the empirical log-quantity distribution for a given destination-year-industry, $q_i(\mu, \sigma^2, \lambda_L, \lambda_R)$ is the model implied $i$-th log-quantity percentile for given parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$, and $N_P$ is the number of percentiles used in estimation. We use the 1st through 99th percentiles of the empirical log-quantity distribution to estimate parameters. In practice, this choice eases computational burden compared to using each data point, without significantly changing the parameter estimates we recover. Furthermore, note that choosing parameters to minimize the sum of squared residuals is equivalent to Head et al.'s (2014) method of recovering parameters from quantile regressions.

Table 2 summaries distribution parameter estimates across 190 observations. As can be seen from the Table, the average sample value of $\sigma$ is 1.64, rejecting a common assumption of Exponentially or Double Exponentially distributed productivity shocks. Furthermore, as can be inferred from the values of the left and right tail parameters, $\lambda_L$ and $\lambda_R$, distributions exhibit substantial heterogeneity in the fatness of both tails. The value of the right tail parameter, $\lambda_R$ varies between 0.37 and 33.57, with about 44 percent of observations exhibiting a fat right tail, i.e. $\lambda_R < 2$. This finding is consistent with the previous empirical research documenting fatness in the right tail of sales or employment distributions across firms.[19] Furthermore, we also find that distributions exhibit fatness in the left tail ($\lambda_L < 2$) in approximately 38 percent of observations.[20]

**Entry Threshold:** Next, we use the fitted distribution to recover the productivity entry thresholds. For each destination-year-industry observation we solve equation (19) for the productivity entry threshold using the data on the corresponding average-to-minimum ratio of export quantity and the distribution parameter estimates.

[18] See Sager and Timoshenko (Forthcoming) for a more thorough characterization of the Double EMG distribution.

[19] See Axtell (2001) and di Giovanni et al. (2011).

[20] Sager and Timoshenko (Forthcoming) document fat left tails in the context of export sales distributions.

Figure 1 provides a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of log-export quantity. Each dot in the Figure corresponds to a destination-year-industry observation. The values of the thresholds are demeaned by a corresponding estimate of $\mu$ of the Double EMG distribution. Hence, the values represent deviations from the mean of the distribution. Figure 1 shows that the greater is the deviation, i.e. the more negative is the value on the y-axis, the lower is the entry threshold. A lower entry threshold relative to the mean implies a smaller size of a marginal exporter relative to an average exporter.

# 5 Quantitative Results

In this section we present the results from quantitatively comparing the models with complete information and with uncertainty. We show that the model with complete information overstates the welfare gains from trade because it features too strong of a selection mechanism.

## 5.1 Estimates of Trade Elasticities

Given estimated distribution parameters and entry thresholds from Section 4.4, we compute the partial trade elasticity, $\eta_{ijk}$, and the extensive margin contribution to the trade elasticity from equation (17). Notice that computation requires estimates of the elasticity of substitution across varieties, $\epsilon_k$. We use estimated elasticities of substitution from Soderbery (2015), which refines estimates in Feenstra (1994) and Broda and Weinstein (2006).[21] Table 3 summarizes average values for and heterogeneity in elasticity estimates.

From Table 3, observe that an average contribution of the extensive margin to trade elasticity is 0.07. In the context of the volume of aggregate trade flows, this magnitude can be understood as follows. Suppose, for example, that a decline in trade costs leads to an increase in trade flows by a million dollars. For an average observation, the new exporters would account for approximately \$65,000 out of a million dollars of the newly created trade.[22]

## 5.2 Comparing Elasticities

To compare the estimates of trade elasticity between the two information environments, we first re-estimate the partial trade elasticity under the assumption of complete information.

[21] Soderbery (2015) estimates the elasticity of substitution values at the HS-10 digit level using the U.S. import data. To use Soderbery (2015) estimates aggregate the elasticities to the HS-6 digit level equally weighing corresponding HS-10 sub-categories for each HS-6 category.

[22] Sager and Timoshenko (Forthcoming) show that this magnitude is a result of an abundance of small exporters in export sales distributions. Other frequently used trade data sets exclude these small firms and, hence, generate much higher extensive margin elasticities.

As discussed in Section 2.4, in a model with complete information the partial trade elasticity depends on the distribution of export sales. Hence, we re-fit the Double EMG distribution to match the distribution of log-export sales, and further use the average-to-minimum ratio of export sales to impute the value of the profitability entry threshold. Panel A in Table 3 provides summary statistics of the elasticity estimates.

**Result 1:** *Under demand uncertainty, the extensive margin contribution to the trade elasticity is larger relative to the complete information environment.*

As can be seen from Panel A in Table 3, the complete information economy yields lower values for the extensive margin elasticity.[23] In a complete information environment, the average contribution of the extensive margin is smaller by two orders of magnitude relative to a model with demand uncertainty. Panel B in Table 3 compares the elasticity estimates across the same observations. In particular, it reports summary statistics of the ratio of the quantity implied trade elasticity relative to the sales implied trade elasticity. We call this ratio the *amplification effect* because demand uncertainty produces a higher contribution of the extensive margin to trade, an order of magnitude of $10^5$.

To motivate this magnitude, consider the following example. Suppose trade increases by a million dollars due to a decline in trade costs. Then, a trade elasticity estimate from a complete information model would attribute approximately \$3,000 out of a million dollars of new trade to trade generated by entrants. In a model with incomplete information, \$65,000 out of a million dollars can be attributed to trade by entrants. Hence, complete information dampens the (already small) contribution of new exporters to trade. Conversely a model with uncertainty amplifies the contribution of the extensive margin to trade.[24]

## 5.3 Role of Demand Dispersion

The magnitude of the uncertainty amplification effect is tightly linked to the extent of variation arising from the demand shocks. Substituting equation (3) into equation (5) and taking the logarithm we obtain

$$\log r_{ijk} = \frac{\epsilon_k - 1}{\epsilon_k} \log q_{ijk} + FE_{jk} + \frac{\theta_{ijk}}{\epsilon_k}, \tag{21}$$

[23] In both models, however, the average partial trade elasticity is around 4 as a result of the overall small contribution of the extensive margin to that elasticity.

[24] While the extensive margin contribution to the partial trade elasticity is larger in the complete information than uncertainty economy, note that the magnitudes are partially generated by our generally small estimates of the extensive margin contribution. The magnitude of the estimates is a feature of our data, which includes the universe of customs declarations and therefore contains smaller firms than most standard datasets. See Sager and Timoshenko (Forthcoming) for a discussion of the potential biases that may contaminate estimation on truncated data.

where $FE_{jk} = \log\left(Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{\epsilon_k - 1}{\epsilon_k}}\right)$. Notice that the distribution of the demand shocks generates a wedge between the distributions of log-export sales and log-export quantity. This wedge is larger when the variance of demand shocks is higher. If the variance of the demand shocks is zero, then the distributions of log-export sales and log-export quantity would coincide, yielding no amplification effect. As the variance of the demand shock rises, the distributions of log-export sales and log-export quantities are more dissimilar. Hence we would expect a larger amplification effect.

Given equation (21), we measure the extent of demand variation in a given destination-year-industry as the difference between the variance of log-export sales and the variance of log-export quantities. Assuming for simplicity that the demand shocks are uncorrelated with log-export quantities and applying the variance operator to both sides of equation (21), we obtain

$$V\left(\frac{\theta_{jk}}{\epsilon_k}\right) = V(\log r_{jk}) - \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^2 V(\log q_{jk}). \tag{22}$$

We first compute the variance of log-export sales, $V(\log r_{jk})$, and the variance of log-export quantity, $V(\log q_{jk})$, across firms within a given destination-year-industry observation.We then use equation (22) to back out the value of the variance of the demand shocks, $V(\theta_{jk}/\epsilon_k)$ for each destination-year-industry observation.

**Result 2:** *The difference in the trade elasticity estimates between environments with demand uncertainty and complete information is larger when demand is more dispersed.*

Figure 2 depicts the relationship between the variance of the demand shocks and the amplification effect. The x-axis measures the variance of the demand shocks, while the y-axis measures the ratio of the export quantity implied relative to the export sales implies estimate of the extensive margin elasticity. The figure confirms that the difference in elasticity estimates between the complete information and uncertainty economies increases with an increase in demand dispersion. In the data, exporters do not have full information about product demand in destination markets and introducing uncertainty into the model leads to a larger extensive margin adjustment.

## 5.4 Welfare Implications

In this section we quantify errors in the measurement of the welfare gains from trade arising from changes in the information environment faced by firms. To do so we proceed by calibrating a model with uncertainty to match the estimated trade elasticities in Section 5.1, first and second order moments of the export sales and quantity data. We quantitatively illustrate that a model with complete information leads to significant errors in the measurement of welfare gains from trade.

For expositional simplicity we conduct our exercise using a symmetric two country environment with one industry. We assume that there is no uncertainty in the home market. In the foreign market, firms face demand shocks as described in Section 2. We further assume that demand shock are drawn from a Normal distribution with mean $m_\theta$ and variance $\upsilon_\theta^2$.

We calibrate the general equilibrium of a model with uncertainty to match moments of the data on export quantities and export sales as follows. For a given distribution of log-export quantity, we recover parameters of the Double EMG productivity distribution (the standard deviation, $\upsilon^2$, the left and right tail parameters, $\xi_L$ and $\xi_R$, respectively) using a truncated Double EMG distribution fit to the log-export quantity data as described in Section 4. We next calibrate the fixed export cost, $f_x$, the variable trade cost, $\tau$, and the mean of the productivity distribution, $m$, to match the average-to-minimum ratio of export quantity, the minimum export quantity, and the average value of export sales. Hence, the calibrated general equilibrium of the model will reproduce the estimates of the trade elasticities from Section 5.1.

We next calibrate the variance of demand shocks, $\upsilon_\theta^2$, to match the difference between the variance of log-export quantity and log-export sales as described in equation (22). We further normalize the mean of the demand shocks as follows: $m_\theta = -\upsilon_\theta^2/(2\epsilon)$. Such normalization of the mean ensures that changes in the variance of the demand shocks have no scale effects on the profitability of firms. Hence, potential difference in the predictions between a model with uncertainty and complete information will arise from solely changes in the information environment and not from changes in the level of demand expectations.[25]

As in Section 5.1, we take the elasticity of substitution estimates from Soderbery (2015). Without the loss of generality we normalize the remaining parameters of the model.[26]

Having calibrated the general equilibrium of a model with uncertainty, our goal is to quantify errors in the measurement of the welfare gains from trade arising from changes in the information environment. Hence, holding all structural parameters at their respective calibrated values we simulate the general equilibrium and trade liberalization starting from that equilibrium using a model with uncertainty and a model with complete information. The results of this simulation are presented in Figure 3. Each dot in Figure 3 represents a simulation for a given observation as defined in Section 4.3.[27]

[25] Notice from equations (4) and (8) that, conditional on the general equilibrium effect $B_{ijk}^{\varphi}$, the export thresholds between a model with uncertainty and complete information differ by a factor of $\left(E_\theta\left(e^{\theta_{ijk}/\epsilon_k}\right)\right)^{\epsilon_k}$. Given the normalization, this expectation equals to unity.

[26] $L = 10^8$, $f_d = 1$, respective values of $J$ are chosen to ensure labor market clearing.

[27] As described in Section 4.3, our sample consists of 190 observations of the distributions of export sales and export quantities across firms for a given destination, year, and industry triplet. We perform the described

Figure 3 depicts the welfare wedge from a 10% decline in the variable trade costs.[28] The welfare wedge is constructed by subtracting the percentage points increase in welfare in the model with uncertainty from the percentage points increase in welfare in a model with complete information. For example, the value of 2 on the y-axis indicates that if a model with uncertainty predicts an x% increase in welfare due to a 10% decline in the variable trade costs, a model with complete information will predict an increase in welfare amounting to x%+2%.

Figure 3 shows errors in computing the welfare gains from trade arising from the effect of information on exporting: A model with complete information overstates the gains from trade, and the magnitude of the bias is larger in more uncertain environments, as measured by the standard deviation of the demand shock.

### 5.5 Mechanism

Uncertainty lowers the gains from trade as compared to the complete information environment because uncertainty dampens the forces that lead to selection of firms into export markets, as demonstrated in Figure 4. Figure 4 depicts the ratio of the number of exporters (left panel) and the average exporter size (right panel) in the initial simulated equilibrium between a model with complete information and a model with uncertainty.

Observe from the left panel of Figure 4 that the equilibrium number of exporters in the model with complete information is always lower than in a model with uncertainty. Hence, there is less entry under complete information. The right panel of Figure 4 indicates that those few firms, which export under complete information, generate average trade flows that are orders of magnitude larger than under uncertainty.

The patterns depicted in Figure 4 are driven by selection based on productivity versus profitability introduced in Section 2. In particular, under complete information, firms condition export decisions on both productivity and demand. Hence, only those firms that will have high realized export profit and export revenue, choose to export. This results in low levels of entry, but high total trade flows. In contrast, under uncertainty, firms selection into exporting occurs based on productivity alone and based on expected profits. This results in more firms trying to export due to incomplete information about their profitability in foreign markets, but lower total trade flows due to some of the firms being unsuccessful in

calibration procedure separately for each observation where such calibration is numerically feasible. A calibration is numerically feasible when moments of the Double EMG distribution exist and are finite, i.e. when $\xi_R/\epsilon > 1$ and when $\upsilon_\theta^2 > 0$. There are 76 such feasible observations.

[28] We have replicated results for changes in trade cost ranging from a 50% decline to a 50% increase in the variable trade costs. While the magnitude of the welfare wedge increases with the extent of trade liberalization, the positive relation between the welfare wedge and the standard deviation of the demand shock is preserved for any level of a decline in trade costs. These results are available upon request.

exporting after demand shocks are realized.

These two forces lead to higher absolute value of the partial elasticity of trade flows with respect to variable trade costs, higher domestic trade share, and hence lower welfare gains from trade under uncertainty compared to complete information.

# 6 Empirical Evidence for Main Results

In this section, we provide a reduced form empirical evidence for the effect of information uncertainty, as measured by demand uncertainty, on the partial trade elasticity. In particular, we use bilateral aggregate trade data and distance data from Head, Mayer, and Ries (2010) and *CEPII GeoDist* database (Mayer and Zignago, 2011) respectively to perform a series of cross-sectional gravity regressions that lend support for testable implications of Result 1 and Result 2 in Section 5.

We consider a number of the standard gravity regressions that are augmented with measures of information uncertainty and demand dispersion as follows:

$$\begin{aligned}\log X_{ij} = \gamma_i + \delta_j \quad &+ \quad \beta_1 \log Distance_{ij} + \\ &+ \quad \beta_2 \log Distance_{ij} \cdot \log Information_j + \\ &+ \quad \beta_3 \log Distance_{ij} \cdot \log Information_j \cdot Dispersion_{ij} + \epsilon_{ij}, \end{aligned} \tag{23}$$

where $X_{ij}$ is the aggregate trade flow from country $i$ to country $j$; $\gamma_i$ and $\delta_j$ are origin and destination country fixed effects respectively; and $Distance_{ij}$ is a measure of distance between country $i$ and country $j$.

Result 1 implies that trade is more elastic with respect to the variable trade cost between a bilateral country pair $ij$ compared to a country pair $ij'$ when country $i$ is more uncertain about demand in country $j$ compared to $j'$. Hence, for a given measure of demand uncertainty or information about demand in country $j$, $Information_j$, our model predicts a negative and statistically significant coefficient $\beta_2$ in regression (23). Hence, coefficient $\beta_2$ measures the amplification effect of uncertainty on trade elasticity.

Result 2 implies that conditional on the information structure, the amplification effect of uncertainty is larger when demand is more dispersed. Hence, for a given measure of the dispersion of demand for country $i$'s goods in country $j$, $Dispersion_{ij}$, our model predicts a negative and statistically significant coefficient $\beta_3$ in regression (23).

Sections 6.1 and 6.2 below describe our measures of $Information_j$ and $Dispersion_{ij}$ respectively, and section 6.3 presents and discusses the regression results.

## 6.1 A Measure of Demand Uncertainty

The ideal measure for demand uncertainty must capture the 'observability' of demand by exporters, i.e. the extent of the information that exporters have about demand in a foreign market, prior to engaging in export activity in that market. We proxy for the availability of information to exporters with diversity measures from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003), which capture the diversity of ethnicity, religion and language within a country.

An implicit link between a population's underlying heterogeneity and a firm's information about demand has been established in Parrotta, Pozzoli, and Sala (2016). Using Danish firm-level data, Parrotta et al. (2016) demonstrate a robust relationship between a firm's work force diversity and the firm's ability to succeed in export markets. The authors find that the diversity of a work force positively correlates with a firm's probability of exporting, export value, the number of export destinations and products. They argue that an ethnically diverse work force improves the ability of a firm to market its products to foreign markets.[29]

Following this literature, we argue that idiosyncratic demand shocks in a demographically diverse export destination are less observable by exporters, since exporters will have less precision in predicting demand from demographics in destinations with greater demographic heterogeneity. Likewise, greater homogeneity in the underlying population would make demand more predictable.[30]

Alesina et al.'s (2003) diversity measures are constructed as one minus the Herfindahl index of observed shares of different groups within a country,

$$Diversity_j \equiv 1 - \sum_{n=1}^{N} s_{nj}^2$$

where $s_{nj}$ is the share of group $n$ in country $j$. This diversity construct is meant to capture the probability that two randomly selected individuals from a country's population do not belong to the same group. Alesina et al. (2003) take the data on shares of linguistic and religious groups within a country from *Encyclopedia Britannica* for the year 2001. The source of ethnic data vary by country and the coverage ranges from 1979 to 2001.[31]

Table 4 provides summary statistics of the three diversity measures that are available

[29] Furthermore, the marketing and industrial management literatures have identified consumer heterogeneity as increasing "demand uncertainty" and consider it an important impediment to forecasting new product sales. For example, Bartezzaghi and Verganti (1995) and Bartezzaghi et al. (1999) show demand tends to be more lumpy and less predictable when a market exhibits great consumer heterogeneity.

[30] In order to relax concerns that more demographically diverse countries may engage in more (less) trade and therefore have a systematically higher (lower) expected amount of sales, we include destination specific fixed effects.

[31] For a more in depth discussion of the data and the three diversity measures see Alesina et al. (2003).

for up to 195 countries. Notably, according to these measures, South Korea is the least ethnically and linguistically diverse country in the sample. Over 99% of the population is comprised of ethnic South Koreans that also speak the Korean language. In contrast, Uganda is the most ethnically and linguistically diverse country in the sample, with ethnic representation from the Ganda (17.8%), Teso (8.9%), Nkole (8.2%), Soga (8.2%), Gisu (7.2%), Chiga (6.8%), Lango (6.0%) and Rwanda (5.8%) ethnic groups that speak thirty different languages (Alesina et al., 2003).[31]

We argue that exporters have more complete information about demand for their products when exporting to South Korea. In South Korea, exporters would only need to know the demand of a single ethnic group, which they will reach with close to 100% probability. In contrast, when exporting to Uganda, exporters would not only need to know the demand of eight ethnic groups, but the exporters are also uncertain about which specific group(s) will see their exported product. Hence exporting to demographically homogeneous countries can be characterized by a more complete information about demand compared to demographically heterogeneous countries.

## 6.2 A Measure of Demand Dispersion

A measure of demand dispersion must capture the variance of demand shocks in an export destination regardless of whether those demand shocks are observable (a model with complete information) or unobservable (a model with uncertainty). We therefore construct a theory consistent measure of demand dispersion. Notice from equations (5) and (9) that under both information environments the variance of log-export sales across firms in country $i$ selling to country $j$ can be written as a weighted sum of the variance of productivity and variance of demand shocks:

$$V(\log r_{ij}) = a_1 V(\varphi) + a_2 V(\theta_{ij}), \tag{24}$$

where, to be consistent with the empirical analysis, we have dropped the industry subscript $k$.[32] Equation (24) implies that for a given origin country $i$, the variance of export sales to destination $j$ versus $j'$ differs only due to the difference in the variance of the demand shocks. Hence, for an origin country $i$, export destinations with larger variance of export sales are associated with a larger variance in demand compared to destination with a smaller variance of export sales. Hence, our constructed measure of demand dispersion is given by the standard deviation of exports sales from country $i$ to country $j$ normalized by the average standard deviation of export sales from country $i$ across destinations $j$ where country

[32] Under uncertainty $a_1 = (\epsilon_k - 1)$ and $a_2 = 1/\epsilon_k$; under complete information $a_1 = (\epsilon_k - 1)$ and $a_2 = 1$.

$i$ exports, i.e.

$$Dispersion_{ij} = \frac{st.dev.(r_{ij})}{average_j(st.dev.r_{ij})}.$$

The data on the standard deviation of export sales are derived from the firm-level customs data for each origin country and come from the World Bank Exporter Dynamics Database (Fernandes et al., 2016). For the sample year 2001, the data cover 20 origin countries. Each country in the sample exports to 149 destinations on average. Overall, the $Dispersion_{ij}$ measure is available for 2,978 bilateral country-pairs.

Figure 5 plots the distribution of the $Dispersion_{ij}$ across export destinations $j$ for an average origin country $i$. Notice that relative to the standard deviation of export sales of an average export destination (normalized to unity), the dispersion measure varies from as low as less than one percent of the average to more than twice the average. Hence the export data exhibit significant variation in the standard deviation of export sales across export destinations. For a given origin country, we attribute these differences to differences in demand dispersion.

## 6.3 Gravity Results

We estimate equation (23) using cross-country bilateral trade flows data from Head, Mayer, and Ries (2010). The bilateral distance is measured as the population weighted distance and come from the *CEPII GeoDist* database (Mayer and Zignago, 2011). Our baseline sample year is 2001 and is chosen based on the availability of the diversity measures. Each regression additionally includes an extended set of gravity controls.[33]

Table 5 presents the OLS estimates of regression (23). Column (1) in Table 5 presents results from a simple gravity regression and serves as a benchmark for the rest of the results. Columns (2), (4), and (6) estimate the effect of information structure on the bilateral trade elasticity and provide a test of Result 1. In all three cases the effect of demand uncertainty, the interaction term between $\log Dist_{ij}$ and $\log Information_j$, is negative. In the case of ethnic and linguistic diversity measures of uncertainty presented in columns (2) and (4)

[33]The gravity controls are dummy variables for a bilateral trade pair sharing a common border, sharing a common official or primary language, language is spoken by at least 9% of the population, sharing a common colonizer post 1945, pair ever in colonial relationship, pair ever in sibling relationship, sharing a common currency, common legal origins before transition, common legal origins after transition, common legal origin changed since transition, origin is GATT/WTO member, destination is GATT/WTO member, origin is donator, destination is donator, origin is a EU member, destination is a EU member, origin country belongs to African, Caribbean and Pacific Group of States and destination country belongs to EU, origin country belongs to EU and destination country belongs to African, Caribbean and Pacific Group of States, dummy for free trade agreement, dummy for regional trade agreement. These data are taken from *CEPII GeoDist* database (Mayer and Zignago, 2011). See the source for greater details.

respectively the effect is statistically significant at the 1% level. We next compute the amplification effect as the interaction term as a percent of the trade elasticity coefficient on $\log Dist_{ij}$. Depending on the specification, uncertainty increases the trade elasticity by 2 to 12 percent for each one percent increase in a diversity measure. These magnitudes are consistent with the model implied amplification effect presented in Table 3. Hence, we find strong reduced form support for Result 1 based on the aggregate cross-sectional trade data.

Columns (3), (5) and (7) provide a further analysis of the effect of demand dispersion on the partial trade elasticity. In all cases, the triple interaction term is negative and statistically significant. These findings provide strong reduced form support for our Result 2 stating that the amplification effect of uncertainty on trade elasticity is stronger when demand is more dispersed.

We finally note that, to construct comprehensive and comparable across countries diversity scores, Alesina et al. (2003) rely on various data sources. While linguistic and religious diversity measures are based on the data from a single source, *Encyclopedia Britannica* for the year 2001, the ethnic diversity data are derived from various sources that vary in terms of their data coverage. The oldest data point in the sample is Tuvalu where the most recent information on ethnicity dates back to 1979. We acknowledge that an ethnic composition of a country can potentially change over time, however drastic changes in diversity are likely to occur over much longer time horizons. We check the robustness of our results by looking at a subsample of countries where the most recent information on ethnic diversity scores are from 1996 onward, i.e. within five years of the sample year of 2001.

The robustness results are presented in Table 6 and are robust with a small caveat. Notice in column (2) that the coefficient on the interaction term is negative, but lacks statistical significance. This is not surprising since the sample size in this specification is reduced to 792 observations and therefore lacks strong identification power.

# 7 Conclusion

This paper draws upon recent research that incorporates uncertainty into standard trade models, by embedding a learning mechanism along the lines of Jovanovic (1982) into trade models with heterogeneous firms to analyze firm behavior such as growth (Arkolakis et al., 2018), export participation (Timoshenko, 2015a), product switching (Timoshenko, 2015b), and pricing decisions (Bastos et al., 2018). However, normative implications of such an alternative information structure, particularly for measurements of the trade elasticities and the welfare gains from trade, are not yet well understood.

This paper examines the impact of information structure on welfare through the partial elasticity of trade flows with respect to variable trade costs. We introduce uncertainty with respect to product demand to an otherwise standard new trade model with heterogeneous firms, as in Melitz (2003). With demand uncertainty, firms must choose how much of their product to export prior to observing their destination specific demand shock and, therefore, make export decisions based only on their productivity.

We find that under demand uncertainty trade flows are more elastic to changes in variable trade costs and the welfare gains from trade are lower compared to an environment with complete information. We show that these results arise because selection into export activity is more stringent when potential exporters have complete information, as potential exporters would not knowingly enter an unprofitable market. When potential exporters face uncertainty about demand in foreign markets, a larger number of firms engage in risky export activity but only few firms become large exporters relative to economies with complete information about demand.

We show this in three ways. First, we use the general structure of the model to prove (under mild conditions) that the extensive margin of the partial trade elasticity, which characterizes the effects of uncertainty on trade flows and welfare, is larger when demand is uncertain.

Second, we quantify the model using Brazilian microdata on export quantities and export sales, and show that the structurally estimated elasticities are systematically larger in the economy with uncertainty than in the complete information economy. Furthermore, we use a stylized symmetric two-country example to show that complete information economies overstate the welfare gains from trade and the magnitude of the bias is larger when the variance of demand is larger. Methodologically, we show that identification of the extensive margin partial trade elasticity under uncertainty requires data on quantities traded while the complete information case can be identified from sales data.

Third, we provide reduced form empirical evidence for our mechanism by estimating an otherwise standard gravity equation with additional controls for information availability and demand dispersion. The reduced form evidence supports the main theoretical and quantitative results of the paper: (i) trade is more elastic when demand is harder to predict and is therefore more uncertain, and (ii) this elasticity is larger in markets with more dispersed demand.

This paper shows that the information structure faced by firms is important for measuring the extensive margin response to a decline in trade costs. In countries or industries in which exporters face high demand uncertainty, by assuming away information asymmetries, trade elasticity estimates will likely understate the true magnitude of extensive margin adjustments.

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# Figures and Tables

Table 1: Properties of the log-export quantity and log-export sales distributions across destination-year-industry observations over 1997-2000.

| Statistic | Mean | Standard Deviation | Min | Max |
|---|---|---|---|---|
| *Panel A: Properties of log-quantity* | | | | |
| Standard Deviation | 2.46 | 0.48 | 1.24 | 3.38 |
| Skewness | 0.09 | 0.40 | -1.00 | 0.96 |
| Interquartile Range | 3.48 | 0.82 | 1.88 | 5.50 |
| Kelly Skew | 0.02 | 0.13 | -0.36 | 0.27 |
| *Panel B: Properties of log-sales* | | | | |
| Standard Deviation | 2.28 | 0.41 | 1.30 | 3.19 |
| Skewness | -0.13 | 0.27 | -0.88 | 0.57 |
| Interquartile Range | 3.09 | 0.62 | 1.74 | 4.47 |
| Kelly Skew | -0.04 | 0.11 | -0.32 | 0.28 |

Note: the statistics are reported across 190 destination-year-industry observations where at least 100 firms export. An industry is defined as a 6-digit HS code. Export quantity is measured as export weight in kilograms.

Table 2: Double EMG distribution parameter estimates.

| Statistic | Mean | Std. Dev. |
|---|---|---|
| $\sigma$ | 1.64 | 0.86 |
| $\lambda_L$ | 1.85 | 4.39 |
| $\lambda_R$ | 8.13 | 7.35 |

[a] The summary statistics are reported across 190 destination-year-industry observations. An industry is defined as a 6-digit HS code.

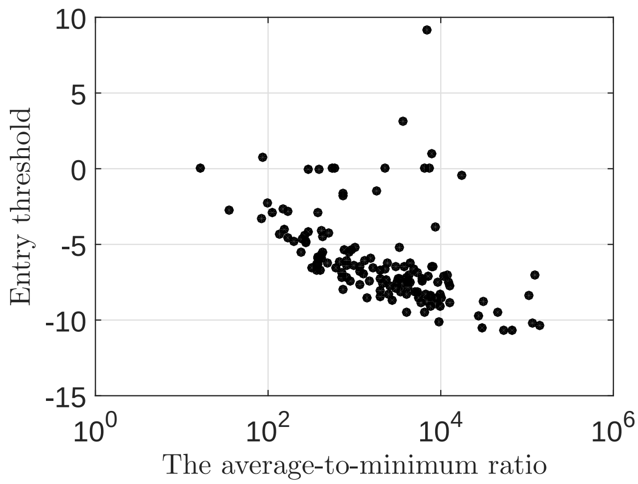
Table 3: Trade elasticity estimates.

| | Extensive Margin Elasticity | | Partial Trade Elasticity, $\eta_{ijk}$ | |
|---|---|---|---|---|
| Measure | Mean | Std. Dev. | Mean | Std. Dev. |
| *Panel A: Estimates of trade elasticity* | | | | |
| Quantity based[a] | 0.07 | 0.34 | 3.38 | 3.64 |
| Sales based | 0.003 | 0.02 | 3.54 | 3.76 |
| *Panel B: Amplification effect* | | | | |
| Amplification effect[b] | $2.8 \cdot 10^5$ | $1.6 \cdot 10^6$ | 1.07 | 0.34 |

[a] The quanity based measure of trade elasticity is based on a model with demand uncertainty. The summary statistics are reported across 175 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > (\epsilon_k - 1)/\epsilon_k$. The elasticities are not defined for $\lambda_R \leq (\epsilon_k - 1)/\epsilon_k$.

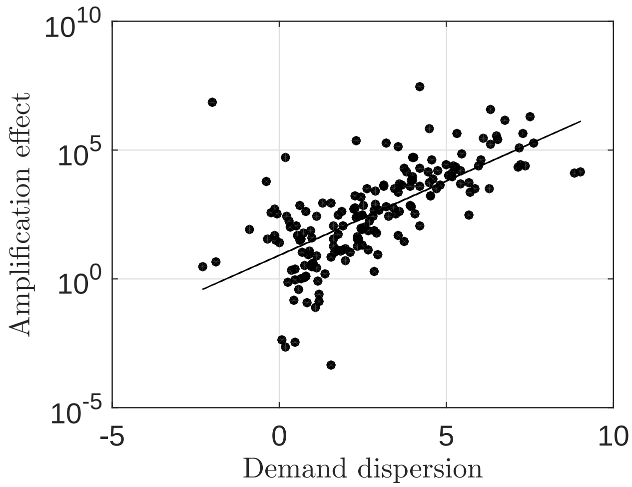
[b] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 175 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures.

Figure 1: The entry threshold and average-to-minimum ratio.



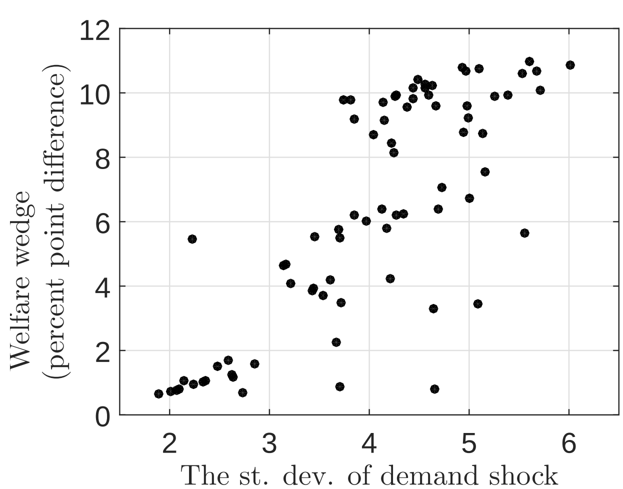

Notes: The figure depicts a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of export quantity for observation with an estimate of the Double EMG tail parameter $\lambda_R > 1$. The threshold is not defined for $\lambda_R \leq 1$. Each dot corresponds to a destination-year-industry observation. Values of the thresholds are demeaned by a corresponding estimate of $\mu$ of the Double EMG distribution.

Figure 2: Amplification effect and demand uncertainty.



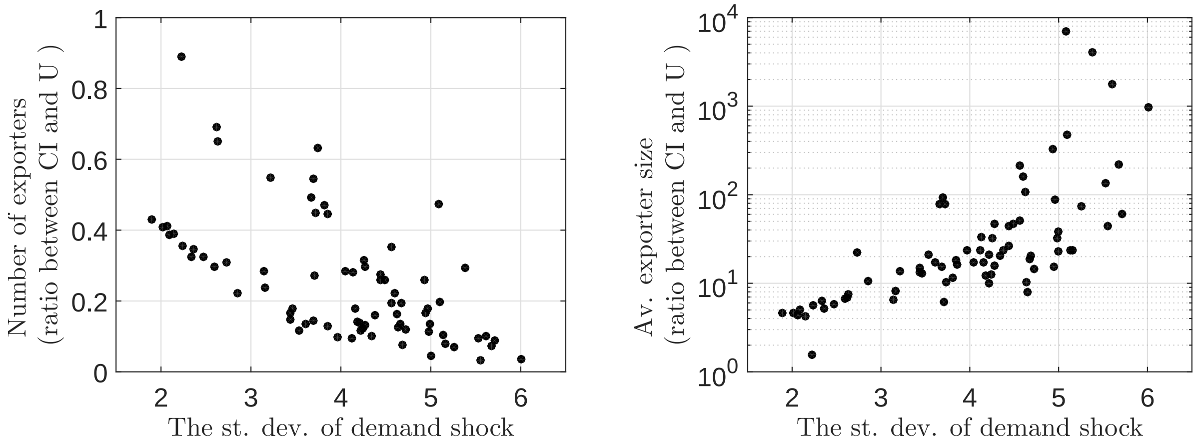

Notes: The figure depicts a scatter plot of the amplification effect and demand uncertainty. The amplification effect is defined as the ratio of the extensive margin elasticity estimates between the quantity based and the sales based measures. Demand dispersion is defined as the variance of the demand shocks estimated using equation (22). Each dot corresponds to a destination-year-industry observation. The solid line is an OLS best fit line.

Figure 3: The welfare wedge from a 10% decline in variable trade costs.



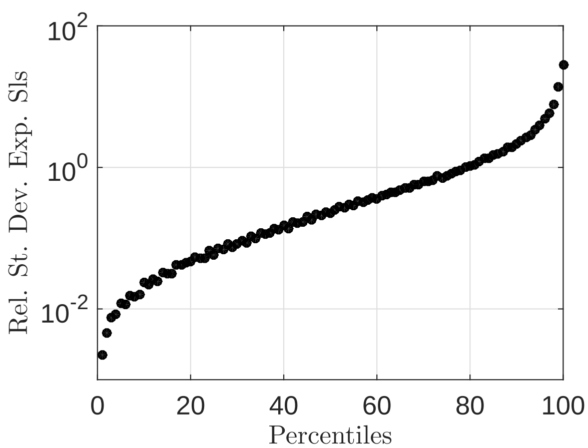

Notes: The figure depicts the percentage point difference in the estimates of the welfare gains from trade from a 10% decline in variable trade costs between a model with complete information and a model with uncertainty. Each dot is a separate observation and represents a simulation result for that observation.

Figure 4: Conterfactual number for exporters and average exporter size.




Notes: Each dot is a separate observation and represents a counterfactual result for that observation. 'CI' stands for 'Complete Information'; 'U' stands for 'Uncertainty'.

Figure 5: Distribution of the demand dispersion measure.




Notes: The figure plots the distribution of the relative to the mean standard deviation of export sales across export destinations for an average origin country.

Table 4: Sample properties of diversity measures.

| Diversity Measure | No. Obs. | Mean | St. Dev. | min | 25th pct | 50th pct | 75th pct | max |
|---|---|---|---|---|---|---|---|---|
| Ethnic | 185 | 0.44 | 0.26 | South Korea (0.002) | El Salvadore (0.20) | Dominican republic (0.43) | Nepal (0.66) | Uganda (0.93) |
| Linguistic | 185 | 0.39 | 0.28 | South Korea (0.002) | Ecuador (0.13) | Mongolia (0.37) | Belize (0.63) | Uganda (0.93) |
| Religious | 196 | 0.44 | 0.23 | Yemen (0.002) | Andorra (0.23) | Haiti (0.47) | Mauritius (0.64) | South Africa (0.86) |

NOTES: Diversity scores are from Alesina et al. (2003). The number in parenthesis beneath a country's name indicates that country's respective score.

Table 5: OLS gravity regressions.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | | Ethnic Div. | | Linguistic Div. | | Religious Div. | |
| $\log Dist_{ij}$ | -1.568[a] | -1.713[a] | -1.612[a] | -1.626[a] | -1.489[a] | -1.595[a] | -1.521[a] |
| | (0.025) | (0.039) | (0.095) | (0.037) | (0.092) | (0.032) | (0.080) |
| $\log Dist_{ij} \cdot \log Information_j$ | | -0.120[a] | -0.192[a] | -0.049[a] | -0.106[a] | -0.025 | -0.117[a] |
| | | (0.023) | (0.052) | (0.016) | (0.039) | (0.019) | (0.042) |
| $\log Dist_{ij} \cdot \log Information_j \cdot Dispersion_{ij}$ | | | -0.005[a] | | -0.004[a] | | -0.014[a] |
| | | | (0.001) | | (0.001) | | (0.002) |
| Amplif. Effect | | 7% | 12% | 3% | 7% | 2% | 8% |
| No. obs. | 22,164 | 20,985 | 2,085 | 21,142 | 2,088 | 21,957 | 2,181 |
| $R^2$ | 0.72 | 0.73 | 0.81 | 0.73 | 0.81 | 0.73 | 0.82 |
| orig. & dest. FE | Y | Y | Y | Y | Y | Y | Y |
| Addit. grav. controls | Y | Y | Y | Y | Y | Y | Y |

NOTES: cross-sectional data on bilateral trade flows for the year 2001. Distance in measured as population weighted distance from *CEPII GeoDist* database (Mayer and Zignago, 2011). The dependent variable is the logarithm of a bilateral trade flows from Head, Mayer, and Ries (2010). $Information_j$ is measured as ethnic diversity in columns (2) and (3); linguistic diversity in columns (4) and (5); religious diversity in columns (6) and (7). Ethnic, religius, linguistic, diversity scores are from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003). $Dispersion_{ij}$ is measured as the standard deviation of the export value per exporter from country $i$ to country $j$ normalized by the average standard deviation of the export value per exporter across all destination $j$ where firms from origin country $i$ export; source the World Bank Exporter Dynamics Database (Fernandes et al., 2016). The amplification effect is comuted by dividing the coefficent of the interraction term by the coeffient on log-distance, and is expressed in percent terms.

[a] statitsically significant at 1% level.

Table 6: OLS gravity regressions, robustness.

| | (1) | (2) |
|---|---|---|
| | Ethnic Div. | |
| $\log Dist_{ij}$ | -1.664$^a$ | -1.394$^a$ |
| | (0.066) | (0.158) |
| $\log Dist_{ij} \cdot \log Information_j$ | -0.101$^a$ | -0.062 |
| | (0.035) | (0.073) |
| $\log Dist_{ij} \cdot \log Information_j \cdot Dispersion_{ij}$ | | -0.005$^a$ |
| | | (0.001) |
| Amplif. Effect | 6% | 4% |
| No. obs. | 8,005 | 792 |
| $R^2$ | 0.75 | 0.84 |
| orig. & dest. FE | Y | Y |
| Addit. grav. controls | Y | Y |

NOTES: cross-sectional data on bilateral trade flows for the year 2001. Distance in measured as population weighted distance from *CEPII GeoDist* database (Mayer and Zignago, 2011). The dependent variable is the logarithm of a bilateral trade flows from Head, Mayer, and Ries (2010). $Information_j$ is measured as ethnic diversity and are from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003). The sample of countries for which ethnic diveristy scores are from 1996-2001. $Dispersion_{ij}$ is measured as the standard deviation of the export value per exporter from country $i$ to country $j$ normalized by the average standard deviation of the export value per exporter across all destination $j$ where firms from origin country $i$ export; source the World Bank Exporter Dynamics Database (Fernandes et al., 2016). The amplification effect is comuted by dividing the coefficent of the interraction term by the coeffient on log-distance, and is expressed in percent terms.
[a] statitsically significant at 1% level.

# A Theoretical Appendix

## A.1 A Model with Demand Uncertainty

In this section we provide derivations for the theoretical results in Section 2. We consider a monopolistically competitive environment as in Melitz (2003) with exogenous entry as in Chaney (2008). We further introduce information asymmetries by constructing a stylized version of the learning model in Timoshenko (2015b).

### A.1.1 Supply

For each destination and industry firms maximize expected profits given by

$$E[\pi(\varphi)] = \max_{q_{ijk}} E_{\theta_{ijk}}\left(p_{ijk}q_{ijk} - \frac{w_i\tau_{ij}}{e^{\varphi}}q_{ijk}\right) - w_i f_{ijk}, \tag{25}$$

subject to the demand equation (2). The expectation over the demand draw, $\theta_{ijk}$, is given by the distribution from which the demand parameter is drawn, $h_{ijk}(.)$. Substituting equation equation (2) into the objective function and applying the expectation operator yields the problem of the firm,

$$\max_{q_{ijk}(\varphi)} q_{ijk}(\varphi)^{\frac{\epsilon_k-1}{\epsilon_k}} E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right) Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{\epsilon_k-1}{\epsilon_k}} - \frac{w_i\tau_{ij}}{e^{\varphi}}q_{ijk}(\varphi) - w_i f_{ijk}. \tag{26}$$

The first order conditions with respect to quantity yield the optimal quantity,

$$q_{ijk}(\varphi) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k\varphi}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}, \tag{27}$$

or equivalently

$$q_{ijk}(\varphi) \quad = \quad B^q_{ijk}\cdot e^{\epsilon_k\varphi},$$

where $B^q_{ijk} = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}$.

A firm's realized revenue is then given by

$$r_{ijk}(\theta_{ijk},\varphi) \quad = \quad \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k-1} e^{(\epsilon_k-1)\varphi+\frac{\theta_{ijk}}{\epsilon_k}}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k-1}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1},$$

or equivalently

$$r_{ijk}(\theta_{ijk},\varphi) \quad = \quad B^r_{ijk}e^{(\epsilon_k-1)\varphi+\frac{\theta_{ijk}}{\epsilon_k}},$$

were $B^r_{ijk} = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k-1}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}$.

### A.1.2 Entry

Firms enter the market as long as expected profit is positive. Hence, the optimal productivity entry threshold, $\varphi^*_{ijk}$, is a solution to the zero-expected profit condition given by

$$E_\theta[\pi(\varphi^*_{ijk})] = 0. \tag{28}$$

Substituting equation (27) into equation (26) and solving equation (28) for $\varphi^*_{ijk}$ yields

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} = \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}}, \tag{29}$$

or equivalently

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} = \frac{B^\varphi_{ijk}}{\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}},$$

where $B^\varphi_{ijk} = (\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}) \,/\, \left(\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk}P_{jk}^{\epsilon_k-1}\right)$.

### A.1.3 Trade Elasticity

The aggregate trade flow from country $i$ to country $j$ in industry $k$ is defined as

$$X_{ijk} = M_{ijk}\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} r_{ijk}(\theta,\varphi)g^\theta_{ijk}(\theta)\frac{g^\varphi_{ijk}(\varphi)}{Prob^\varphi_{ijk}(\varphi > \varphi^*_{ijk})}d\theta d\varphi, \tag{30}$$

where $M_{ijk}$ is the mass of firms exporting from country $i$ to country $j$ in industry $k$. Given the exogenous entry assumption, the mass of firms is given by

$$M_{ijk} = J_i \times Prob^\varphi_{ijk}(\varphi > \varphi^*_{ijk}), \tag{31}$$

where $J_i$ is the exogenous mass of entrants. Equation (30) can then be simplified as follows:

$$\begin{aligned} X_{ijk} &= J_i\int_{\varphi^*_{ijk}}^{+\infty}\int_{-\infty}^{+\infty} q_{ijk}(\varphi)p_{ijk}(\theta,\varphi)g^\theta_{ijk}(\theta)g^\varphi_{ijk}(\varphi)d\theta d\varphi = \\ &= J_i\int_{\varphi^*_{ijk}}^{+\infty} q_{ijk}(\varphi)\frac{\epsilon_k}{\epsilon_k-1}\frac{w_i\tau_{ij}}{e^\varphi E_\theta\left(e^{\frac{\theta}{\epsilon_k}}\right)}\left(\int_{-\infty}^{+\infty} e^{\frac{\theta_{ijk}}{\epsilon_k}}g^\theta_{ijk}(\theta)d\theta\right)g^\varphi_{ijk}(\varphi)d\varphi = \\ &= J_i\int_{\varphi^*_{ijk}}^{+\infty} q_{ijk}(\varphi)\frac{\epsilon_k}{\epsilon_k-1}\frac{w_i\tau_{ij}}{e^\varphi}g^\varphi_{ijk}(\varphi)d\varphi = \end{aligned}$$

$$= \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k - 1)\varphi} g^{\varphi}_{ijk}(\varphi) d\varphi.$$

Define $\phi_k = (\epsilon_k - 1)\varphi$ with the corresponding probability density function denoted by $g^{\phi}_{ijk}(.)$ and $\phi^*_{ijk} = (\epsilon_k - 1)\varphi^*_{ijk}$. Then, the aggregate trade flow can be equivalently written as

$$X_{ijk} \quad = \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\phi^*_{ijk}}^{+\infty} e^{\phi} g^{\phi}_{ijk}(\phi) d\phi. \quad (32)$$

Differentiate the logarithm of equation (32) with respect to $\log \tau_{ij}$ to obtain

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} \quad = \quad (1 - \epsilon_k) - \frac{e^{\phi^*_{ijk}} g^{\phi}_{ijk}(\phi^*_{ijk})}{\int_{\phi^*_{ijk}}^{+\infty} e^{\phi} g^{\phi}_{ijk}(\phi) d\phi} \frac{\partial \phi^*_{ijk}}{\partial \log \tau_{ijk}}. \quad (33)$$

Differentiate equation (29) with respect to $\tau_{ij}$ to obtain

$$\frac{\partial \phi^*_{ijk}}{\partial \log \tau_{ij}} \quad = \quad \epsilon_k - 1. \quad (34)$$

Combine equations (33) and (34) to obtain the partial elasticity of trade flows with respect to the variable trade costs being given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} \quad = \quad (1 - \epsilon_k) \left(1 + \frac{g^{\phi}_{ijk}(\phi^*_{ijk}) e^{\phi^*_{ijk}}}{\int_{\phi^*_{ijk}}^{+\infty} e^{\phi} g^{\phi}_{ijk}(\phi) d\phi}\right). \quad (35)$$

### A.1.4 Estimation Approach

Consider the following change of notation: let $\tilde{\varphi} \equiv \epsilon_k \varphi$. Denote by $g^{\tilde{\varphi}}(.)$ the probability distribution function of $\tilde{\varphi}$. Given the change in notation, $g^{\tilde{\varphi}}(.)$ is the distribution of $\varphi$, $g^{\varphi}_{ijk}(.)$, scaled by the elasticity of substitution, $\epsilon_k$.

With the change in notation, equations (29) and (32) can be written as

$$e^{\frac{\epsilon_k - 1}{\epsilon_k}\tilde{\varphi}^*} \quad = \quad \frac{\epsilon_k w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}} \quad (36)$$

$$X_{ijk} \quad = \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} \left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{\tilde{\varphi}^*_{ijk}}^{+\infty} e^{\frac{\epsilon_k - 1}{\epsilon_k}\phi} g^{\tilde{\varphi}}(\varphi) d\varphi. \quad (37)$$

Differentiating equation (36) and (37) with respect to $\tau_{ij}$, the partial trade elasticity can be expressed as

$$\eta_{ijk} = (1-\epsilon_k)\left(1+\frac{\epsilon_k}{\epsilon_k-1}\frac{g^{\tilde{\varphi}}_{ijk}(\tilde{\varphi}^*_{ijk})e^{\frac{\epsilon_k-1}{\epsilon_k}\tilde{\varphi}^*_{ijk}}}{\int_{\tilde{\varphi}^*_{ijk}}^{+\infty} e^{\frac{\epsilon_k-1}{\epsilon_k}\varphi} g^{\tilde{\varphi}}_{ijk}(\varphi)d\varphi}\right).$$

The distribution $g^{\tilde{\varphi}}_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export quantity. From equation (27), the optimal quantity can we written as

$$q_{ijk}(\tilde{\varphi}) = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} e^{\tilde{\varphi}}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}. \tag{38}$$

Hence, the distribution of log-export quantity is given by the distribution of $\tilde{\varphi}$. Given the distribution of $g^{\tilde{\varphi}}_{ijk}(.)$, the scaled productivity entry threshold, $\tilde{\varphi}^*_{ijk}$, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities. From equation (38) the average export quantity, $\tilde{q}_{ijk}$, and the minimum export quantity, $q^{\min}_{ijk}$, are given by

$$\begin{aligned}
\tilde{q}_{ijk} &= \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k}\left(E\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}\int_{\tilde{\varphi}^*_{ijk}}^{+\infty}\frac{e^{\varphi}g^{\tilde{\varphi}}_{ijk}(\varphi)}{Prob^{\tilde{\varphi}}_{ijk}(\varphi>\tilde{\varphi}^*_{ijk})}d\varphi \\
q^{\min}_{ijk} &= \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k}\left(E_\theta\left(e^{\frac{\theta_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}e^{\tilde{\varphi}^*_{ijk}}.
\end{aligned}$$

Hence, the average-to-minimum ratio, $\tilde{q}_{ijk}/q^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-\tilde{\varphi}^*_{ijk}}\int_{\tilde{\varphi}^*_{ijk}}^{+\infty}\frac{e^{\varphi}g^{\tilde{\varphi}}_{ijk}(\varphi)}{Prob^{\tilde{\varphi}}_{ijk}(\varphi>\tilde{\varphi}^*_{ijk})}d\varphi. \tag{39}$$

## A.2 A Model with Complete Information

In this section, for comparison purposes, we develop theoretical results in a model with complete information. The information structure only affects the supply side of the economy. Hence, on the demand side, the utility of a representative consumers is still given by equation (1), and the demand for a given variety is given by equation (2).

### A.2.1 Supply

In contrast to a model with uncertainty, in a model with complete information firms make their market participation and quantity decisions after observing their productivity and demand shocks.

For each destination and industry firms maximize profits given by

$$\max_{q_{ijk}} p_{ijk}q_{ijk} - \frac{w_i\tau_{ij}}{e^{\varphi}}q_{ijk} - w_i f_{ijk}, \tag{40}$$

subject to the demand equation (2). The first order conditions with respect to quantity yield the optimal quantity being given by

$$q_{ijk}(\theta_{ijk},\varphi) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k\varphi+\theta_{ijk}}(\tau_{ij}w_i)^{-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}. \tag{41}$$

Notice that in contrast to equation (27), in a complete information environment the quantity choice is determined by a combination of a supply and a demand shocks, i.e. by a firm's profitability. Using equations (2) and (41), a firm's optimal sales are further given by

$$r_{ijk}(\theta_{ijk},\varphi) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k-1} e^{(\epsilon_k-1)\varphi+\theta_{ijk}}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}, \tag{42}$$

or equivalently

$$\begin{aligned} r_{ijk}(\theta_{ijk},\varphi) &= B_{ijk}e^{(\epsilon_k-1)\varphi+\theta_{ijk}} \\ r_{ijk}^{CI}(z_{ijk}) &= B_{ijk}e^{z_{ijk}}, \end{aligned}$$

where $B_{ijk} = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}(\tau_{ij}w_i)^{1-\epsilon_k}Y_{jk}P_{jk}^{\epsilon_k-1}$, $z_{ijk} = (\epsilon_k - 1)\varphi + \theta_{ijk}$, and 'CI' stands for Complete Information.

#### A.2.2 Entry

Given the optimal profits, firms enter the market as long as the profit is positive. Hence, the optimal any demand draw, $\theta_{ijk}$, productivity entry threshold, $\varphi^*_{ijk}(\theta_{ijk})$, is a solution to the zero-profit condition given by

$$\pi(\varphi^*_{ijk}(\theta_{ijk})) = 0. \tag{43}$$

Substituting equation (41) into equation (40) and solving equation (43) for $\varphi^*_{ijk}(\theta_{ijk})$ yields

$$e^{(\epsilon_k-1)\varphi^*_{ijk}(\theta_{ijk})} = \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}Y_{jk}P_{jk}^{\epsilon_k-1}e^{\theta_{ijk}}}. \tag{44}$$

Notice that in contrast to the incomplete information environment discussed in Section A.1.1 and equation (29), the productivity entry threshold depends on the realized value of demand parameter, $\theta_{ijk}$. Firms with a higher demand parameter have a lower productivity entry threshold.

Equation (44) can be viewed as defining an entry boundary in the space of $(\theta_{ijk},\varphi)$ or as defining the profitability entry threshold $z^*_{ijk}$. The profitability entry thresholds is given by the sum of $(\epsilon_k - 1)\varphi_{ijk}$ and $\theta_{ijk}$ such that equation (44) holds:

$$e^{z^*_{ijk}} \quad = \quad \frac{\epsilon_k w_i f_{ijk}(w_i\tau_{ij})^{\epsilon_k-1}}{\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} Y_{jk} P_{jk}^{\epsilon_k-1}} \equiv B^{\varphi}_{ijk}. \tag{45}$$

Hence, in a model with complete information, selection into exporting occurs based on profitability rather than productivity as is the case in a model with uncertainty.

### A.2.3 Trade Elasticity

The aggregate trade flow from country $i$ to country $j$ in industry $k$ is given by

$$\begin{aligned} X_{ijk} &= J_i \int_{\varphi^*_{ijk}(\theta_{ijk})}^{+\infty} \int_{-\infty}^{+\infty} q_{ijk}(\theta,\varphi) p_{ijk}(\theta,\varphi) g^{\theta}_{ijk}(\theta) g^{\varphi}_{ijk}(\varphi) d\theta d\varphi \\ &= J_i \left(\frac{\epsilon_k}{\epsilon_k - 1}\frac{\tau_{ij} w_i}{P_{jk}}\right)^{1-\epsilon_k} Y_{jk} \int_{\varphi^*_{ijk}(\theta_{ijk})}^{+\infty} \int_{-\infty}^{+\infty} e^{(\epsilon_k-1)\varphi+\theta_{ijk}} g^{\theta}_{ijk}(\theta) g^{\varphi}_{ijk}(\varphi) d\theta d\varphi \end{aligned} \tag{46}$$

Define $z_{ijk} = (\epsilon_k - 1)\varphi + \theta_{ijk}$. From equation (45) the entry into exporting occurs when $z_{ijk} > z^*_{ijk}$. Using this change of variables, equation (46) can be be written as

$$X_{ijk} \quad = \quad J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k-1} (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1} \int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz, \tag{47}$$

where $g^z_{ijk}(.)$ is the distribution of profitability $z_{ijk}$.

Compare the expressions for the aggregate trade flow between the two information environments, equation (32) versus equation (47). Notice, that in the incomplete information environment, the aggregate trade flows are determined by the distribution of *productivity*, $g^{\varphi}_{ijk}(\varphi)$, while in the complete information environment the aggregate trade flows are determined by the distribution of *profitability*, $g^z_{ijk}(z)$.

Following the same differentiation steps as in Section A.2.3, the partial elasticity of trade flows with respect to the variable trade costs is given by

$$\begin{aligned} \eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} \quad &= \quad (1-\epsilon_k)\left(1 + \frac{g^z_{ijk}(z^*_{ijk}) e^{z^*_{ijk}}}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz}\right) = \\ &= \quad (1-\epsilon_k)\left(1 + \frac{g^z_{ijk}(z^*_{ijk})}{Prob^z_{ijk}(z > z^*_{ijk})}\left(\frac{\tilde{r}_{ijk}}{r^{\min}_{ijk}}\right)^{-1}\right). \end{aligned}$$

The last equality hold due to equation (49) below.

### A.2.4 Estimation Approach

The distribution $g^z_{ijk}(.)$ can be directly recovered from the empirical distribution of the log-export sales. From equation (42), the optimal sales can we written as

$$r_{ijk}(z_{ijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} e^{z_{ijk}}(\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}. \tag{48}$$

Hence, the distribution of log-export sales is given by the distribution of $z_{ijk}$. Given the distribution of $g^z_{ijk}(.)$, the profitability entry threshold, $z^*_{ijk}$, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of export quantities. From equation (38) the average export sales, $\tilde{r}_{ijk}$, and the minimum export sales, $r^{\min}_{ijk}$, are given by

$$\tilde{r}_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{z^*_{ijk}}^{+\infty} e^z \frac{g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz$$

$$r^{\min}_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (\tau_{ij}w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{z^*_{ijk}}.$$

Hence, the average-to-minimum ratio, $\tilde{r}_{ijk}/r^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-z^*_{ijk}} \int_{z^*_{ijk}}^{+\infty} \frac{e^z g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz. \tag{49}$$

To contrast the two information environments, notice that while equations for estimating the entry thresholds are similar, equation (39) versus (49), different data are used for estimation. In the environment with demand uncertainty the relevant distributions and entry thresholds are identified from the empirical export quantity distributions, while in the complete information framework, log export sales identify the necessary parameters.

# B Proofs of Propositions

**Proposition 1** *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

*(i) $\gamma(x) \equiv [e^x g(x)]/\int_x^{+\infty} e^z g(z) dz$ is an increasing function of $x$.*

*(ii) Denote the extensive margin elasticity associated with $g(x)$ as $\gamma(x)$. Let $\tilde{g}(x)$ be a mean preserving spread of $g(x)$, with extensive margin elasticity $\tilde{\gamma}(x)$. There exists $x^*$ such that $\tilde{\gamma}(x) < \gamma(x)$ for all $x > x^*$, $\tilde{\gamma}(x) = \gamma(x)$ if $x = x^*$, and $\tilde{\gamma}(x) > \gamma(x)$ for all $x < x^*$.*

**Proof of Proposition 1**

**Part (i)** First, define $h(x) = (e^x g(x))/E$, where $E = \int_{-\infty}^{+\infty} e^x g(x) dx$. Notice that $h(x)$ is positive for all $x$ and that $\int_{-\infty}^{+\infty} h(x) dx = 1$. Hence, $h(x)$ is a probability density function.

The corresponding cumulative density function is given by $H(x) = \int_{-\infty}^{x} e^z g(z)dz/E$. The corresponding survival function is given by $1 - H(x) = \int_{x}^{+\infty} e^z g(z)dz/E$.

Next, function $\gamma(x)$ can then be written as

$$\gamma(x) \quad = \quad \frac{e^x g(x)}{\int_x^{+\infty} e^z g(z)dz} = \frac{h(x)}{1 - H(x)}.$$

Hence, $\gamma(x)$ is a hazard rate associated with the distribution $H(x)$. By Theorem 10 in Rinne (2014), the hazard rate $\gamma(x)$ is monotonically increasing in $x$ if and only if its logarithmic survival function, $\log(1 - H(x))$, is concave. Notice that by part (iii) of A1, $\log(1 - H(x))$ is a concave function of $x$. Hence, $\gamma(x)$ is increasing in $x$. For completeness, we reproduce the proof of this result below.

Notice that

$$\gamma(x) = -\frac{d \log(1 - H(x))}{dx}.$$

Hence,

$$\frac{d\gamma(x)}{dx} = -\frac{d^2 \log(1 - H(x))}{dx^2}.$$

Since $\log(1 - H(x))$ is a concave function of $x$, $d^2 \log(1 - H(x))/dx^2 < 0$. Therefore, $d\gamma(x)/dx > 0$.

**Part (ii)** Function $\tilde{\gamma}(x)$ is given by

$$\tilde{\gamma}(x) \quad = \quad \frac{e^x \tilde{g}(x)}{\int_x^{+\infty} e^z \tilde{g}(z)dz} = \frac{\tilde{h}(x)}{1 - \tilde{H}(x)},$$

where $\tilde{g}(.)$ is a mean preserving spread of $g(.)$, $\tilde{h}(x) = [e^x \tilde{g}(x)]/\int_{-\infty}^{+\infty} e^x \tilde{g}(x)dx$, and $\tilde{H}(x)$ is the corresponding cumulative distribution function.

$\gamma(x) > \tilde{\gamma}(x)$ if and only if $H(x) > \tilde{H}(x)$ as follows for the following set of equivalent inequalities:

$$\begin{aligned}
\gamma(x) = -\frac{d \log(1 - H(x))}{dx} \quad &> \quad -\frac{d \log(1 - \tilde{H}(x))}{dx} = \tilde{\gamma}(x) \\
d \log(1 - H(x)) \quad &< \quad d \log(1 - \tilde{H}(x)) \\
\int d \log(1 - H(x)) \quad &< \quad \int d \log(1 - \tilde{H}(x)) \\
\log(1 - H(x)) \quad &< \quad \log(1 - \tilde{H}(x)) \\
H(x) \quad &> \quad \tilde{H}(x).
\end{aligned}$$

We will now show in three steps that $H(x)$ crosses $\tilde{H}(x)$ once from below, and therefore there exists $x^*$ such that $H(x) > \tilde{H}(x)$ holds for $x > x^*$, and therefore (ii) holds.

Step 1: Denote by $X$ and $\tilde{X}$ random variables distributed according to $g(x)$ and $\tilde{g}(x)$ respectively. Since $\tilde{g}(x)$ is a mean preserving spread of $g(x)$, it holds that $\tilde{X} = X + \hat{X}$, where $\hat{X}$ is distributed according to $\hat{g}(x)$ with mean zero, and $\hat{X}$ is independent from $X$. Hence, $\tilde{g}(.)$ is a convolution of $g(.)$ and $\hat{g}(.)$ and can be written as

$$\tilde{g}(x) = \int_{-\infty}^{+\infty} g(x-u)\hat{g}(u)du.$$

Step 2: Denote by $X^h$, $\tilde{X}^h$, $\hat{X}^h$ random variables distributed according to $h(x)$, $\tilde{h}(x)$, and $\hat{h}(x)$ respectively, where $\hat{h}(x) = [e^x \hat{g}(x)] / \int_{-\infty}^{+\infty} e^x \hat{g}(x)dx$. Similarly, it can be show that $\tilde{h}(.)$ is a convolution of $h(.)$ and $\hat{h}(.)$:

$$\begin{aligned}
\int_{-\infty}^{+\infty} h(x-u)\hat{h}(u)du &= \frac{\int_{-\infty}^{+\infty} e^{x-u} g(x-u) e^u \hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right] \cdot \left[\int_{-\infty}^{+\infty} e^x \hat{g}(x)dx\right]} = \\
&= \frac{\int_{-\infty}^{+\infty} e^x g(x-u)\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right] \cdot \left[\int_{-\infty}^{+\infty} e^x \hat{g}(x)dx\right]} = \\
&= \frac{e^x \tilde{g}(x)}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right] \cdot \left[\int_{-\infty}^{+\infty} e^x \hat{g}(x)dx\right]} = \tilde{h}(x).
\end{aligned}$$

Thus, it hold that $\tilde{X}^h = X^h + \hat{X}^h$, where $\tilde{X}^h$ and $\hat{X}^h$ are independent.

Step 3: Consider a random variable $\bar{X} = X^h + \hat{X}^h - E(\hat{X}^h)$ with the cumulative distribution function denoted by $\bar{H}(x)$. $\bar{X}$ is a mean preserving spread of $X^h$ and therefore the two corresponding cumulative distribution functions satisfy the single-crossing property whereby $H(x) = \bar{H}(x)$ if $x = E(X^h)$; $H(x) < \bar{H}(x)$ for $x < E(X^h)$, and $H(x) > \bar{H}(x)$ for $x > E(X^h)$.

Next, notice that $\tilde{X}^h = \bar{X} + E(\hat{X}^h)$. Therefore the cumulative distribution function of $\tilde{X}^h$ is a shift of the cumulative distribution function of $\bar{X}$ along the x-axis, namely $\tilde{H}(x) = \bar{H}(x - E(\hat{X}^h))$. Hence $\tilde{H}(x)$ preserves the same single-crossing property with respect to $H(x)$. Namely $\exists x^*$ that that $H(x) = \tilde{H}(x)$ if $x = x^*$; $H(x) < \tilde{H}(x)$ for $x < x^*$, and $H(x) > \tilde{H}(x)$ for $x > x^*$. ■

**Corollary 1** *Let $g(x)$ be a probability density function satisfying A1. $\forall a \in \mathbb{R}$ there exists $x^*(a)$ such that $\gamma(x) > \tilde{\gamma}(x+a)$ if $x > x^*(a)$; $\gamma(x) = \tilde{\gamma}(x+a)$ if $x = x^*(a)$, and $\gamma(x) < \tilde{\gamma}(x+a)$ if $x < x^*(a)$.*

**Proof of Corollary 1**

Notice that part (ii) of Proposition 1 implies a single crossing property of $\gamma(.)$ and $\tilde{\gamma}(.)$. This property is preserved under an affine transformation of the abscissa for either of the functions. Therefore, $\gamma(x)$ also crosses $\tilde{\gamma}(x+a)$ from above for some $x^*(a)$.■

# C Robustness

In this section we demonstrate the robustness of our theoretical and quantitative results to the way we choose to model a firm's decision under uncertainty. In the main text we assume that in a model with uncertainty, firms choose export quantities before demand shocks are realized. This assumption is consistent with the majority of the literature on learning such as Timoshenko (2015b), Arkolakis et al. (2018), Berman et al. (Forthcoming). In contrast to this literature, in this section we assume that firms choose prices before demand shocks are realized. Below, we present an alternative representation of the model with uncertainty in line with this assumption. In this model, given that firms choose prices, the price data contain information necessary to identify the partial elasticity of trade flows with respect to variable trade costs. We subsequently quantify trade elasticities according to this insight. Our quantitative results are unaffected by the change in the firm's choice variable.

The intuition for this equivalence lies in the fact that the price and quality are inversely related through sales. In a model with uncertainty where firms choose quantities, the empirical distribution of quantity identifies the underlying theoretical distribution of productivities. In a model with uncertainty where firm choose prices, the optimal price equals to the inverse of productivity. Therefore, theoretical productivity distribution is identified by the empirical distribution of the inverse of the prices, which is proportional to the empirical distribution of quantities through the equation quantity = sales/price.

## C.1 Alternative Model of Demand Uncertainty

The economic environment and demand are the same as in Section 2.

$$c_{ijk}(\omega) = e^{\theta_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}, \tag{50}$$

### C.1.1 Supply

For each destination and industry firms maximize expected profits given by

$$E[\pi(\varphi)] = \max_{p_{ijk}} E_{\theta_{ijk}} \left( p_{ijk} q_{ijk} - \frac{w_i \tau_{ij}}{e^{\varphi}} q_{ijk} \right) - w_i f_{ijk},$$

subject to the demand equation (50). The expectation over the demand draw, $\theta_{ijk}$, is given by the distribution from which the demand parameter is drawn, $h_{ijk}(.)$. Substituting equation

equation (50) into the objective function and applying the expectation operator yields the problem of the firm,

$$\max_{p_{ijk}(\varphi)} p_{ijk}(\varphi)^{1-\epsilon_k} E\left(e^{\theta_{ijk}}\right) Y_{jk} P_{jk}^{\epsilon_k-1} - \frac{w_i \tau_{ij}}{e^{\varphi}} E\left(e^{\theta_{ijk}}\right) p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1} - w_i f_{ijk}.$$

The first order conditions with respect to price yield the optimal price,

$$p_{ijk}(\varphi) \;=\; \left(\frac{\epsilon_k}{\epsilon_k-1}\right) \frac{w_i \tau_{ij}}{e^{\varphi}}. \tag{51}$$

A firm's realized revenue is then given by

$$r_{ijk}(\theta_{ijk}, \varphi) \;=\; e^{\theta_{ijk}(\omega)} \left(\frac{\epsilon_k}{\epsilon_k-1} \frac{w_i \tau_{ij}}{e^{\varphi}}\right)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}.$$

### C.1.2 Entry

Firms enter the market as long as expected profit is positive. Hence, the optimal productivity entry threshold, $\varphi^*_{ijk}$, is a solution to the zero-expected profit condition given by

$$E[\pi(\varphi^*_{ijk})] \;=\; 0,$$

and is given by

$$e^{(\epsilon_k-1)\varphi^*_{ijk}} \;=\; \frac{\epsilon_k^{\epsilon_k} w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k-1}}{(\epsilon_k-1)^{\epsilon_k-1} Y_{jk} P_{jk}^{\epsilon_k-1} E\left(e^{\theta_{ijk}}\right)}. \tag{52}$$

### C.1.3 Trade Elasticity

The aggregate trade flow from country $i$ to country $j$ in industry $k$ is defined as

$$X_{ijk} \;=\; M_{ijk} \int_{\varphi^*_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(\theta, \varphi) h_{ijk}(\theta) \frac{g_{ijk}(\varphi)}{Prob_{ijk}(\varphi > \varphi^*_{ijk})} d\theta d\varphi, \tag{53}$$

where $M_{ijk}$ is the mass of firms exporting from country $i$ to country $j$ in industry $k$. Given the exogenous entry assumption, the mass of firms is given by

$$M_{ijk} \;=\; J_i \times Prob_{ijk}(\varphi > \varphi^*_{ijk}),$$

where $J_i$ is the exogenous mass of entrants. Equation (53) can then be simplified as follows:

$$\begin{aligned} X_{ijk} \;&=\; J_i \int_{\varphi^*_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(\varphi) h_{ijk}(\theta) g_{ijk}(\varphi) d\theta d\varphi = \qquad (54) \\ &=\; J_i \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} E\left(e^{\theta_{ijk}}\right) (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1} \int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi} g_{ijk}(\varphi) d\varphi. \end{aligned}$$

Differentiate equation (54) with respect to $\tau_{ij}$ to obtain

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = (1-\epsilon_k)\frac{X_{ijk}}{\tau_{ijk}} - \frac{X_{ijk}}{\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi} g_{ijk}(\varphi)d\varphi} e^{(\epsilon_k-1)\varphi^*_{ijk}} g_{ijk}(\varphi^*_{ijk})\frac{\partial \varphi^*_{ijk}}{\partial \tau_{ijk}}. \quad (55)$$

Differentiate equation (52) with respect to $\tau_{ij}$ to obtain

$$\frac{\partial \varphi^*_{ijk}}{\partial \tau_{ij}} = \frac{1}{\tau_{ij}}. \quad (56)$$

Combine equations (55) and (56) to obtain the partial elasticity of trade flows with respect to the variable trade costs being given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = (1-\epsilon_k)\left(1 + \frac{e^{(\epsilon_k-1)\varphi^*_{ijk}} g_{ijk}(\varphi^*_{ijk})}{(\epsilon_k-1)\int_{\varphi^*_{ijk}}^{+\infty} e^{(\epsilon_k-1)\varphi} g_{ijk}(\varphi)d\varphi}\right). \quad (57)$$

### C.1.4 Estimation Approach

From equation (51), the distribution $g_{ijk}(.)$ can be directly recovered from the empirical distribution of the logarithm of the inverse of export price as follows:

$$\log\left(1/p_{ijk}(\varphi_{ijk})\right) = B^p_{ijk} + \varphi_{ijk}. \quad (58)$$

Hence, the distribution of the logarithm of the inverse of export price is given by the distribution of $\varphi_{ijk}$. Given the distribution of $g_{ijk}(.)$, the productivity entry threshold, $\varphi^*_{ijk}$, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of the inverse of export prices. From equation (51) the average of the inverse of export price, $\widetilde{1/p}_{ijk}$, and the minimum of the inverse of export price , $(1/p)^{\min}_{ijk}$, are given by

$$\widetilde{1/p}_{ijk} = \frac{\epsilon_k-1}{\epsilon_k}(\tau_{ij}w_i)^{-1}\int_{\varphi^*_{ijk}}^{+\infty} \frac{e^{\varphi} g_{ijk}(\varphi)}{Prob^{\varphi}_{ijk}(\varphi > \varphi^*_{ijk})} d\varphi$$

$$(1/p)^{\min}_{ijk} = \frac{\epsilon_k-1}{\epsilon_k}(\tau_{ij}w_i)^{-1} e^{\varphi^*_{ijk}}.$$

Hence, the average-to-minimum ratio, $\widetilde{1/p}_{ijk}/(1/p)^{\min}_{ijk}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-\varphi^*_{ijk}}\int_{\varphi^*_{ijk}}^{+\infty} \frac{e^{\varphi} g_{ijk}(\varphi)}{Prob^{\varphi}_{ijk}(\varphi > \varphi^*_{ijk})} d\varphi. \quad (59)$$

## C.2 Trade Elasticity Estimates

Table C1 replicates results in Table 3 and shows that the quantitative magnitude of the trade elasticities and the amplification effect remains robust to the alternative firm-level choice variable under uncertainty.

Table C1: Trade elasticity estimates, the log of the inverse of prices.

| | Extensive Margin Elasticity | | Partial Trade Elasticity, $\eta_{ijk}$ | |
|---|---|---|---|---|
| Measure | Mean | Std. Dev. | Mean | Std. Dev. |
| *Panel A: Estimates of trade elasticity* | | | | |
| Price based [a] | 0.03 | 0.13 | 2.88 | 3.51 |
| Sales based[b] | $1.7 \cdot 10^{-4}$ | $8.8 \cdot 10^{-4}$ | 3.82 | 3.86 |
| *Panel B: Amplification effect* | | | | |
| Amplification effect [c] | $1.1 \cdot 10^{4}$ | $5.1 \cdot 10^{4}$ | 1.03 | 0.15 |

[a] The summary statistics are reported across 109 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[b] The sales based measure of the trade elasticity is based on a model with complete information. The summary statistics are reported across 124 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[c] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 77 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures.