Development, Volatility and Intertemporal Distortions

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Abstract

Why is macroeconomic volatility in poor countries so much higher than in rich countries? And to what extent can policy improve welfare by mitigating this volatility? In this paper I evaluate the extent to which frictions in financial markets generate more volatile macroeconomic outcomes in poor countries. I motivate my focus on financial frictions with a business cycle accounting exercise using a diverse sample of 89 countries. I identify sources of the empirical relationship between output levels and the volatility of output, consumption and investment. I find that a relevant model of cross country volatility should give a central role to distortions affecting capital accumulation. Accordingly, this paper presents a two-sector stochastic growth model in which firms face constraints to accessing external funds. Firms must collateralize the value of their capital stock against debt issuances. Because contract enforcement is more costly in poor countries, lenders extend less credit and firms are more debt constrained. When firms are debt constrained, the price of capital amplifies the effect of productivity shocks and generates an inefficiently large reallocation of capital across sectors. Consequently, poor countries exhibit higher volatility of investment, output and consumption. To quantitatively evaluate the mechanism, I calibrate model parameters to cross country average data. The only source of cross country heterogeneity is the steady state level of productivity. The model is consistent with the cross-country relationship between income and volatility, suggesting that differences in productivity levels can account for differences in volatility. I then compute constrained efficient allocations in which a social planner understands the effects of investment decisions on the price. Constrained efficient allocations generate a smaller correlation between income levels and volatility, reflecting a disproportionately large reduction in volatility for poor countries; and, lastly, yield large welfare improvements in poor countries, which can be mostly attributed to decreases in consumption volatility.

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1 Introduction

In the literature on economic development there is a central relationship, due to Lucas (1988), between cross-country output levels and volatility of output growth rates: "within the advanced countries, growth rates tend to be very stable over long periods of time," where within poor countries "there are many examples of sudden, large changes in growth rates, both up and down." Understanding output volatility is important because volatility has been shown to have a negative effect on long-run growth (Ramey and Ramey (1995)). Accordingly, an understanding of the sources of output volatility is crucial for the design of development promoting policies.

In this paper, I undertake the dual objective of understanding why poor countries are so much more volatile than rich countries, not only in terms of output but also with respect to consumption and investment volatility, and measuring the extent to which government policy can improve welfare by mitigating volatility. First I perform an accounting exercise to provide an explanation for the cross country income-volatility relationship. I then use the results of this accounting exercise to discipline the construction of a model that is quantitatively capable of reproducing the main features of the cross country data. The model features financial frictions that impede capital accumulation and risk sharing. I use this model to perform policy experiments that allow me to compute the welfare gains from implementing optimal government policy.

In accounting for the income-volatility relationship at the macroeconomic level, I find that cross-country data point to two predominant sources of variation. First is the volatility in productivity growth rates. The volatility in productivity, measured as the Solow residual, has a strong positive relationship with volatility in output and furthermore declines as a function of output levels. I show that cross country differences in productivity account well for the variation in GDP volatility, but are not sufficient to additionally account for consumption and investment volatility. Therefore, I must introduce a second source of variation: variation in the effectiveness with which economies intertemporally substitute resources. From the perspective of Neoclassical theory, such variation arises as a result of distortions that impede capital accumulation or the use of a savings technology. This finding is reminiscent of a classic result from the empirical literature on cross-country income differences: investment rates are highly correlated with income and the relationship is very robust to empirical specifications. This accounting exercise draws heavily on previous contributions from Hevia (2009) and Lama (2011), who extend the Business Cycle Accounting methodology to study external shocks in emerging markets. One of this paper's contribution is to use the accounting framework to study cross country relationships.

A candidate theory for the cross country relationship between income and volatility, therefore, must incorporate primitives and frictions that induce variation in both productivity and rates of intertemporal substitution. The standard macroeconomic model with these features employs frictional financial markets to distort capital accumulation within firms, e.g. Kiyotaki and Moore (1997). Furthermore, two sets of empirical observations complement this suggestion for a candidate theory. First, financial crises are a pervasive feature of modern economies. Reinhart and Rogoff (2009) document over 400 instances of economic crises for a diverse sample of 70 countries, particularly ranging from poor to rich countries. During these crises, output falls over nine percent on average and employment falls seven percent on average, constituting a large amount of average macroeconomic volatility. Furthermore, Ranciere, Tornell, and Westermann (2008) provide evidence that countries that experience financial crises tend to exhibit above average growth in output during their recoveries. This is the so called "Boom-Bust Cycle" that is prevalent in emerging economies. Accordingly, the data suggest that a large class of episodes in which output volatility is high can be, in part, attributed to financial distortions.

Second, how well financial intermediaries such as banks are able to pool and diversify firm level risks systematically varies across countries. Using firm-level data, Larrain (2006) argues that economies in which firms have a larger aggregate debt obligation - to finance investments and operations - tend to have lower aggregate output volatility. His empirical work suggests that a two standard deviation (roughly 50 percent) increase in outstanding credit claims as a fraction of GDP reduces aggregate volatility by at least 25 percent. As a root cause, firms in these economies tend to better pool and diversify idiosyncratic risks, and utilize more short term debt to smooth output.

Motivated by the results of the accounting exercise and the above empirical observations, which suggest financial disturbances are seemingly capable of generating large amounts of volatility, I develop an otherwise standard two-sector growth model in which debt constraints limit credit availability to firms. Two main ingredients allow the model to generate larger volatility in poor countries than in rich countries: endogenous prices and a non-homotheticity operating through the debt constraints.

In the model, firms face debt constraints because they cannot commit to their repay debt to lenders. Prices enter debt constraints because firms must collateralize the *value* of their productive capital against debt issuance. Individual firms take prices as given and do not internalize how their own investment and borrowing decisions affect those prices and subsequently the tightness of debt constraints. In other words, this economy features a pecuniary externality:[1] when debt constraints bind, leveraged firms' cannot pay their existing liabilities and must sell their capital in order to satisfy their budget and debt constraints. During the sale, capital prices become volatile as they adjust to clear capital markets, thereby inducing investment and output volatility. I show that, as a result of the pecuniary externality, firms' debt issuance and the amount of price volatility are both inefficiently high.

The second ingredient, a non-homothetic debt constraint, implies that the cost of financial intermediation is larger in poor countries. The higher cost induces a wealth effect that results in less overall lending and, consequently, firms in poor economies face more credit rationing. But because binding debt constraints generate large price adjustments, the upshot is that poor economies exhibit more macroeconomic volatility than do rich economies.

I quantitatively evaluate the mechanism using cross country data. To discipline the exercise, I minimize the use of country specific parameterization in the quantitative exercise.[2] In particular, only the steady state level of aggregate productivity varies across countries in the model. All other parameters are set the same value across countries, including those governing stochastic processes and financial constraints.

I find that the calibrated model is consistent with the negative relationships between income and aggregate volatility. The result suggests that, in the presence of financial frictions, cross country variation in steady state productivity is sufficient to generate the pattern of volatility that we observe in the data.

[1] In the context of the current environment, this type of externality was first analyzed by Kehoe and Levine (1993). See Lorenzoni (2008) for more recent work on the connection between the externality and aggregate volatility.

[2] A large amount of the literature on growth and financial frictions performs comparative statics with respect to a sole parameter that governs the tightness of borrowing constraints. Differences in this parameter are meant to proxy for a host of country-specific determinants of credit availability. One problematic aspect of such an exercise is that without empirically identifying a source of cross-country differences in credit claims, it is difficult to know whether such a parameter is invariant to changes in the model environment. The goal here is to propose an endogenous economic mechanism in which credit availability does depend on the economic environment, in a way that is consistent with the observed cross-country data.

Finally, due to the presence of the pecuniary externality, government policy can play a role in mitigating the effect distortions have on volatility. Constrained efficient allocations can be implemented as a tax on capital investment, which lowers firms' incentive to lever and mitigates the volatility associated with binding debt constraints. I find that constrained efficient allocations exhibit lower volatility, especially in poor countries. Furthermore, welfare improvements associated with constrained efficient outcomes relative to competitive outcomes are largely attributable to lower consumption volatility.

The paper is organized as follows. The remainder of this section relates this work to the broader literature. Section 2 presents the relevant empirical observations and presents the accounting exercise. Section 3 describes the model and characterizes the competitive equilibrium and constrained efficient allocations. Section 4 details the quantitative results. Section 5 concludes.

Related Literature This paper is related to a large literature on the role of financial frictions in amplifying an propagating macroeconomic disturbances. The present focus on endogenous market incompleteness draws from Kehoe and Levine's (1993) seminal contribution, as well as subsequent work by Alvarez and Jermann (2000) and Albuquerque and Hopenhayn (2004). Most relevant to this paper is Rampini and Viswanathan (2012), who provide conditions under which collateral constraints decentralize a long-term, optimal contract between lenders and firms who cannot commit to repay debt.

This paper also draws upon ideas contained in another vein of the financial frictions literature, which studies the role of pecuniary externalities in amplifying macroeconomic shocks. This paper quantifies a theoretical mechanism that is close to that in Lorenzoni (2008). In other quantitative work, Bianchi (2011) calibrates a model with pecuniary externalities and exogenous restrictions on asset trade. In his framework, agents can only hold non-contingent bonds, whereas in this paper debt is state-contingent and debt constraints are the endogenous outcome of an underlying limited commitment friction.

This paper is related to the literature on cross country macroeconomic disparities. In particular, this paper pertains to recent work that infers sources of cross country productivity differences from price data. Hsieh and Klenow (2007) argue that cross country differences in sectoral steady state productivities, not investment and capital distortions, generate cross country differences in investment rates. This paper complements their work by studying the implications of steady state productivity differences for macroeconomic volatility.

In the same literature, Castro, Clementi, and MacDonald (2009) prove that productivity differences across countries arise from differences in financial frictions, thereby providing and endogenous mechanism to explain Hsieh and Klenow's (2007) findings. For Castro et al. (2009), differences in the variance of sectoral productivity shocks and cross country differences in financial constraint parameters generate productivity differences. This paper takes a similar approach, by also allowing sectoral shock processes to differ. However, this paper studies cross country volatility and does so without assume differences in financial constraint parameters - either across sectors or across countries. Instead, this paper demonstrates that differences in steady state productivity can endogenously generate variation in the strength of financial frictions.

In a related paper, Buera, Kaboski, and Shin (2011) construct a quantitative two-sector model that also studies the effect of collateral constraints, although their paper focuses on long-run implications. The current paper studies an environment with aggregate uncertainty that is meant to study time series data.

Lastly, this paper follows Greenwood and Jovanovic (1990), Bencivenga and Smith (1991) and Acemoglu and Zilibotti (1997) in modeling wealth effects induced by the cost of financial intermediation. In their papers, agents face fixed costs of intermediation (or for financial investment) that introduce a non-homotheticity in agents' budget sets. This non-homotheticity generates differences in income volatility for economies with different levels of development. This paper applies their basic idea to a model of endogenous borrowing constraints.

2 Empirical Accounting Exercise

In this section I describe the accounting exercise that I use to identify the sources volatility in rich versus poor countries. I extend the Business Cycle Accounting framework of Chari, Kehoe, and McGrattan (2007) to a small open economy following recent contributions by Hevia (2009) and Lama (2011). Relative to this previous work, my main contribution is to study a large number of countries, 89 in total, to decompose cross country relationships as opposed to movements in a single country's time-series data. The Business Cycle Accounting framework allows me to measure the extent to which Neoclassical theory deviates from empirical observations. I will quantify the importance of each of these measured deviations, or "wedges," in generating the observed relationships between volatility and income.

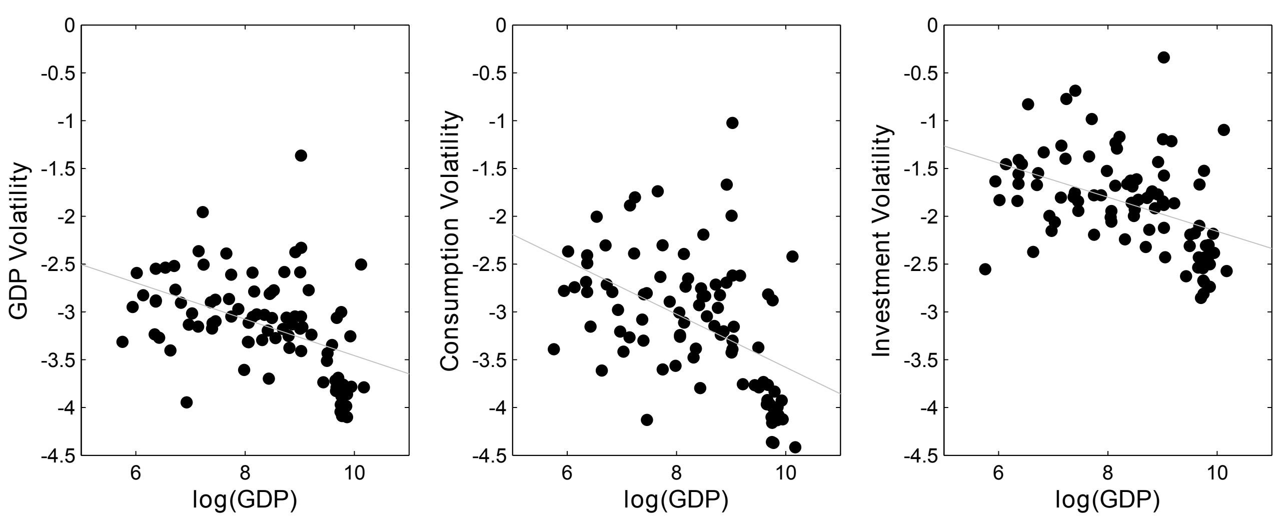
In the remainder of this section, I will first provide empirical evidence on the relationship between income and volatility. Given the empirical observations, I then present the model that I use to measure wedges in the data. I will then describe how to measure wedges using an estimation procedure. Lastly I present results from the procedure.

2.1 Data on Income and Volatility

To motivate the research question, this section presents evidence on the relationship between income and volatility. I consider 89 countries from the Penn World Tables v7 to construct the following measures of income and volatility over the period 1970-2009.[3] I study data on real GDP per capita, real consumption per capita and real investment per capita. All variables are deflated using purchasing-power parity (PPP) from the Penn World Tables v7.

Figure 1 provides scatter plots of the relationship between income (x-axis) and the volatility of GDP, consumption and investment, respectively (y-axis). Linear regression lines are plotted. The volatility measure in Figure 1 is constructed as the log of the standard

Figure 1: Cross Country Relationship Between Income and Volatility



deviation of growth rates. For example, GDP volatility in country i is constructed as:

$$\text{Volatility} \equiv \frac{1}{2} \log \left(\frac{1}{40} \sum_{t=1970}^{2009} \left(\frac{GDP_{i,t-1} - GDP_{i,t-1}}{GDP_{i,t-1}} - \mathbb{E}_t \left[\frac{GDP_{i,t-1} - GDP_{i,t-1}}{GDP_{i,t-1}} \right] \right)^2 \right)$$

[3]I further explain data sources and issues in the next section, page 12.

I use the time series average of $\log(GDP)$ as the income measure. The partial correlations between income and volatility are presented in Table 1. Table 1 presents the coefficients from regressing measures of volatility on $\log(GDP)$. The "time-aggregation" presented in the table corresponds to computing a country's income and volatility over non-overlapping time periods. For example, 10-year aggregation refers to computing $\log(GDP)$ averaged over 1970-1979, 1980-1989, 1990-1999 and 2000-2009. Likewise, the volatility measure is computed over these non-overlapping decades. I perform this regression with and without country-fixed effects, labeled as "OLS" and "FE" respectively. Standard errors are in parenthesis.

Table 1: Cross Country Relationship Between Income and Volatility

Dependent Variable	5-Year Aggregation OLS	5-Year Aggregation FE	10-Year Aggregation OLS	10-Year Aggregation FE	40-Year Aggregation OLS
GDP Volatility	-0.18***	-0.73***	-0.16***	-0.60**	-0.19***
	(0.03)	(0.24)	(0.03)	(0.28)	(0.04)
Consumption Volatility	-0.28***	-0.38	-0.26***	-0.38	-0.28***
	(0.03)	(0.25)	(0.03)	(0.30)	(0.05)
Investment Volatility	-0.12***	-0.79***	-0.10***	-1.29***	-0.18***
	(0.03)	(0.28)	(0.03)	(0.35)	(0.04)
Observations	712	712	356	356	89

Independent Variable: $\log(GDP)$ averaged over corresponding aggregation period.
*** significant at 1%; ** significant at 5%; * significant at 10%.

I infer the following stylized facts from the figures and tables.

1. GDP, consumption and investment volatility are negatively related to income;

2. the negative relationships are robust to different time aggregations and the inclusion of country fixed effects;[4]

3. the negative relationships are significant at the 1% level in nearly all cases;

4. investment volatility is larger than GDP and consumption volatility; and

5. for the 40-year time aggregation, between the poorest and richest country GDP is 2.3 times more volatile, consumption is 3.4 times more volatile and investment is 2.2 times more volatile.[5]

[4]The fixed effects regression coefficient shows that countries exhibit a negative relationship between GDP growth and changes in volatility. This result is reminiscent of Ramey and Ramey (1995), who argue that income volatility causes a negative impact on income growth.

[5]These numbers are constructed from the predicted values of the OLS regression at a 40-year aggrega-

2.2 Environment

I consider a small open economy version of an otherwise standard one-sector growth model. The model is populated by a continuum of identical households and a continuum of identical firms.

Firms operate a Cobb-Douglas production technology that exhibits labor augmenting productivity growth $(1+g_z)$ and shocks z_t:

$$y_t = k_t^{\alpha}((1+g_z)^t z_t l_t)^{1-\alpha}$$

and choose capital and labor $\{k_t, l_t\}_{t=0}^{\infty}$ to maximize profits, $y_t - w_t l_t - r_t k_t$, each period.

Given an initial allocation of wealth (k_0, b_0), the representative household chooses consumption, investment and bond holdings $\{c_t, x_t, b_{t+1}\}_{t=0}^{\infty}$ to maximize its discounted expected utility, with discount factor $\beta < 1$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \psi \log(1-l_t)\right] N_t$$

where N_t is the measure of households. I assume that a constant growth rate for population:

$$N_{t+1} = (1+g_n)N_t$$

A household's utility maximization is subject to a budget constraint

$$c_t + (1+\tau_{xt})x_t + (1+g_n)(1+g_z)Q_t b_{t+1} + g_t \leq (1-\tau_{lt})w_t l_t + r_t k_t + b_t + T_t$$

and a law of motion for capital (denote capital depreciation by δ)

$$(1+g_n)(1+g_z)k_{t+1} = (1-\delta)k_t + x_t$$

where households take prices $\{w_t, r_t, Q_t\}_{t=0}^{\infty}$ and taxes $\{\tau_{xt}, \tau_{lt}, T_t\}_{t=0}^{\infty}$ as given. Above, I have detrended individual variables by population and productivity growth.

I must make an additional assumption on Q_t in order to ensure stationarity of equilibrium. Following Schmitt-Grohe and Uribe (2003) I assume that $Q_t = q_t(R + \nu(\exp(b_{t+1} -$

tion. For example, since the highest value of $\log(GDP)$ is 10.2 and the lowest value is 5.8, then for the regression of GDP volatility on $\log(GDP)$ a coefficient of -0.19 yields a ratio of predicted values: $\exp(-0.19 \cdot 5.8) / \exp(-0.19 \cdot 10.2) = -2.3$.

$b_{ss}) - 1))^{-1}$, where R is the world interest rate, b_{ss} is the steady state level of bond holdings and q_t is a random variable. This specification ensures stationarity by allowing the price Q_t to increase with the quantity of bond holdings. However, the parameter ν is set to approximately zero so that bond holdings have a negligible impact on the price while still inducing stationarity.

Equilibrium Conditions I impose an additional condition: the government finances its consumption, g_t, and transfers, T_t, by collecting taxes, $\{\tau_{xt}, \tau_{lt}\}$ and must satisfy a balanced budget condition:

$$g_t + T_t = \tau_{xt} x_t + \tau_{lt} w_t l_t$$

Accordingly, the resource constraint for this economy is:

$$c_t + x_t + (b_t - (1 + g_n)(1 + g_z) Q_t b_{t+1}) + g_t = y_t$$

Equilibrium conditions are summarized by the resource constraint, the household's intratemporal condition,

$$\frac{\psi c_t}{1 - l_t} = (1 - \tau_{lt})(1 - \alpha) \frac{y_t}{l_t}$$

Euler equation for investment,

$$(1 + \tau_{xt}) = \frac{\beta}{1 + g_z} \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)(1 + \tau_{xt+1}) \right) \right]$$

and Euler equation for bond holdings[6]

$$q_t \approx \frac{\beta R}{1 + g_z} \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \right]$$

Constructing Wedges For each of these equilibrium conditions I construct a corresponding wedge that measures the extent to which the condition is violated by the data. Specifically, I construct sequences of wedges $\{z_t, \tau_{lt}, \tau_{xt}, q_t, g_t\}_{t=1970}^{2009}$ from the time series national accounts data. Using national accounts data, time series for $\{z_t, \tau_{lt}\}$ can be constructed directly from the production function and the intratemporal condition respectively.

Let a "d" superscript denote data. Then the wedge constructs will satisfy the following

[6] The bond wedge can be thought of as shocks to the interest rate on debt, consistently with Neumeyer and Perri (2005). These authors provide evidence of a strong relationship between interest rates and business cycle volatility in emerging economies, and provide a model in which interest rate shocks account for a third of Argentinian output volatility.

conditions for each t:

$$z_t^d \equiv \left(\frac{y_t^d}{(k_t^d)^\alpha (l_t^d)^{1-\alpha}} \right)^{1/(1-\alpha)}$$

$$(1-\tau_{lt}^d) \equiv \frac{\psi}{(1-\alpha)} \cdot \frac{l_t^d}{1-l_t^d} \cdot \frac{c_t^d}{y_t^d}$$

$$g_t^d \equiv y_t^d - c_t^d - x_t^d - nx_t^d$$

where nx_t^d is net exports data and capital data is constructed using the perpetual inventory method, $k_{t+1} = (1-\delta)k_t + x_t^d$. The initial allocation of capital and bond holdings (e.g. at $t = 1970$) are set to the model's steady state values.

The remaining investment and bond wedges cannot be constructed directly from equilibrium conditions. Instead, I compute policy functions and construct a sequence for $\{\tau_{xt}^d, q_t^d\}_{t=1970}^{2009}$ that ensures the policy functions for investment and bond holdings match the corresponding data on investment and net exports, respectively:

$$x_t^d = x(k_t^d, b_t^d, s_t^d)$$

$$q_t^d \approx \frac{R}{(1+g_n)(1+g_z)} \frac{b_t^d - nx_t^d}{b(k_t^d, b_t^d, s_t^d)}$$

where $s_t^d \equiv (z_t^d, \tau_{lt}^d, \tau_{xt}^d, q_t^d, g_t^d)$, and $x(\cdot)$ and $b(\cdot)$ are the policy functions for investment and bond holdings.

2.3 Decomposition

When fed into the model simultaneously, these constructed wedges ensure that the model *exactly* reproduces the consumption, investment, net export, output and labor data. Therefore this methodology decomposes the time series data for $\{c_t^d, x_t^d, nx_t^d, y_t^d, l_t^d\}_{t=1970}^{2009}$ into the contribution of the five wedges $\{z_t^d, \tau_{lt}^d, \tau_{xt}^d, q_t^d, g_t^d\}_{t=1970}^{2009}$.

Counterfactual Exercises I will decompose the cross country relationship between income and volatility into the contribution of the wedges. To do this I will conduct a series of counterfactual exercises in which I feed a subset of the wedges into the model (setting the remaining wedges to their steady state values) and compute the counterfactual equilibrium allocation. I repeat this counterfactual exercise for each country until I have a panel of counterfactual data, corresponding to the allocations implied by a feeding a particular subset of wedges into the model. Using this panel of counterfactual data, I can then compute the cross country correlation between income and volatility.

After computing the income-volatility correlations implied by each subset of wedges, I find the subset that best matches the actual data - in the sense of minimizing a statistic of model fit.

Data The data on consumption, investment, net exports, output and population come from the Penn World Tables v7. Data on number of employed workers comes from the International Labor Organization. Data on the ratio of foreign bonds to GDP comes from Lane and Milesi-Ferretti (2007). The sample consists of 89 countries,[7] ranging from sub-Saharan African countries to G8 countries. A country was included in the sample if it had no missing data over the 1970-2009 period, for any of the three data sets. The data are observed at an annual frequency. Prior to computation, all data were depopulated and all NIPA data from the Penn World Tables were additionally deflated by the PPP price of consumption goods and detrended by the linear trend of the Solow residual.

Calibration and Estimation The model has six parameters that are set in advance and are the same for each country in the sample:

Parameters	β	α	δ	ψ	R	ν
Values	0.94	0.33	0.10	2.24	1.02	0.0001

The parameters for (α, δ, β) are standard in the real business cycle literature. The parameter for δ corresponds to a 2.5% depreciation rate at a quarterly frequency, which is a standard value for the US. Although the parameters for the world interest rate and utility of leisure (R, ψ) are chosen to be standard values, the particular values are unimportant. This is because (R, ψ) will affect the levels for $\{q_t, \tau_{lt}\}$ but not the growth rates, which are the object of interest in this exercise.

Growth rates (g_n, g_z) were obtained as linear trends of the population data and the constructed Solow residual, respectively. Following Lama (2011), I pin down the steady state value of bond holdings using the mean ratio of foreign bonds to GDP from Lane and Milesi-Ferretti (2007).

Following Chari et al. (2007), I assume that the wedges follow a vector autoregressive

[7] A full list of countries is contained in Appendix A.

process of the form:

$$\log(s_{t+1}) = \log(\bar{s}) + P\log(s_t) + \epsilon_{t+1}$$

where $\log(s_t) \equiv (\log z_t, \tau_{lt}, \tau_{xt}, \log q_t, \log g_t)$, $\bar{s}$ is the vector of mean values for s_t, P is a matrix that describes the dependence of shocks on lagged values and ϵ_{t+1} is a vector of iid Gaussian innovations with mean zero and covariance matrix V.

Again I follow Chari et al. (2007) by using a maximum likelihood procedure to estimate $(\bar{s}, P, V)$ that maximizes the likelihood that the model generates the data on consumption, investment, net exports, output and employment. The solution to the model is computed using first-order linear approximation.[8]

Results Table 2 presents a summary of results from the accounting exercise. A full table of results can be found in Appendix B.

Table 2: Counterfactual Correlations (VAR Process)

	Correlations[a]			% Deviations[b]			RMSE(θ)
θ	GDP	CON	INV	GDP	CON	INV	
Data	-0.44	-0.48	-0.40	—	—	—	—
z_t	-0.52	-0.08	-0.10	118	17	24	0.29
z_t, q_t	-0.47	-0.29	-0.25	106	61	64	0.14
z_t, q_t, τ_{lt}	-0.41	-0.45	-0.10	93	94	26	0.17
z_t, τ_{xt}, q_t	-0.43	-0.13	-0.29	98	26	72	0.21

(a) Correlations are between average $\log(GDP)$ and the log of the standard deviation of GDP growth, consumption growth and investment growth in each country.

(b) Percent deviations between model correlation and data correlation.

Table 2 presents the income-volatility correlations implied by feeding particular wedges into the growth model. The row labeled "Data" presents the income-volatility correlations as measured from the data sample. Rows three through six indicate the subset of wedges fed into the model for computing counterfactual correlations. Columns two through four present correlations from the data (row 2) and from a subset of model specifications (rows 3 - 6). Columns five through seven present the deviation of the counterfactual correlation from the data. The last column presents a statistic of model fit, the root mean squared error (RMSE)[9], which is constructed as the square root of

[8]Some experimentation with second-order linear approximation appears to give similar results. Future work will present the results of this exercise with higher-order linear approximations.

[9]Note that if I change the measure of model fit to "mean absolute errors," the order does not change. In

the average squared deviation of the counterfactual correlations from correlations in the data. Mechanically, if θ is defined as the subset of wedges fed into the model and $\{v_{it}(\theta)\}_{t=1970}^{2009}$ is the time series data generated from a model with θ-wedges, then the model correlation is given by:

$$\kappa(GDP_{it}(\theta), v_{it}(\theta)) \equiv corr_i\left(\mathbb{E}_t[\log(GDP_{it}(\theta))], \log\left(std_t\left(1 - v_{it}(\theta)/v_{it-1}(\theta)\right)\right)\right)$$

$$v \in \{CON, INV, GDP\}$$

and the root mean squared error is constructed as:

$$RMSE(\theta) = \sqrt{\sum_{v \in \{CON, INV, GDP\}} \left(\kappa(GDP_{it}(\theta), v_{it}(\theta)) - \kappa(GDP_{it}^{data}, v_{it}^{data})\right)^2}$$

First take the row labeled z_t. This row shows the income-volatility correlations for panel data generated by a model with only productivity fed in. Of all the single-wedge specifications, productivity has the lowest RMSE. Furthermore, the productivity-only specification does remarkably well at explaining GDP volatility across countries. As can be seen in the Appendix Table (section B), the productivity-only specification is the only single-wedge specification to generate the correct negative sign on all three correlations. However, the productivity-only specification is less successful at explaining consumption and investment volatility across countries: it can only explain about $20 - 25\%$ of these additional correlations. For this reason, I turn to a two-wedge specification to see if I can improve the overall model fit.

If a productivity wedge is the best-fitting wedge across single-wedge specifications, then what additional wedge can improve overall model fit? I select a two-wedge specification that includes productivity and provides the largest improvement in the RMSE. The z_t, q_t two-wedge specification satisfies these criteria. The row labeled z_t, q_t in Table 2 presents the correlations and RMSE for this two-wedge specification. Relative to the productivity-only specification, adding the bond wedge yields only a slight improvement in explaining GDP volatility, but more than doubles the explanatory power for consumption and investment volatility (to around 63%). On one hand, the result suggests that productivity alone accounts well for GDP and including a second wedge does

fact the two measures produce very similar values. Since the RMSE is a quadratic scoring rule, models with large deviations from the data are penalized more heavily than in the mean absolute error criterion. The fact that the two criteria do not yield very different outcomes suggests that models have relatively small errors.

not help on that dimension. On the other hand, including a second wedge does in fact improve the model fit.

In the last two rows of Table 2, I show that adding a third wedge to the (z_t, q_t) specification does not improve model fit. In the second to last row, adding a labor wedge, τ_l, worsens the RMSE error slightly. However, it is interesting that the addition yields a very close fit to the consumption volatility correlation on top of the GDP volatility correlation. But this improvement in the consumption volatility correlation comes at the cost of lowering the investment volatility correlation, hence the worse RMSE. Finally, in the last row, adding an investment wedge, τ_x, also worsens the RMSE. (The last row was added for comparison to a robustness exercise in the next section.) Interestingly, adding the investment wedge generates a better fit for the investment volatility correlation at the cost of the consumption volatility correlation - the opposite effect of adding a labor wedge.

Robustness To check whether to results are sensitive to assumptions on the stochastic process for wedges, I redo the exercise with a restriction on the P vector. I now assume that wedges follow a correlated first-order autoregressive process, so that P is restricted to be a diagonal matrix. A summary of the results are presented in Table 3.

Table 3: Counterfactual Correlations (AR1 Process)

	Correlations[a]			Deviations[b]			RMSE(θ)
θ	GDP	CON	INV	GDP	CON	INV	
Data	-0.44	-0.48	-0.40	—	—	—	—
z_t	-0.86	-0.66	-0.58	195	137	147	0.28
z_t, τ_{xt}	-0.55	-0.60	0.08	124	125	-21	0.29
z_t, τ_{xt}, q_t	-0.72	-0.74	-0.54	164	154	135	0.23
$z_t, \tau_{xt}, \tau_{lt}$	-0.64	-0.78	0.19	145	163	-49	0.40

(a) Correlations are between average $\log(GDP)$ and the log of the standard deviation of GDP growth, consumption growth and investment growth in each country.

(b) Percent deviations between model correlation and data correlation.

The overall message of this robustness exercise is that the main results of the previous section do not substantially change. In Table 3, first notice that the RMSE are generally larger (also see the full table in Appendix B). This is the result of restricting P: it should not be surprising that an additional restriction on stochastic process estimates yields an overall worse model fit. In particular, this restriction causes the model to over predict most correlations.

Choosing among single-wedge specifications, as before, the productivity-only specification yields the best RMSE. If we add a second wedge, now the productivity plus investment wedge instead of the productivity plus bond wedge specification provides the best RMSE among two-wedge specifications. Note that the second lowest RMSE among two-wedge specifications is the productivity and bond wedge specification. Among three-wedge specifications, the specification with the overall lowest RMSE includes productivity, investment and bond wedges. If instead we were to add a labor wedge, which is shown in the last row of Table 3, we see that the RMSE is substantially worse relative to both the (z, τ_x) and (z, τ_x, q) specifications. In the previous exercise, in which P was unrestricted, adding a labor wedge yielded a good fit to the data. However, changing the specification for P undoes that result. What remains is that some combination of productivity and intertemporal wedges (investment and bond wedges) robustly yield the best model fit across the two specifications of stochastic processes.

Discussion I will examine the data through the lens of a model, in order to elucidate the decomposition results and to perform policy experiments. The results of the accounting exercise inform theory by disciplining the class of viable models that can explain patterns of cross country volatility. In order for a model to explain the data, it must induce the same wedges that we observe empirically. To this end, I will consider a parsimonious representation of the data by selecting a subset of wedges.

The results suggest that cross country differences in productivity (z_{it}) are the single-most important economic margin, especially for explaining the the relationship between income and GDP volatility. However, adding intertemporal distortions - either investment or bond wedges - provide the largest value added to improving the explanatory power of the model. This is because intertemporal wedges bring the growth model closer to matching the relationship between income and both consumption and investment volatility. In the VAR specification, adding a bond wedge improves the RMSE by fifteen points - which is the largest improvement out of any alternative specification with two wedges. In the AR(1) specification, adding both intertemporal wedges improves the model more than any other three-wedge or two-wedge specification. Overall, across the two different specifications of the stochastic processes governing wedges, a combination of productivity and intertemporal wedges generate the lowest RMSE's in a parsimonious way.

Accordingly, for it to be consistent with the data, a theory of cross country volatility differences must incorporate primitive shocks and frictions that induce productivity, investment and bond wedges (z, τ_x, q). As is well known (c.f. Chari, Kehoe, and McGrattan (2005), Hevia (2009)), models with financial frictions in the tradition of Kiyotaki and Moore (1997) or Mendoza (2008) induce investment and bond wedges. Furthermore, distortions that impede the reallocation of capital across firms or sectors are capable of inducing productivity wedges (c.f. Castro, Clementi, and MacDonald (2009)). In the next section, I produce a theory that features these distortions.

3 Two-Sector Growth Model with Financial Frictions

In this section I describe the economic environment. The model integrates endogenously incomplete financial markets into a two-sector neoclassical growth model. I outline the model in steady state and provide a competitive equilibrium definition. I then prove a series of propositions to characterize financial contracts and highlight the key economic mechanisms at work in the model. Lastly, I define and characterize constrained efficient allocations and illustrate how efficient allocations improve outcomes relative to competitive equilibrium allocations.

3.1 Model Environment

The model framework is that of a standard two-sector neoclassical growth model. There are four distinct types of agents: a representative consumer, a representative consumption sector firm, a representative investment sector firm, and a foreign lender. I will first present the consumer's problem. I will then describe the environment in which firms operate, which contains this model's novel features.

Representative Consumer The model is populated by a unit continuum of identical, infinitely-lived consumers with standard preferences over streams of consumption, given by:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t) \right]$$

where $\beta < 1$ is the discount factor. The consumer inelastically supplies his labor endowment (normalized to unity) in exchange for a wage denominated in terms of the consumption good. The consumer can hold two financial assets: ownership shares of consumption sector firms and investment sector firms. An ownership share costs v_{it} and pays a dividend d_{it} in each period, where $i \in \{c, x\}$ denotes the consumption and investment sector firms respectively. The consumer's budget constraint is:

$$c_t + \sum_{i \in \{c,x\}} v_{it} a_{it+1} \leq w_t + \sum_{i \in \{c,x\}} (d_{it} + v_{it}) a_{it}$$

where a_{it} is the consumer's ownership share in each productive sector $i \in \{c, x\}$.

As is standard, consumer ownership of firms implies that the equilibrium price of an ownership share in either sector must equal the consumer's valuation of the firm's stream of dividends:

$$v_{it} = \mathbb{E}_t \sum_{j=t}^{\infty} m_{t+j} d_{it+j}$$

where $m_{t+j} = \beta^j u'(c_{t+j})/u'(c_t)$ is the consumer's stochastic discount factor. Accordingly, I assume that firms' optimization is consistent with consumers' optimization.

Firms In each sector there is a unit continuum of infinitely-lived firms. Firms in each sector are identical. All firms, regardless of the sector they inhabit, are endowed with the same preferences and production technology.

As just discussed, firms are risk neutral and have preferences over dividends subject to a given stochastic discount factor. Firms are subject to limited liability and therefore dividends are non-negative. Firms are endowed with a production technology $F(k, zl)$ that transforms capital k and labor l into a sector specific good. The functional form of the production technology does not differ across sectors. Productivity, z, is sector specific and follows a correlated first-order autoregressive process described by the transition function $\pi(\mathbf{z}, \mathbf{z}')$ on $\mathcal{Z}^2$, where $\mathbf{z} \equiv (z_c, z_x)$ is the cross-sectional distribution of productivity. Productivity is given by $z_i = A_i \exp(\tilde{z}_i)$, $i \in \{c, x\}$. In the expression, $\tilde{z}_i$ is the mean-zero, stochastic component of productivity and A_i is the average level of productivity.

Firms can purchase or sell capital at a price p, which is the relative price of the capital good denominated in terms of the consumption good. Firms take today's price, p and the vector of tomorrow's prices, $p(\mathbf{z}')$, for all $\mathbf{z}' \in \mathcal{Z}^2$, as given. Capital is either purchased from another firm (in either sector) that is selling its existing capital, or an investment sector firm that produces and sells new capital. Any capital that is purchased in period t cannot be used in production or be re-sold until period $t + 1$.

In order to finance dividend payouts, capital purchases or payment of existing liabilities, firms can issue one-period, state-contingent debt, denoted $b_i'(\mathbf{z}')$. At a price $q(\mathbf{z}')$, the firm can borrow a unit of the consumption good and promises to repay a unit tomorrow if state $\mathbf{z}'$ is realized.

Given the above description, the a sector $i \in \{c, x\}$ firm's budget constraint is:

$$d_i + pk_i' - \sum_{\mathbf{z}'} q(\mathbf{z}')b_i'(\mathbf{z}') \leq n_i$$

where n_i is the firm's net worth. Net worth, n_i, is defined as the sum of firm revenues and the value of after-depreciation capital, net of the wage bill and any outstanding debt obligations. Therefore, tomorrow's net worth is defined for consumption sector firms as:

$$n_c'(\mathbf{z}') \equiv F(k_c', z_c' l_c') + (1-\delta)p(\mathbf{z}')k_c' - w'l_c' - b_c'(\mathbf{z}')$$

and for investment sector firms as:

$$n_x'(\mathbf{z}') \equiv p(\mathbf{z}')F(k_x', z_x' l_x') + (1-\delta)p(\mathbf{z}')k_x' - w'l_x' - b_x'(\mathbf{z}')$$

where δ is the depreciation rate on capital, which I assume does not vary across sectors. Notice that since the investment sector produces capital, investment sector firms' revenue depends on the relative price.

Financial Contracts Firms cannot fully commit to their promises: firms may default on their debt repayment. Therefore, lenders must structure financial contracts in a way that provides firms with the incentive to repay. I now describe the financial environment.

Firms can borrow funds from a unit continuum of lenders. Lenders are risk neutral and discount the future with a factor $1/R < \beta$. Lenders have a large endowment of funds at all states and dates. Lenders are able to commit to financial contracts, which consist of state-contingent transfers between the lender and firm. Lenders do not have access to the production technology and are therefore willing to trade any set of state-contingent claims as long as their expected rate of return is equal to R and firms credibly repay their debt.

However, firms can default on their debt obligations by not repaying debt. In this case, firms privately consuming all current revenues and keep the non-collateralized fraction, $1-\theta$, of their capital stock. In order to repossess the remaining fraction θ of the capital stock, a lender must pay a fixed cost ξ that can be interpreted as a variable haircut (e.g. the larger the value of collateral, the smaller the percentage discount). After default, firms cannot be excluded from future financial contracts and therefore can issue debt at future dates and states.

As a result of firms' lack of commitment in upholding promised repayments, lenders are not willing to provide financing in excess of the value of collateralized capital less the fixed cost of accessing their claim to collateralized capital. Therefore, the commitment problem gives rise to the following sequence of debt constraints:

$$b_i'(\mathbf{z}') \leq \max\{0, \theta(1-\delta)p(\mathbf{z}')k_i' - \xi\}$$

Following Rampini and Viswanathan (2012), I can equivalently write debt constraints as enforcement constraints that restrict long-term, state-contingent contracts. Such enforcement constraints ensure optimal contracts provide firms with the incentive to repay its outstanding debt. However, their proof requires slight modification in this environment because of the presence of the fixed cost, ξ.

Proposition 1 (Endogenous Debt Constraints): A firm's allocation consisting of dividends, $d(n,z,p)$, capital capital purchase, $k'(n,z,p)$, and debt issuance, $b'(n,z,p;\mathbf{z}')$, satisfies the debt constraint $b(\mathbf{z}') \leq \theta(1-\delta)p(\mathbf{z}')k' - \xi$ if and only if the allocation satisfies enforcement constraints given by:

$$v(n,z,p) \geq \hat{v}(\hat{n},z,p) \equiv \hat{d}(n,z,p) + \sum_{\mathbf{z}'} \pi(\mathbf{z},\mathbf{z}')m(\mathbf{z},\mathbf{z}')\hat{v}(n'(\mathbf{z}'),\mathbf{z}',p(\mathbf{z}'))$$

where $\hat{v}(\cdot)$ is the firm's value after defaulting on debt repayment, $\hat{n} = n - \theta(1-\delta)pk$ is net worth after collateralized capital seizure and $\{\hat{d},\hat{k}',\hat{b}'(\mathbf{z}')\}$ is the post-default allocation.

Proof: The proof for Proposition 1 is contained in Appendix C.1.

Formulation of Firm's Problem A firm's state is its net worth, the distribution of sectoral productivity shocks and today's price of capital: $s_i \equiv (n_i, \mathbf{z}, p)$. The aggregate state is then $s \equiv (s_c, s_x)$. I denote by $\Gamma(\cdot)$ firms' forecast of future prices from the current aggregate state s.

The cum-dividend value of the firm is:

$$v_i(n_i,\mathbf{z},p) = \max_{d,k',b(z')} d + \sum_{\mathbf{z}'} \pi(\mathbf{z},\mathbf{z}')m(\mathbf{z},\mathbf{z}')v_i(n_i'(\mathbf{z}_i'),\mathbf{z}',p(\mathbf{z}'))$$

subject to limited liability, $d \geq 0$, the consumer's stochastic discount factor, $m(\mathbf{z}, \mathbf{z}')$, the budget constraint

$$d_i + pk_i' - \sum_{\mathbf{z}'} q(\mathbf{z}') b_i'(\mathbf{z}') \leq n_i$$

the debt constraint

$$b_i'(\mathbf{z}') \leq \max\{0, \theta(1-\delta)p(\mathbf{z}')k_i' - \xi\}$$

and the law of motion for prices

$$p(\mathbf{z}') = \Gamma(s, \mathbf{z}')$$

Equilibrium Before defining a competitive equilibrium, I present market clearing conditions. First, ownership shares in the firm sum to one, e.g. $a_{ct} = a_{xt} = 1$ for all t. Next, the consumption goods market clearing condition is given by:

$$c + \sum_{i \in \{c,x\}} \left(b_i - \sum_{z'} q(\mathbf{z}') b_i(\mathbf{z}') \right) = F(k_c, z_c l_c)$$

As in a standard decentralization of the two-sector growth model (c.f. Danthine and Donaldson (2002)), all savings decisions are made by firms. In this environment firms' net debt determines the consumer's aggregate savings (or debt). Because firms borrow from external lenders, aggregate net debt does not necessarily sum to zero.

The capital market clearing condition is given by:

$$\sum_{i \in \{c,x\}} k_i' = (1-\delta) \sum_{i \in \{c,x\}} k_i + F(k_x, z_x l_x)$$

This condition covers cases in which either both sectors demand newly produced capital, or one of the two-sectors sells a portion of its capital stock, $k_i' < (1-\delta)k_i$. Notice that if both sectors wanted to sell a portion of its capital stock, then the equilibrium price must adjust downward to provide at least one-sector with incentives to hold capital.

As in the standard growth model, the labor market clearing condition requires that the quantity of labor demanded by firms equals the inelastic supply by consumers, $l_c + l_x = 1$. Lastly, an arbitrage condition on lending ensures that the price of external financing equals R^{-1} on average. This implies that $q(\mathbf{z}') = R^{-1}\pi(\mathbf{z}, \mathbf{z}')$.

I can now define general equilibrium.

Definition (Equilibrium): A **recursive competitive equilibrium** consists of consumer allocations $\{c(s), a'_c(s), a'_x(s)\}_{s\in\{\mathbb{R}^2_+\times\mathcal{Z}^2\times\mathbb{R}_{++}\}}$ and stochastic discount factor $\{m(s,\mathbf{z}')\}_{s,\mathbf{z}'}$, consumption sector and investment sector firm allocations $\{d_i(s_i), k'_i(s_i), b'_i(s_i,\mathbf{z}')\}_{i,\mathbf{z}'}$ and value functions $\{v_i(s_i)\}_i$ for $i \in \{c, x\}$, prices $\{w(s), p(s), q(s,\mathbf{z}')\}_{\mathbf{z}'}$ and law of motion $\Gamma(s,\mathbf{z}')$ that satisfy:

1) consumer optimization,

2) consumption sector and investment sector firm optimization,

3) consumption goods market clearing, capital market clearing and labor market clearing,

4) lender's arbitrage condition,

5) firm shares equal one, $a_{it} = 1$, for all t, and

6) the perceived law of motion is consistent with the actual law of motion.

3.2 Competitive Equilibrium

In this section I discuss properties of the competitive equilibrium allocation. This section is divided into four parts each corresponding to a separate aspect of the economic mechanism employed by this model. The four parts illustrate how (1) low net worth firms engage in risky debt decisions, (2) the debt constraints distort intertemporal margins, (3) debt constraints distort the allocation of capital across sectors thereby distorting aggregate productivity and (4) prices are essential for generating the a cross country relationship between income and volatility.

Optimal Financial Contracts Optimality conditions for the firm are as follows. The envelope condition implies, $v_n(n,\mathbf{z},p) \geq 1$. The first order condition with respect to state-contingent debt holdings implies:

$$\mu_i(\mathbf{z}') = R^{-1}v^i_n(n_i,\mathbf{z},p) - m(\mathbf{z},\mathbf{z}')v^i_n(n'_i(\mathbf{z}'),\mathbf{z}',p(\mathbf{z}'))$$

where $\mu_i(\cdot)$ is the sector $i \in \{c, x\}$ firm's multiplier on the debt constraint. The Euler equation for firm $i \in \{c, x\}$ is given by:

$$v_n^i(n_i, \mathbf{z}, p) = \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') m(\mathbf{z}, \mathbf{z}') v_n^i(n_i'(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}')) \frac{p_i(\mathbf{z}') F_k(k_i', z_i' l_i') + (1-\theta)(1-\delta) p(\mathbf{z}')}{p - R^{-1}\theta(1-\delta) \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') p(\mathbf{z}')}$$

where $p_i(\mathbf{z}')$ equals one for consumption sector firms and equals $p(\mathbf{z}')$ otherwise.

Rampini and Viswanathan (2012) provide a characterization of a similar economy that does not contain endogenous capital prices or a stochastic discount factor governing firms' preferences. However, much of their characterization is valid in this environment. Characterization of the value function and allocation are as follows.

Proposition 2 (Allocation): Given prices $(p, p(\mathbf{z}'))$ and stochastic discount factor $m(\mathbf{z}, \mathbf{z}')$, the firm's decision rules $\{d_i(n, \mathbf{z}, p), k_i(n, \mathbf{z}, p), n_i'(n, \mathbf{z}, p; \mathbf{z}')\}$ exhibit the following characteristics.

1. For all $(\mathbf{z}, p) \in \mathcal{Z}^2 \times \mathbb{R}_{++}$, $\exists\, \bar{n}(\mathbf{z}, p)$ such that for all $n \geq \bar{n}(\mathbf{z}, p)$, $v_n(n, \mathbf{z}, p) = 1$.
2. Fix $(\mathbf{z}, p)$. For any $n \in \mathbb{R}_+$, $d(n, \mathbf{z}, p) = \max\{0, n - \bar{n}(\mathbf{z}, p)\}$.
3. Fix $(\mathbf{z}, p)$. For all $n < \bar{n}(\mathbf{z}, p)$, $k'(n, \mathbf{z}, p)$ is increasing in n.
4. Fix $(\mathbf{z}, p)$. For all $n \geq \bar{n}(\mathbf{z}, p)$, $k'(n, \mathbf{z}, p) = k'(\bar{n}(\mathbf{z}, p), \mathbf{z}, p)$ and $n'(n, \mathbf{z}, p; \mathbf{z}') = n'(\bar{n}(\mathbf{z}, p), \mathbf{z}, p; \mathbf{z}')$.
5. For all $(\mathbf{z}, p)$, $\exists \underline{n}(\mathbf{z}, p)$ such that for all $n < \underline{n}(\mathbf{z}, p)$, $\mu(\mathbf{z}') > 0$ for all $\mathbf{z}' \in \mathcal{Z}^2$.

Proposition 3 (Value Function): Given $(\mathbf{z}, p) \in \mathcal{Z}^2 \times \mathbb{R}_{++}$, the value function $v(n, \mathbf{z}, p)$ is continuous, strictly increasing and strictly concave for all $n < \bar{n}(\mathbf{z}, p)$.

Proof: These proofs can be found in Rampini and Viswanathan (2012).

The main idea of the characterization is that the debt constraint impedes intertemporal consumption smoothing for firms by limiting the amount of available funds after a positive productivity shock. In the absence of debt constraints, the firm would increase borrowing when it anticipates high future productivity. However debt constraints limit the availability of funds for such an optimal investment. In order to overcome debt constraints in highly productivity states, the firm will issue more debt against low productivity states and thereby increase its overall available funds at the cost of hedging risk in low states.

Most importantly, once net worth becomes sufficiently large, the firm will stop investing in additional capital and start paying out dividends. On the other hand, if net worth is sufficiently small, the firm will maximally borrow against all future states. That is, the firm will not hedge risk by transferring resources from high to low states, as the first order condition with respect to debt suggests is optimal. When net worth is this low, the firm's capital stock is also low. Therefore the firm attains a higher marginal value across all future states by by purchasing an additional unit of capital when its marginal productivity is large than it gains from reducing debt in a low state.

Intertemporal Distortions Financial frictions distort the intertemporal substitution of resources. In the empirical accounting exercise in Section 2, I showed that cross country differences in this margin were key to understanding why poor countries exhibit higher macroeconomic volatility. In order to demonstrate how the present economic environment with financial frictions distorts the intertemporal margin, I will now provide a theoretical mapping from the present environment to the one-sector growth model with "wedges."

To show that the model maps induces an investment wedge, there are two approaches. The more complicated is to guess and verify a functional equation for the investment wedge. The problem with this approach is that the mapping is may not admit a tractable functional form. To make the argument more transparent, I will allow for both an investment wedge and a capital wedge. A capital wedge is effectively a time varying tax on capital income that enters the growth model's capital Euler equation multiplicatively with the marginal product of capital.

PROPOSITION 4 (MAPPING TO INTERTEMPORAL WEDGES): Consider a competitive equilibrium of the economy with financial frictions. Define the investment and capital wedges as follows:

$$
\begin{aligned}
1+\tilde{\tau}_x(\mathbf{z}) &= p v_n^c(n_c, \mathbf{z}, p) \\
1+\tilde{\tau}_x(\mathbf{z}') &= p(\mathbf{z}') v_n^c(n_c(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}')) \\
1-\tilde{\tau}_k(\mathbf{z}') &= v_n^c(n_c(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}')) + \frac{\theta(1-\delta)p(\mathbf{z}')\mu_c(\mathbf{z}')}{\beta[U_c(s')/U_c(s)]F_k(k_c', z_c' l_c')}
\end{aligned}
$$

Define the bond wedge as:

$$
\tilde{q}_t = \beta R \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') \frac{U_c(s')}{U_c(s)} = \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') \frac{v_n^c(n_c, \mathbf{z}, p) - R\mu_c(\mathbf{z}')}{v_n^c(n_c(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}'))}
$$

Then the competitive equilibrium allocation of the economy with financial frictions satisfies the capital Euler equation and bond Euler equation for the one-sector growth model with wedges.

Proof: The proof immediately follows from a comparison of the intertemporal optimality conditions of the two economies. ■

In this mapping, the investment wedge is equivalent to the the utility cost of purchasing capital. The investment wedge closely resembles the relative price of capital. The capital wedge is equivalent to marginal value of net worth plus a term that captures the shadow value of relaxing debt constraints. Thus a binding debt constraint distorts the capital Euler equation in the one-sector growth model.

The bond holding wedge can be recovered from the first order condition with respect to debt. This wedge measures the average stochastic discount factor. In the model with financial frictions, when debt constraints are slack we know that $\mu_i(\mathbf{z}') = 0$. This corresponds to a higher marginal rate of substitution. When debt constraints bind, however, $\mu_i(\mathbf{z}') > 0$ and the firm would like to substitute consumption from the future to the present. This corresponds to a lower marginal rate of substitution. Thus, the bond wedge generates larger distortions when the demand for debt is high. When thought of as the price of debt, a smaller value of the bond wedge corresponds to a higher interest rate on debt.

Cross Country Volatility The key element that generates cross country differences in the model is a non-homotheticity, ξ, in debt constraints. Following Greenwood and Jovanovic (1990), a sufficient condition for financial distortions to endogenously differ across rich and poor economies is the existence of a fixed cost for accessing a risk-sharing technology. Drawing upon their insight, this model generates differences in financial conditions without assuming differences in parameters governing financial constraints, but instead through steady state productivity differences that induce wealth effects.

In the following proposition, I demonstrate that lower steady state productivity generates higher investment volatility. The reason is that the fixed cost of financial intermediation, ξ, generates a wealth effect that effectively increases the resource cost of external financing in low productivity countries relative to high productivity countries. Since productivity z has an average level A, the fixed cost of financial intermediation, ξ, creates a non-homothetic debt constraint, $b'(\mathbf{z}')/A \leq \theta(1-\delta)k'/A - (\xi/A)$. As a result, a firms in low productivity (poor) countries face tighter debt constraints.

When $\xi > 0$ and debt constraints bind, investment growth is elastic with respect to the down payment on capital purchases. The down payment is defined as today's price of a unit of capital net of the expected resale value of a unit of collateralized capital tomorrow:

$$p - R^{-1}\theta(1-\delta)\sum_{\mathbf{z}'}\pi(\mathbf{z},\mathbf{z}')p(\mathbf{z}')$$

Volatility in the down payment is amplified by larger fixed costs. Hence, since lower steady state productivity increases the effective intermediation cost, (ξ/A), the mechanism generates larger investment volatility in low productivity countries relative to high productivity countries.

Furthermore, in order for the level of productivity to affect volatility, there must be a sectoral price. If the price has zero variance, then the model does not generate a different investment growth volatility across rich and poor countries unless we assume different stochastic processes across countries. Therefore, prices play a central role in the model: they generate cross country variation in aggregate volatility amplifying the response of investment to shocks, more so in poor than in rich economies.

To provide a tractable illustration of this mechanism, I prove this result within a simplified version of the economy. The simplified economy has a finite time horizon ($t = 0, 1, 2$) in which firms consume all net worth in the final period. Firms operate an Ak production technology and are subject to an *iid* productivity shock that is realized at $t = 1$. For the sake of explication, I allow the price of capital to follow an exogenous stochastic process instead of being determined in competitive markets. This stochastic price is realized at $t = 1$. These assumptions on prices simplify the explication considerably without sacrificing or altering the fundamental mechanism.

PROPOSITION 5 (CROSS COUNTRY VOLATILITY): Consider the "simplified environment." Take a stochastic process $s \overset{iid}{\sim} \pi(s)$ and non-degenerate random variables $p_1(s)$ and $z_1(s)$ for prices and productivity, respectively.

(a) If the down payment, $p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)$ has the same sign as $cov(1/p_1(s), z_1(s)/p_1(s))$, then the variance of investment growth increases as a function of ξ.

(b) If $p_1(s) = \bar{p} \in \mathbb{R}_+$ for all s, then the variance of investment growth is inelastic to changes in ξ.

Proof: The proof for Proposition 5 is contained in Appendix C.3.

Whether the model can match the negative conditional variance of investment growth that we observe in the data is primarily quantitative question that I leave to the quantitative section of the paper.

Sectoral Reallocation A key part of the economic mechanism in this paper is that debt constraints generate an inefficient allocation of capital across sectors. In this section I characterize a productivity wedge on total output and demonstrate how debt constraints distort the allocation of capital across sectors. Furthermore, I show how this distortion generates smaller and more volatile productivity relative to an unconstrained economy.

In a standard decentralization of the one-sector growth model, allocations can be supported with production in two-sectors as in the current model. In this decentralization, the relative price of investment adjusts to ensure that capital-labor ratios equalize across sectors. In the current environment, debt constraints prevent such equalization and thereby distort the allocation of capital across sectors. To study the cross-sectional distribution of capital, I turn to the deterministic steady state of the model.[10] The deterministic steady state is fully tractable and therefore useful for characterizing equilibrium behavior.

In the following proposition I show that economies with tighter debt constraints exhibit larger distortions to the distribution of capital across sectors. In particular, tighter debt constraints imply that the investment sector holds too much capital relative to the consumption sector. As a result, GDP decreases as debt constraints become tighter. When debt constraints bind, the demand for capital increases and therefore the price adjusts by increasing. Because a higher price generates an increase in the net worth of the investment sector firm and decreases the consumption sector firms' incentives to purchase capital, a price increase triggers a reallocation of capital to the investment sector.

PROPOSITION 6 (INTERSECTORAL DISTORTIONS): Take some $\theta \in (0,1)$ and some small $\varepsilon > 0$. Denote $(k_c(\theta), k_x(\theta))$ as the deterministic steady state allocations of capital in the consumption and investment sectors, respectively, and $p(\theta)$ as the deterministic steady state price. Capital allocations and the price are denoted as functions of θ. Let $\theta' \equiv \theta + \varepsilon$. Then $p(\theta') < p(\theta)$ and $k_c(\theta')/k_x(\theta') > k_c(\theta)/k_x(\theta)$.

Proof: The proof for Proposition 6 is contained in Appendix C.2.

If the investment sector's productivity shocks exhibit higher variance than the consumption sector's, then the debt constraint should also generate a higher variance of aggregate productivity.[11] The basic reason for this can be seen as a straightforward extension of Proposition 6: the reallocation of capital to the investment sector, induced by a tighter

[10] The deterministic steady state is defined as a competitive equilibrium in which productivity is set to its steady state level at the initial date and is fully persistent.

[11] Castro et al. (2009) provide empirical support of this view. They show that the variance of sales growth, for US firms represented in COMPUSTAT, is higher in investment goods producing sectors than in other

debt constraint, increases the share of aggregate capital that is subject to the investment sector's higher variance productivity shocks.

Even without a fixed cost of financial intermediation ($\xi = 0$), differences in the steady state productivity of the consumption sector, A_c, generate differences in the share of capital in the investment sector - and therefore generate differences in the volatility of aggregate productivity. However, the fixed cost amplifies the these effects. As I demonstrated in Proposition 5, when $\xi > 0$ investment growth is elastic with respect to the down payment on capital. This elasticity is what generates a negative conditional variance of investment growth. In terms of the volatility of aggregate productivity, $\xi > 0$ again creates a dependence of growth rates on the down payment. The difference relative to investment volatility is that for aggregate productivity volatility the down payment is purely an amplification mechanism.

In the following proposition, I demonstrate that debt constraints distort measured productivity. I further show that because poor countries (e.g. countries with low steady state productivity in the consumption sector) have tighter debt constraints, measured aggregate productivity is more volatile in poor countries than in rich countries. In order to provide tractable results that more clearly illustrate the mechanism, I return to the "simplified environment" of Proposition 5 in the last section.

Proposition 7 (Productivity Distortions): Consider the "simplified" version of this economy. Let $F(k, Az) = Azk$. Define aggregate capital as $K = k_c + k_x$ and define total output as $F(k_c, A_c z_c) + F(k_x, A_x z_x)$. Then define aggregate productivity as Z, given by:

$$\sum_{i\in\{c,x\}} F(k_i, A_i z_i) = \left[\sum_{i\in\{c,x\}} A_i z_i \frac{k_i}{k_c + k_x}\right] \cdot (k_c + k_x) \equiv ZK$$

and let Z' denote aggregate productivity in the next period. Under regularity conditions on the down payment on capital purchases (given in Appendix C.4),

$$\frac{\partial}{\partial A_c} var\left(Z'/Z\right) < 0$$

that is, high productivity countries exhibit lower volatility of aggregate productivity .

sectors. The result holds when controlling for observable qualities, such as the age and size of the firms, firm fixed effects and sector fixed effects. Furthermore, Hsieh and Klenow (2009) show that a distorted allocation of capital across firms can distort the level of aggregate productivity. This paper extends their results to show how the distorted allocation of capital across firms can generate higher volatility of aggregate productivity as well.

Proof: The proof for Proposition 7 is contained in Appendix C.4.

3.3 Efficiency

In this section I define and characterize the constrained efficient allocation. I show that, in fact, the competitive equilibrium allocation is constrained inefficient. As I discuss, this is because the the constrained efficient allocation accounts for changes in prices as a result of changes in firm decisions. Relative to the constrained efficient allocation, private firms in competitive markets will issue too much debt which leads to fire sales of capital. Lastly, the constrained efficient allocation can be implemented with sector-specific, state-contingent investment and income taxes.

Efficiency In a competitive equilibrium, firms take the price of capital as given. To study constrained efficient allocations, I consider a social planner that can choose firms' allocations.[12] The planner must choose allocations that are feasible for individual firms at prices that are determined by the competitive capital market. However, unlike firms, the social planner does not take prices as given. The social planner understands how changes in capital and debt across sectors affects capital prices. Consequently, the social planner will choose an allocation that partially undoes capital market distortions.

Definition (Constrained Efficiency): A competitive equilibrium allocation is **constrained efficient** if and only if a social planner who chooses firms' allocations in both sectors subject to firms' constraints, but lets markets clear competitively, cannot improve the representative consumer's utility.

From the capital market clearing conditions:

$$\sum_i k_i'(n_i, \mathbf{z}, p) = (1-\delta)\sum_i k_i + F(k_x, z_x)$$

$$\sum_i k_i'(n_i'(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}')) = (1-\delta)\sum_i k_i'(n_i, \mathbf{z}, p) + F(k_x'(n_x, \mathbf{z}, p), z_x')$$

and from the stationarity of the decision rules, market clearing prices can be expressed

[12] This efficiency concept follows those of Lorenzoni (2008), Bianchi (2011) and others. Furthermore, it satisfies Kehoe and Levine's (1993) definition of conditional constrained efficiency: if the planner were to take the competitive price as given, then the planner's allocation would be equivalent to the competitive equilibrium allocation. Hence, the allocation is efficient conditional on prices.

as functions of state vectors: $p(\mathbf{z}') \equiv p(\{k_i', b_i'(\mathbf{z}'), \mathbf{z}'\}_i)$ and $p \equiv p(\{k_i, b_i, \mathbf{z}\}_i)$. Since the social planner knows how prices relate to the distribution of capital and debt across sectors, the social planner can take derivatives with respect to prices.

Accordingly, the social planner's problem is to choose firms' dividends, capital purchases and debt issuance in order to jointly maximize the value of firms in both sectors:

$$\sum_{i\in\{c,x\}} v^i(k_i, b_i, \mathbf{z}) = \max_{\{d_i, k_i', b_i'(\mathbf{z}')\}} (d_c + d_x) + \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') m(\mathbf{z}, \mathbf{z}') \sum_{i\in\{c,x\}} v^i(k_i', b_i'(\mathbf{z}'), \mathbf{z}')$$

The social planner takes the price *function* as given and is constrained by firms' budget constraints, debt constraints, limited liability and the consumer's stochastic discount factor. Notice that since prices are now functions, I have rearranged the state vector to consist of $s \equiv (k, b, z)$. This is an equivalent representation of the state vector in the competitive environment because values for (k, b, z) and $p(k, b, z)$ are sufficient to recover the value of net worth.

Optimality conditions for the social planner's problem consist of first order conditions for debt in the consumption and investment sector firms, respectively:

$$\mu_c(\mathbf{z}') = \frac{R^{-1} v_b^c(k_c, b_c, \mathbf{z})}{1 - \theta(1-\delta) p_b^c(\mathbf{z}') k_c'} - \frac{1 - (1-\delta) p_b^c(\mathbf{z}') k_c'}{1 - \theta(1-\delta) p_b^c(\mathbf{z}') k_c'} \cdot m(\mathbf{z}, \mathbf{z}') v_b^c(k_c', b_c'(\mathbf{z}'), \mathbf{z}')$$

$$\mu_x(\mathbf{z}') = \frac{R^{-1} v_b^x(k_x, b_x, \mathbf{z})}{1 - \theta(1-\delta) p_b^x(\mathbf{z}') k_x'} - \frac{1 - p_b^x(\mathbf{z}') \left(F(k_x', z_x') + (1-\delta) k_x'\right)}{1 - \theta(1-\delta) p_b^x(\mathbf{z}') k_x'} \cdot m(\mathbf{z}, \mathbf{z}') v_b^x(k_x', b_x'(\mathbf{z}'), \mathbf{z}')$$

where for $i \in \{c, x\}$,

$$p_b^i(\mathbf{z}') \equiv \frac{\partial p(\{k_i', b_i'(\mathbf{z}'), \mathbf{z}'\}_i)}{\partial b_i'(\mathbf{z}')}$$

$$p_k^i(\mathbf{z}') \equiv \frac{\partial p(\{k_i', b_i'(\mathbf{z}'), \mathbf{z}'\}_i)}{\partial k_i'}$$

and Euler equations for the consumption and investment sector firms, respectively:

$$1 = \mathbb{E}_{\mathbf{z}} m(\mathbf{z}, \mathbf{z}') \frac{v_b^c(k', b'(\mathbf{z}'), \mathbf{z}')}{v_b^c(k, b, \mathbf{z})} \cdot \frac{F_k(k', z') + (1-\theta)(1-\delta)(1 + \psi_c(k', b'(\mathbf{z}'), \mathbf{z}')) p(\mathbf{z}')}{(1 + \psi_c(k, b, \mathbf{z})) p - R^{-1} \theta (1-\delta) \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}')(1 + \psi_c(k', b'(\mathbf{z}'), \mathbf{z}')) p(\mathbf{z}')}$$

$$1 = \mathbb{E}_{\mathbf{z}} m(\mathbf{z}, \mathbf{z}') \frac{v_b^x(k', b'(\mathbf{z}'), \mathbf{z}')}{v_b^x(k, b, \mathbf{z})} \cdot \frac{(1 + \Psi(k', b'(\mathbf{z}'), \mathbf{z}')) F_k(k', z') + (1-\theta)(1-\delta)(1 + \psi_x(k', b'(\mathbf{z}'), \mathbf{z}')) p(\mathbf{z}')}{(1 + \psi_x(k, b, \mathbf{z})) p - R^{-1} \theta (1-\delta) \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}')(1 + \psi_x(k', b'(\mathbf{z}'), \mathbf{z}')) p(\mathbf{z}')}$$

where for $i \in \{c, x\}$,

$$\psi_i(k_i', b_i'(\mathbf{z}'), \mathbf{z}') \equiv \frac{p_k^i(\mathbf{z}')k_i' + \theta(1-\delta)p(\mathbf{z}')p_b^i(\mathbf{z}')k_i'}{p(\mathbf{z}') + \theta(1-\delta)p(\mathbf{z}')p_b^i(\mathbf{z}')k_i'}$$

$$\Psi(k', b'(\mathbf{z}'), \mathbf{z}') \equiv \frac{F(k_x', z_x')}{F_k(k_x', z_x')k_x'} \cdot \psi_x(k_x', b_x'(\mathbf{z}'), \mathbf{z}')$$

By comparing the optimality conditions for price taking firms in competitive equilibrium and the social planner, I can prove the following proposition:

PROPOSITION 8 (INEFFICIENCY): The competitive equilibrium allocation is generically inefficient.

Proof: The proof for Proposition 8 is contained in Appendix C.5.

The basic insight is that if prices respond to different capital and debt allocations, which the capital market clearing conditions imply prices should, then $\psi(\cdot)$ and $\Psi(\cdot)$ are nonzero for at least some state state vector. But then the competitive equilibrium allocation violates the social planner's Euler equations.

Decentralization In this environment, the constrained efficient allocation can be implemented through investment taxes. Optimal taxation is fully consistent with the fact that debt constraints generate intertemporal distortions, as evidenced by the mapping of distortions into "investment wedges" in the one-sector growth model (see page 24). In this sense, the model makes predictions about constrained efficient intertemporal distortions. In the quantitative section I measure the income-volatility relationship in the presence of optimal taxation in order to study these predictions.

PROPOSITION 9 (DECENTRALIZATION): The constrained efficient allocation can be implemented with a sector-specific, state-contingent investment tax,

$$\tau^i(k, b, \mathbf{z}) = \psi_i(k, b, \mathbf{z}), \qquad i \in \{c, x\},$$

a state-contingent income tax on sales of new capital,

$$\omega^x(k, b, \mathbf{z}) = -\Psi(k, b, \mathbf{z}),$$

and lump sum rebates of tax revenues such that consumption sector firms and investment sector firms receive, respectively,

$$T^c(k,b,\mathbf{z}) = \psi_c(k,b,\mathbf{z})p(k'_c(k,b,\mathbf{z}) - (1-\delta)k)$$

$$T_x(k,b,\mathbf{z}) = -\Psi(k,b,\mathbf{z})pF(k,z) + \psi_x(k,b,\mathbf{z})p(k'_x(k,b,\mathbf{z}) - (1-\delta)k)$$

Proof: The proof for Proposition 9 is contained in Appendix C.6.

Over Borrowing The main source of the inefficiency in the model is that private agents are price takers and do not account for how their individual decisions affect prices. In this section, I describe how the social planner's incentives to insure against risk differ from those of individual firms in competitive equilibrium.

Consider the social planner's optimality condition for debt in the consumption sector. The condition shows that the planner has an additional incentive to reduce debt relative to private agents and thereby increase insurance across states. Unlike private firms, the planner accounts for the indirect effect of debt issuance on the price, in particular, prices in the debt constraint and in tomorrow's net worth. Suppose that tomorrow's price is a decreasing function of debt issuance, $p^i_b(\mathbf{z}') < 0$.[13] Then reducing debt by a unit in state $\mathbf{z}'$ increases available resources by $(1-\delta)p^i_b(\mathbf{z}')k'_i$ with a marginal utility of $m(\mathbf{z},\mathbf{z}')v^i_b(k'_i, b'_i(\mathbf{z}'), z'_i)$. Lowering debt also relaxes the debt constraint by $\theta(1-\delta)p^i_b(\mathbf{z}')k'_i$ with an associated marginal utility of $\mu_i(\mathbf{z}')$. Hence, the planner realizes an additional benefit to reducing debt that private firms do not. In other words, the competitive equilibrium allocation exhibits over-borrowing relative to the constrained efficient allocation.

Looking to the investment sector's optimality conditions, the social planner has an additional incentive to decrease investment sector debt issuance. This is because the investment sector receives revenues from the sale of capital and therefore price taking investment firms demand more capital when prices are high. Consider the planner's optimality condition for debt in the investment sector. Lowering debt will increase tomorrow's capital price, resulting in larger revenues and net worth. The increase is given by $p^x_b(\mathbf{z}')F(k'_x, z'_x)$ with a marginal utility of $m(\mathbf{z},\mathbf{z}')v^i_b(k'_i, b'_i(\mathbf{z}'), z'_i)$. Notice that a de-

[13]This conjecture stems from Proposition 2. Because capital purchases are an increasing function of net worth, an increase in debt issuance will lower tomorrow's demand for capital and therefore reduce tomorrow's price.

crease in debt still relaxes the debt constraint and increases the value of depreciated capital in tomorrow's net worth.

4 Quantitative Analysis

In this section I turn to the quantitative evaluation of the model and policy experiments. I present the calibration strategy and then detail the quantitative exercises along with results.

4.1 Calibration

A period in the model represents a year, corresponding to the annual Penn World Tables data.

Specifications Assume that preferences take a log specification, $U(c) = \log(c)$, which corresponds to the specification used in the empirical accounting exercise.

A Priori Parameters For comparability to the empirical accounting exercise, I will use the same parameters where possible. I use the same parameter values for $(\beta, \alpha, \delta, R)$:

Parameters	β	α	δ	R
Values	0.94	0.33	0.10	1.02

Debt Constraint Parameters For parameters governing the financial constraint, (θ, ξ), I use the model's long-run restrictions to obtain conditions that map data to parameters. Parameters (θ, ξ) can be recovered from the deterministic steady state's investment rate and debt-to-GDP ratio, respectively (see the Proposition 10 below). I use data from the median country in my sample to pin down these parameters.

Productivity Parameters The remaining parameters are the country-specific steady state values of productivity, $(A_{ic}, A_{ix})_{i=1}^{89}$, and the parameters governing the stochastic processes for sector-specific productivities, $(\rho_c, \rho_x, \sigma_c, \sigma_x, \sigma_{cx})$. I assume that the productivity shocks follow a vector autoregression.

$$\log(z_{ct}) = \rho_c \log(z_{ct-1}) + \sigma_c \varepsilon_{ct} + \sigma_{cx} \varepsilon_{xt}$$

$$\log(z_{xt}) = \rho_x \log(z_{xt-1}) + \sigma_{cx} \varepsilon_{ct} + \sigma_x \varepsilon_{xt}$$

$$(\varepsilon_{ct}, \varepsilon_{xt}) \overset{iid}{\sim} N(0, I)$$

where I is the identity matrix and $(\sigma_c, \sigma_x, \sigma_{cx})$ are the standard deviation of consumption sector innovations, standard deviation of investment sector innovations, and the covariance of the innovations, respectively. I will use data from the median income country in the sample to estimate these stochastic process parameters.

To obtain these parameters, I must first construct time-series for consumption sector productivity. As is standard, I construct the consumption sector productivity time-series as the detrended Solow residual, given data on GDP, labor and capital (constructed from a perpetual inventory method). I can directly compute the autocorrelation from this productivity time series, thereby obtaining ρ_c. I directly compute the time-series mean of the consumption sector productivity series to obtain A_c in a given country. To compute A_x, I use the model's long-run restrictions as detailed in following proposition.

Proposition 10 (Calibration): Given parameters $(\beta, \alpha, \delta, R, A_c)$ and data from the median income country on the average investment rate $(x/y)_{data}$, the average ratio of net exports to GDP $(nx/y)_{data}$ and the steady state relative price of investment p_{data}, the parameters (θ, ξ) are pinned down by the following conditions:

$$(x/y)_{data} = \left(\frac{A_x}{A_c}\right)^{1-\alpha} \left(\frac{1}{(\delta^{-1}\kappa(\theta^*)^{\alpha-1} - 1)}\right)^{\alpha}$$

$$\xi^* = \frac{1}{2}\left(\theta^*(1-\delta)\frac{\delta^{1-\alpha}}{\kappa(\theta^*)^{\alpha-1} - \delta} - \frac{(nx/y)_{data}}{1 - R^{-1}}\right)\left(A_x^{\alpha} A_c^{1-\alpha}(\delta^{-1}\kappa(\theta^*)^{\alpha} - \kappa(\theta^*))^{\alpha}\right)$$

$$A_x = \frac{A_c}{(\delta^{-1}\kappa(\theta^*) - 1)} \cdot \frac{1}{p_{data}^{1/1-\alpha}}$$

where $\kappa(\cdot)$ is a function of model parameters. Furthermore, A_x can be computed for each country by using that country's (A_c^{data}, p^{data}) and fixing θ^*.

Proof: The proof for Proposition 10 is contained in Appendix C.2, under the heading "Calibration." ■

I structurally estimate the remaining productivity parameters, $(\rho_x, \sigma_c, \sigma_x, \sigma_{cx})$ by using an indirect inference procedure (c.f. Smith (1993)). I estimate the following vector autoregression using data on consumption sector (Solow residual) productivity and the relative price of investment from the median income country in the sample:

$$\log(z_{ct}) = \rho_c \log(z_{ct-1}) + \sigma_c \varepsilon_{ct} + \sigma_{cp} \varepsilon_{xt}$$

$$\log(p_t) = \rho_p \log(p_{t-1}) + \sigma_{cp} \varepsilon_{pt} + \sigma_p \varepsilon_{pt}$$

$$(\varepsilon_{ct}, \varepsilon_{pt}) \overset{iid}{\sim} N(0, I)$$

Indirect inference chooses the model parameters as to minimize the difference between the VAR parameters implied by the model and those estimated from the observed productivity and price time series. Hence, the indirect inference strategy is to use the empirical relationship between productivity and relative prices to identify parameters underlying the model equivalent of productivity and prices.

4.2 Algorithm

The standard numerical algorithm for compute the solution to a state-contingent, optimal contracting problem consists of using numerical solvers to find the allocation for each element of the state vector. However, in the present setting with 89 countries, the standard technique would be prohibitively time consuming.

I develop a new numerical algorithm to solve for allocations and value functions quickly. The algorithm is a generalization of the endogenous gridpoint method, see Carroll (2006), to environments with Arrow securities. The algorithm resolves two difficulties facing the standard endogenous gridpoint method with respect to the present class of models.

First, the firm is risk neutral and as a result I cannot use the Euler equation to invert the utility function and recover the consumption allocation. Instead I guess the functional values of the multiplier associated with the budget constraint and update this multiplier using an endogenous grid for the firm's net worth. This is similar to Barillas and Fernandez-Villaverde's (2007) extension of the endogenous gridpoint method to value function iteration techniques.

Second, this environment features state-contingent assets instead of non-contingent bonds as in the standard endogenous gridpoint method. Consequently, computing the vector of state-contingent debt decisions requires solving a static fixed point problem that is nested between updates of the multiplier on the endogenous grid. To solve the nested static fixed point problem I use logic similar to Hintermaier and Koeniger (2010) by computing the unconstrained allocation (assuming that the debt constraint does not bind in any state) and then imposing a binding debt constraint if the debt constraint is violated.

My generalization of the endogenous gridpoint method is more generally applicable to computing optimal contracts in a large class of mechanism design problems. Furthermore, separating the computations into a static portfolio allocation step and a value function iteration step (on the endogenous grid) renders the algorithm applicable for solving a large class of environments with state-contingent asset trade. A detailed pseudo algorithm is contained in Appendix D.

4.3 Cross Country Correlations

I compute the competitive equilibrium allocation for each country, allowing steady state productivities (A_c, A_x) to vary by country. I then simulate model data for all 89 countries. I choose the firms' initial state vectors as the deterministic steady state allocations. I then simulate the economy for 140 periods, dropping the first 100 periods to ensure that the economy is in stochastic steady state. I repeat this procedure 100 times and compute the average correlations between model income and model volatility (of income, consumption and investment). I compare the model simulated correlations with those in the data. Final computations are forthcoming.

4.4 Welfare Calculations

I compute the constrained efficient allocation for each country. I follow the same simulation strategy as used for simulating the competitive equilibrium allocation. I then compare the income-volatility correlations between the competitive equilibrium allocation and the constrained efficient allocation. Lastly, I compute the representative consumer's welfare implied by the equilibrium and efficient allocations. In the spirit of Lucas (2003), I compute the amount of competitive equilibrium consumption that the representative consumer would require to make him indifferent between the competitive equilibrium allocation and the constrained efficient allocation. The proportional increase is given by a function of all possible states, denoted by $\gamma(\{k_i, b_i, z_i\}_i)$. Given the logarithmic utility specification, the function $\gamma(\cdot)$ satisfies:

$$V^{sp}(\{k_i, b_i, z_i\}_i) = \log\left(1 + \gamma(\{k_i, b_i, z_i\}_i)\right) + V^{ce}(\{k_i, b_i, z_i\}_i)$$

where $V^{sp}(\cdot)$ is the expected present value of consumer's utility given by the constrained efficient allocation and $V^{ce}(\cdot)$ is the value evaluated at the competitive equilibrium allocation. This constitutes a measure of the welfare gains attributable to correcting the pecuniary externality and reducing aggregate volatility. Final computations are forthcoming.

5 Conclusion

This paper provides an explanation for the higher volatility of output, consumption and investment in poor relative to rich countries and quantifies the effects of welfare improving fiscal policy across countries.

To provide an empirical account of the income-volatility relationship, I performed an accounting exercise that decomposes time-series movements in national accounts data into movements in "wedges," which are defined as the numerical deviation of Neoclassical theory from the data. The accounting exercise demonstrated that cross country variation in productivity and distortions to the intertemporal substitution of resources are the key economic margins for explaining the income-volatility relationship. This paper is the first to use the Business Cycle Accounting framework to decompose cross country relationships.

Turning to a theoretical study, I construct a quantitative model that is capable of matching the main features of the data. Disciplined by the results of the accounting exercise, the model embeds financial frictions in a two-sector Neoclassical growth model. I show that financial frictions generate the same distortions as the accounting exercise demonstrates are required by theory: the model generates intertemporal distortions due to debt constraints and productivity distortions through the inefficient allocation of capital across sectors. In the model, cross country variation in the volatility of output, consumption and investment is the result of differences in steady state productivity and not differences in the variance of shocks or exogenous differences in financial frictions. The model suggests that TFP differences across countries are seemingly capable of additionally explaining a portion of cross-country volatility.

Lastly, the relative price of goods across production sectors generates an inefficiency in the model. The inefficiency allows optimal government policy to play a non-trivial role in mitigating volatility across countries. I show that the constrained efficient allocation can be decentralized as an investment tax, in direct correspondence to the "investment wedge" in the empirical accounting exercise. As a result, I interpret the income-volatility relationship implied by the constrained efficient allocation as a measure of the potential for government policy to reduce volatility. As a theoretical contribution, this paper is the first to study constrained efficiency in the presence of pecuniary externalities when agents face endogenously incomplete markets.

In future work, I will focus on two extensions of the current framework. First, I will allow for within-sector heterogeneity across firms. As demonstrated by Hsieh and Klenow (2009), distortions affecting the allocation of capital and labor across firms generates distortions in measured aggregate productivity. Midrigan and Xu (2010) measure such "productivity wedges" in a heterogeneous agent economy by assuming that agents face collateral constraints. Under certain conditions, they show that productivity wedges can be large. In the context of the financial frictions in this paper, heterogeneous firms could generate larger productivity wedges in poor countries, which is supported by the empirical evidence in Hsieh and Klenow (2009).

Second, in a work-in-progress, I provide a theoretical explanation for Ramey and Ramey's (1995) result that high income volatility generates low GDP growth. In Sager (2012), I construct a model in which heterogeneous entrepreneurs face financial constraints, similar to Midrigan and Xu (2010) or Buera et al. (2011), and additionally invest in organizational capital (c.f. Klette and Kortum (2004) and Atkeson and Burstein (2010)). Organizational capital investments improve a firm's productivity and, in the aggregate, determines the economy's output growth rate. Economies with more severe financial frictions exhibit higher aggregate output volatility and lower growth rates. This preliminary result suggests that Ramey and Ramey's (1995) relationship between volatility and growth can be explained by features of financial markets.

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Appendix

A Listing of Sample Countries

Table 4: Countries in Sample, 89 in Total

Country Name	PWT ID	Country Name	PWT ID
Albania	ALB	Korea, Republic of	KOR
Angola	AGO	Madagascar	MDG
Argentina	ARG	Malawi	MWI
Australia	AUS	Malaysia	MYS
Austria	AUT	Mali	MLI
Bahrain	BHR	Malta	MLT
Bangladesh	BGD	Mexico	MEX
Belgium	BEL	Morocco	MAR
Bolivia	BOL	Mozambique	MOZ
Brazil	BRA	Netherlands	NLD
Bulgaria	BGR	New Zealand	NZL
Burkina Faso	BFA	Niger	NER
Cambodia	KHM	Nigeria	NGA
Cameroon	CMR	Norway	NOR
Canada	CAN	Oman	OMN
Chile	CHL	Pakistan	PAK
China	CHN	Panama	PAN
Colombia	COL	Philippines	PHL
Congo, Dem. Rep.	ZAR	Poland	POL
Costa Rica	CRI	Portugal	PRT
Cote d'Ivoire	CIV	Romania	ROM
Cyprus	CYP	Senegal	SEN
Denmark	DNK	Singapore	SGP
Dominican Republic	DOM	South Africa	ZAF
Ecuador	ECU	Spain	ESP
Egypt	EGY	Sri Lanka	LKA
Ethiopia	ETH	Sudan	SDN
Finland	FIN	Sweden	SWE
France	FRA	Switzerland	CHE
Germany	GER	Syria	SYR
Ghana	GHA	Taiwan	TWN
Greece	GRC	Tanzania	TZA
Guatemala	GTM	Thailand	THA
Hong Kong	HKG	Trinidad &Tobago	TTO
Hungary	HUN	Tunisia	TUN
Iceland	ISL	Turkey	TUR
India	IND	Uganda	UGA
Indonesia	IDN	United Kingdom	GBR
Iran	IRN	United States	USA
Ireland	IRL	Uruguay	URY
Israel	ISR	Venezuela	VEN
Jamaica	JAM	Vietnam	VNM
Japan	JPN	Zambia	ZMB
Jordan	JOR	Zimbabwe	ZWE
Kenya	KEN		

B Business Cycle Accounting Results

Table 5: Counterfactual Correlations, All Specifications

	Correlations						RMSE	
	VAR Specification			AR(1) Specification				
	GDP	CON	INV	GDP	CON	INV	VAR	AR(1)
Data	-0.44	-0.48	-0.40	-0.44	-0.48	-0.40	0	0
z	-0.52	-0.08	-0.10	-0.86	-0.66	-0.58	**0.29**	**0.28**
τ_l	0.15	0.74	0.34	0.17	0.34	0.40	0.89	0.75
τ_x	-0.63	0.48	-0.73	-0.51	0.43	-0.63	0.59	0.54
q	-0.60	0.13	-0.61	-0.27	0.48	-0.51	0.38	0.57
g	0.80	0.82	0.59	0.90	0.87	0.78	1.18	1.29
z, τ_l	-0.42	-0.21	-0.08	-0.89	-0.78	-0.69	0.24	0.36
z, τ_x	-0.51	0.13	-0.46	-0.55	-0.60	0.08	0.35	**0.29**
z, q	-0.47	-0.29	-0.25	-0.41	-0.66	0.16	**0.14**	0.34
z, g	-0.43	0.49	-0.03	-0.35	0.42	-0.08	0.60	0.55
τ_l, τ_x	-0.52	0.67	-0.69	-0.05	0.46	-0.30	0.69	0.59
τ_l, q	-0.56	0.43	-0.64	-0.09	0.09	-0.17	0.54	0.41
τ_l, g	0.42	0.82	0.46	0.82	0.87	0.66	1.03	1.23
τ_x, q	-0.32	0.41	-0.28	0.52	0.63	0.32	0.53	0.94
τ_x, g	-0.05	0.85	-0.60	0.45	0.88	-0.49	0.81	0.94
q, g	-0.19	0.69	-0.60	0.50	0.86	-0.45	0.70	0.94
z, τ_l, τ_x	-0.34	-0.08	-0.29	-0.64	-0.78	0.19	0.24	**0.40**
z, τ_l, q	-0.41	-0.45	-0.10	-0.53	-0.77	0.26	**0.17**	0.42
z, τ_l, g	-0.45	0.33	-0.07	-0.77	-0.10	-0.55	0.50	0.30
z, τ_x, q	-0.43	-0.13	-0.29	-0.72	-0.74	-0.54	**0.21**	**0.23**
z, τ_x, g	-0.50	0.67	-0.63	-0.41	0.65	-0.62	0.68	0.67
z, q, g	-0.61	0.21	-0.56	-0.41	0.54	-0.69	0.42	0.61
τ_l, τ_x, q	0.21	0.51	0.04	0.64	0.35	0.54	0.73	0.96
τ_l, τ_x, g	-0.23	0.80	-0.61	0.24	0.86	-0.54	0.76	0.87
τ_l, q, g	-0.36	0.69	-0.63	0.35	0.82	-0.50	0.69	0.88
τ_x, q, g	0.56	0.77	0.06	0.88	0.88	0.67	0.96	1.26
τ_l, τ_x, q, g	0.43	0.72	-0.03	0.88	0.85	0.60	0.88	1.23
z, τ_x, q, g	-0.35	0.47	-0.35	0.51	0.87	0.18	0.55	1.01
z, τ_l, q, g	-0.65	-0.19	-0.55	-0.59	-0.41	-0.48	0.23	**0.11**
z, τ_l, τ_x, g	-0.49	0.24	-0.59	-0.53	0.18	-0.52	0.43	0.39
z, τ_l, τ_x, q	-0.47	-0.38	-0.20	-0.84	-0.84	-0.70	**0.13**	0.36
z, τ_l, τ_x, q, g	-0.44	-0.48	-0.40	-0.44	-0.48	-0.40	0	0

C Proofs

C.1 Proof for Proposition 1 (Endogenous Debt Constraints)

Consider the following game between a lender and a firm. A continuum of competitive lenders offer financial contracts to a firm, at which time the firm accepts or rejects.. Contracts consist of state-contingent payments $\{\rho(n, \mathbf{z}, p)\}$. If the firm accepts the offered contract, it chooses decision rules $\{d, k'\}$ to maximize its expected discounted value of dividends. Both firms and lenders take prices $\{p, p(\mathbf{z}')\}$ and the stochastic discount factor $m(\mathbf{z}, \mathbf{z}')$ as given.

The firm's problem can be written as:

$$v(n, \mathbf{z}, p) = \max_{d, k', \rho} \; d + \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') m(\mathbf{z}, \mathbf{z}') v(n'(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}'))$$

subject to limited liability, $d \geq 0$, the budget constraints,

$$d + pk' + \rho \leq n \equiv F(k, zl) - wl + (1 - \delta)pk - b$$

the lender's participation constraints,

$$b(n, \mathbf{z}, p) = \rho + R^{-1} \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') b(n'(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}')) \geq 0$$

and enforcement constraints,

$$v(n, \mathbf{z}, p) \geq \hat{v}(\hat{n}, \mathbf{z}, p) = \max_{\hat{d}, \hat{k}', \hat{\rho}} \; \hat{d} + \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') m(\mathbf{z}, \mathbf{z}') \hat{v}(\hat{n}'(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}'))$$

where $\hat{v}(\cdot)$ is the value of the firm after a default and $\{\hat{d}, \hat{k}', \hat{\rho}\}$ are the associated decision rules.

Upon default the lender can seize firm's collateralized capital, $\theta(1 - \delta)k$ thereby leaving the firm with new net worth, $\hat{n} = n - \theta(1 - \delta)pk$. After a default, the lender can pay the firm a fixed cost to obtain ownership of seized collateral. This fixed cost constitutes a variable haircut on collateral: the fixed cost is lower in percentage terms when the value of collateral is larger. For the lender, the value of default is the resale value of collateralized capital net the fixed cost, $\hat{b}(n, \mathbf{z}, p) = -\xi + \theta(1-\delta)pk$. The participation constraint requires that $\hat{b}(n, \mathbf{z}, p) \geq 0$, so if $\xi > \theta(1-\delta)pk$, then the lender does not pay the fixed cost and neither the firm nor the lender owns or operates the seized capital.

The enforcement constraints support a no-default equilibrium path, on which firms attain at least as high of a value as attained after defaulting. The following lemma proves that the debt constraint equivalently supports the no-default equilibrium, as in Rampini and Viswanathan (2012).

LEMMA: $b(n, \mathbf{z}, p)$ is incentive feasible in the firm's problem if and only if it satisfies the debt constraint $b(n, \mathbf{z}, p) \leq \theta(1-\delta)pk - \xi$.

Proof: Take an incentive feasible $b(n, \mathbf{z}, p)$. Suppose that in state $(n, \mathbf{z}, p)$ the debt constraint is violated, $b(n, \mathbf{z}, p) > \theta(1-\delta)pk - \xi$. If the firm deviates from the no-default equilibrium, then it can make a profit. Upon default, the firm does not repay its promised debt, ρ, but loses $\theta(1-\delta)k'$ of its post depreciation capital stock.

The firm can immediately sign a new financial contract in which it borrows

$$\hat{\rho} = -R^{-1} \sum_{\mathbf{z}'} \pi(\mathbf{z}, \mathbf{z}') b(n'(\mathbf{z}'), \mathbf{z}', p(\mathbf{z}'))$$

at the time of default and sets $\hat{\rho}(n, z, p) = \rho(n, z, p)$ for all subsequent states and dates. By construction $\hat{\rho}$ satisfies the lender's participation constraint. The firm has two sources of additional funds, those from signing a new contract ($\hat{\rho}$) and those from its old lender if the lender chooses to exercise its option to seize control of collateralized capital (ξ). Using these funds the firm can repurchase lost capital and pay dividends with the remainder. Under this construction the firm's continuation value does not change, only dividends at the time of default change. The change in the firms value after this one-shot deviation is given by:

$$
\begin{aligned}
\hat{v}(n, \mathbf{z}, p) - v(n, \mathbf{z}, p) &= \rho(n, \mathbf{z}, p) - \hat{\rho}(n, \mathbf{z}, p) - (\theta(1-\delta)pk - \xi) \\
&= b(n, \mathbf{z}, p) - (\theta(1-\delta)pk - \xi)
\end{aligned}
$$

Therefore, in order to provide the firm with incentives to repay its debt obligations, lenders only offer contracts that satisfy $v(n, \mathbf{z}, p) \geq \hat{v}(n, \mathbf{z}, p)$ at each $(n, \mathbf{z}, p)$. Equivalently, contracts must satisfy the debt constraint. Conversely, if an allocation satisfies the debt constraint then default at some state $(n, \mathbf{z}, p)$ makes the firm worse off. ■

C.2 Proofs for Proposition 6 (Intersectoral Distortions) and Proposition 10 (Calibration)

Here I study the deterministic steady state of the economy. Suppose that the production technology exhibits decreasing returns to scale, of the form $F(k, A) = A^{1-\alpha}k^{\alpha}$. Assume that the number or workers employed in each sector is inelastic and equal to one.

Consumption Sector I will first compute allocations for the consumption sector. Using the Euler equation:

$$
\begin{aligned}
v_n^c(n_c, A_c, p) &= \beta v_n^c(n_c, A_c, p) \frac{\alpha A_c^{1-\alpha} k_c^{\alpha-1} + (1-\theta)(1-\delta)p}{p - R^{-1}\theta(1-\delta)p} \\
k_c &= \frac{A_c}{p^{1/(1-\alpha)}} \left(\frac{\alpha\beta}{1 - R^{-1}(1-\delta)(1+\beta R(1-\theta))} \right)^{1/(1-\alpha)} \\
&\equiv p^{-1/(1-\alpha)} A_c \kappa
\end{aligned}
$$

Using both the first order condition with respect to debt and the envelope condition, I find that the debt constraint always binds in steady state:

$$
R\mu_c = (1-\beta R) v_n^c(n_c, A_c, p) \geq 1 - \beta R > 0
$$

Since $\mu_c > 0$, the debt constraint gives the value of net worth:

$$\begin{aligned} n_c &= A_c^{1-\alpha} k_c^{\alpha} + (1-\theta)(1-\delta) p k_c + \xi \\ &= p^{-\alpha/(1-\alpha)} A_c \kappa^{\alpha} + (1-\theta)(1-\delta) p^{-\alpha/(1-\alpha)} A_c \kappa + \xi \\ &= p^{-\alpha/(1-\alpha)} A_c \left[\kappa^{\alpha} + (1-\theta)(1-\delta)\kappa\right] + \xi \end{aligned}$$

From the budget constraint and debt constraint:

$$d_c = n_c - p k_c$$

Investment Sector I now compute allocations for the investment sector. Using the Euler equation for investment sector firms:

$$\begin{aligned} v_n^x(n_x, A_x, p) &= \beta v_n^x(n_x, A_x, p) \, \frac{\alpha p A_x^{1-\alpha} k_x^{\alpha-1} + (1-\theta)(1-\delta) p}{p - R^{-1}\theta(1-\delta) p} \\ k_x &= A_x \left(\frac{\alpha\beta}{1 - R^{-1}(1-\delta)(1+\beta R(1-\theta))} \right)^{1/(1-\alpha)} \\ &\equiv A_x \kappa \end{aligned}$$

Again, the optimality condition for debt implies that the steady state debt constraint binds. Next, use the debt constraint to compute net worth:

$$\begin{aligned} n_x &= p A_x^{1-\alpha} k_x^{\alpha} + (1-\theta)(1-\delta) p k_x + \xi \\ &= p A_x \left[\kappa^{\alpha} + (1-\theta)(1-\delta)\kappa\right] + \xi \end{aligned}$$

Notice that the price of capital positively enters the investment sector firm's net worth, while it decreases the consumption sector firm's net worth.

Prices Using the market clearing condition for capital, I now solve for p.

$$A_x^{1-\alpha} k_x^{\alpha} = \delta(k_x + k_c)$$

$$A_x\left(\kappa^\alpha - \delta\kappa\right) = \delta p^{-1/(1-\alpha)} A_c \kappa$$

$$p = \left(\frac{\delta\kappa}{\kappa^\alpha - \delta\kappa} \cdot \frac{A_c}{A_x}\right)^{1-\alpha}$$

Given the expression for prices, now solve for the capital allocations across sectors:

$$k_c = A_x(\delta^{-1}\kappa^\alpha - \kappa)$$

$$k_x = A_x\kappa$$

Distribution of Capital Lastly, notice that κ is a negative function of the parameter θ. When $\theta = 1$, debt constraints do not bind. Therefore, I will perform a comparative statics exercise with respect to θ to induce changes in the debt constraint's tightness.[14] Take some $\theta \in (0, 1)$. If $\theta' = \theta + \varepsilon$ for $\varepsilon > 0$ small, then $\kappa(\theta') < \kappa(\theta)$. Because an increase in κ induces an increase in the price, we then know that $p(\theta') < p(\theta)$. That is, decreasing the tightness of the debt constraint (increasing θ) lowers the price.

The quantity of capital held by the consumption sector relative to the investment sector is given by:

$$\frac{k_c}{k_x} = \frac{\kappa^{\alpha-1} - \delta}{\delta}$$

Since κ is decreasing as a function of θ, a decrease in θ induces a reallocation of capital from the consumption sector to the investment sector (k_c/k_x decreases).

Calibration From the deterministic steady state conditions, I obtain values for the financial constraint parameters (θ, ξ).

The deterministic steady state investment rate is:

$$\frac{x}{y} = \frac{A_x^{1-\alpha} k_x^\alpha}{A_c^{1-\alpha} k_c^\alpha}$$

[14] The same qualitative results obtain if, instead, I perform comparative statics with respect to R. This is an appealing alternative parameter to perturb since it the multiplier μ is a function of R.

$$= \left(\frac{A_x}{A_c}\right)^{1-\alpha} \left(\frac{1}{(\delta^{-1}\kappa^{\alpha-1} - 1)}\right)^{\alpha}$$

Since κ is a function of θ, I can solve this equation for the value of θ that ensures the model's deterministic steady state equals the investment rate x/y from the data.

The deterministic steady state debt-to-GDP ratio is:

$$\begin{aligned}
\frac{nx}{y_c} &= \frac{(1-R^{-1})(b_c + b_x)}{A_c^{1-\alpha} k_c^{\alpha}} \\
&= (1-R^{-1})\theta(1-\delta) \cdot \frac{p}{A_c^{1-\alpha}} \cdot \frac{k_c + k_x}{A_x \cdot (\delta^{-1}\kappa^{\alpha} - \kappa)} - \frac{2(1-R^{-1})\xi}{A_c^{1-\alpha} k_c^{\alpha}} \\
&= (1-R^{-1}) \left(\theta(1-\delta)\frac{\delta^{-\alpha}\kappa}{\delta^{-1}\kappa^{\alpha} - \kappa} - \frac{2\xi}{A_x^{\alpha} A_c^{1-\alpha} (\delta^{-1}\kappa^{\alpha} - \kappa)^{\alpha}}\right)
\end{aligned}$$

where I use the debt constraint (given that $\mu_c > 0$ and $\mu_x > 0$) to substitute for b_c and b_x. Given a value for θ and data on net exports divided by GDP, denoted $(nx/y)^{data}$, I can solve for the value of ξ from the above equation.

$$\xi^* = \frac{1}{2}\left(\theta(1-\delta)\frac{\delta^{-\alpha}\kappa}{\delta^{-1}\kappa^{\alpha} - \kappa} - \frac{(nx/y)^{data}}{1-R^{-1}}\right)\left(A_x^{\alpha} A_c^{1-\alpha} (\delta^{-1}\kappa^{\alpha} - \kappa)^{\alpha}\right)$$

I identify investment sector productivity from the expression for prices. Given the time-series mean of detrended relative price of investment (the price of investment divided by the price of consumption in the Penn World Tables), denoted p_{data}, I find A_x as follows:

$$A_x = \frac{A_c}{(\delta^{-1}\kappa(\theta^*) - 1)} \cdot \frac{1}{p_{data}^{1/1-\alpha}}$$

Note that θ^* is solved for using the mean level of prices from the median country. ■

C.3 Proof for Proposition 5 (Cross Country Volatility)

Environment I describe the simplified environment.[15] Let time be finite, $t = 0, 1, 2$. There is a single, risk neutral firm that maximizes its discounted stream of dividends. The firm operates a production technology described by $F(k, z) = zk$. An *iid* stochastic process s is described by density function $\pi(s)$. A shock hits at $t = 1$ and at no other time. Productivity and prices are functions of this underlying stochastic process, denote these functions as $z_1(s)$ and $p_1(s)$ respectively. Assume that $z_1(s)$ and $p_1(s)$ have positive variance. Also assume that $z_1(s) = A\tilde{z}(s)$, where $\tilde{z}(s)$ is a zero-mean random variable and A is steady state productivity (e.g. the mean of $z_1(s)$ is A).

The firm chooses dividends, capital investment and debt to maximize its value subject to budget constraints, debt constraints and limited liability constraints (as before):

$$v_0(n_0, p_0) = \max_{d_t, k_{t+1}, b_{t+1}} d_0 + \sum_s \pi(s) \left[\beta d_1(s) + \beta^2 d_2(s) \right]$$

subject to

$$d_0 + p_0 k_1 - R^{-1} \sum_s \pi(s) b_1(s) \leq n_0$$

$$d_1(s) + p_1(s) k_2(s) - R^{-1} b_2 \leq z_1(s) k_1 + (1 - \delta) p_1(s) k_1 - b_1(s)$$

$$d_2(s) = A k_2(s) + (1 - \delta) p_2 k_2(s) - b_2$$

$$b_1(s) \leq \max\{0, \theta(1 - \delta) p_1(s) k_1 - \xi\}$$

$$b_2 \leq \max\{0, \theta(1 - \delta) p_2 k_2(s) - \xi\}$$

$$d_t \geq 0, \quad k_t \geq 0$$

I assume that productivity and the price at $t = 2$ are non-stochastic. In order to focus on the initial investment decision, I assume that $p_2 = 0$.

[15] This environment is very close to that in Rampini and Viswanathan (2010). The main difference is the addition of ξ in the debt constraint. This difference motivates my focus on comparative statics with respect to changes in the steady state level of productivity.

Optimality Let $(\lambda_0, \beta\pi(s)\lambda_1(s), \beta^2\pi(s)\lambda_2(s))$ be the multipliers on the $t = 0, 1, 2$ budget constraints, respectively. Let $(\pi(s)\mu_1(s), \beta\pi(s)\mu_2(s))$ be the multipliers on the debt constraints for debt issued against second and third period capital, respectively. Lastly let $(\phi_0, \beta\pi(s)\phi_1(s))$ be the multipliers on the limited liability constraints. The first order conditions for the firm's problem are given by:

$$\lambda_0 = 1 + \phi_0 \geq 1$$

$$\lambda_1(s) = 1 + \phi_1(s) \geq 1$$

$$p_0\lambda_0 = \sum_s \pi(s) \left[\beta\lambda_1(s) \left(z_1(s) + (1-\delta)p_1(s)\right) + \theta(1-\delta)p_1(s)\mu_1(s)\right]$$

$$p_1(s)\lambda_1(s) = \sum_s \pi(s) \left[\beta\lambda_2(s) \left(A + (1-\delta)p_2\right) + \theta(1-\delta)p_2\mu_2(s)\right]$$

$$R\mu_1(s) = \lambda_0 - \beta R\lambda_1(s)$$

$$R\mu_2(s) = \lambda_1(s) - \beta R\lambda_2(s)$$

The Euler equations as:

$$\lambda_0 = \beta \sum_s \pi(s)\lambda_1(s) \frac{z_1(s) + (1-\theta)(1-\delta)p_1(s)}{p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)} \equiv \beta \sum_s \pi(s)\lambda_1(s)r_1(s)$$

$$\lambda_1(s) = \beta\lambda_2(s)\frac{A}{p_1(s)} \equiv \beta\lambda_2(s)r_2(s)$$

where I have defined $r_1(s)$ and $r_2(s)$ as return functions. In order to ensure that we are in an interesting region of the parameter space, one in which there is positive capital investment, I make the following assumption on primitives.

Assumption: Assume that $r_2(s) > R$ for all s and $\beta = R^{-1}$.

Following Rampini and Viswanathan (2010) I will now show that if $r_2(s) > R$ then $d_0 = d_1(s) = b_2 = 0$ for all s. From the first order condition with respect to b_2 and the above Euler equations:

$$\beta R\lambda_2(s) + R\mu_2(s) = \lambda_1(s) = \beta r_2(s)\lambda_2(s) \geq \beta R\lambda_2(s)$$

Therefore $\mu_2(s) > 0$ and $b_2 = 0$. Using the first order condition with respect to $b_1(s)$ and the Euler equations:

$$\lambda_0 \geq \beta R \lambda_1(s) = \beta R \cdot (R\mu_2(s) + \beta R \lambda_2(s)) > (\beta R)^2 \lambda_2(s) \geq \beta R$$

Since $\beta R = 1$ by assumption, $\lambda_0 > 1$ and $\lambda_1(s) > 1$ for all s. Therefore $d_0 = d_1(s) = 0$.

If we further assume that $\sum_s \pi(s) r_1(s) r_2(s) > R \max_s\{r_2(s)\}$, then we will restrict attention to the region of the parameter space for which $k_1 > 0$. In this case, trivially $\lambda_2(s) = 1$, $\mu_1(s) > 0$ and

$$n_1(s) \equiv z_1(s)k_1 + (1-\theta)(1-\delta)p_1(s)k_1 + \xi > 0$$

Lastly, use the budget and debt constraints to find:

$$k_1 = \frac{n_0 - R^{-1}\xi}{p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)}$$

$$k_2(s) = \frac{n_1(s) - R^{-1}\xi}{p_1(s)}$$

$$n_1(s) = r_1(s)(n_0 - R^{-1}\xi) + \xi$$

Volatility Given the allocation $\{k_1, k_2(s)\}$, the investment growth is given by:

$$\frac{k_2(s)}{k_1} = \left((1-\theta)(1-\delta) + \frac{p_0 - R^{-1}\theta(1-\delta)\sum_{\hat{s}} \pi(\hat{s})p_1(\hat{s})}{p_1(s)} \cdot \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\right) + \frac{z_1(s)}{p_1(s)}$$

and the variance of investment growth is:

$$var\left(\frac{k_2(s)}{k_1}\right) = \left(\left(p_0 - R^{-1}\theta(1-\delta)\sum_{\hat{s}} \pi(\hat{s})p_1(\hat{s})\right) \cdot \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\right)^2 var\left(\frac{1}{p_1(s)}\right) + var\left(\frac{z_1(s)}{p_1(s)}\right)$$
$$+ \left(\left(p_0 - R^{-1}\theta(1-\delta)\sum_{\hat{s}} \pi(\hat{s})p_1(\hat{s})\right) \cdot \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\right) cov\left(\frac{1}{p_1(s)}, \frac{z_1(s)}{p_1(s)}\right)$$

Now, given that $z_1(s) = A\tilde{z}(s)$, divide the debt constraint (against $t = 1$) by steady state productivity:

$$\frac{b_1(s)}{A} \leq \theta(1-\delta)p_1(s)\frac{k_1(s)}{A} - \frac{\xi}{A}$$

Clearly, the fixed cost of financial intermediation, ξ, generates a non-homotheticity in the model. Lower productivity corresponds to a higher effective fixed cost, ξ/A. Therefore, in the expression above, in order for volatility to differ across countries, the variance of investment growth must change with respect to changes in ξ. The expression for the variance of investment growth clearly shows that prices must vary in order for ξ to have an effect.

Formally, if $p_1(s) = p_1 \in \mathbb{R}_+$ for all s, then

$$var\left(\frac{k_2(s)}{k_1}\right) = var(z_1(s)) \qquad \text{and} \qquad \frac{\partial}{\partial\xi} var\left(\frac{k_2(s)}{k_1}\right) = 0.$$

Otherwise, if $p_1(s)$ is a stochastic process, then

$$\frac{\partial}{\partial\xi} var\left(\frac{k_2(s)}{k_1}\right) \neq 0$$

In this simplified environment, sufficient conditions for low productivity (high ξ) countries to have larger investment growth variance than high productivity (low ξ) countries are:

1. the fixed cost, ξ, and $var(1/p_1(s))$ are positive,

2. initial net worth is sufficiently large, $n_0 > R^{-1}\xi$, and

3. the down payment on capital, $p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)$, and the covariance, $cov(1/p_1(s), z_1(s)/p_1(s))$ have the same sign.

If these conditions hold then the variance of investment growth increases with an increase in ξ (e.g. a lower steady state productivity):

$$\frac{\partial}{\partial\xi} var\left(\frac{k_2(s)}{k_1}\right) = \left(p_0 - R^{-1}\theta(1-\delta)\sum_{\hat{s}} \pi(\hat{s})p_1(\hat{s})\right) \cdot \frac{(1-R^{-1})}{n_0 - R^{-1}\xi}\left(1 + \frac{R^{-1}\xi}{n_0 - R^{-1}\xi}\right)$$

$$\times \left[2 \left(p_0 - R^{-1}\theta(1-\delta) \sum_{\hat{s}} \pi(\hat{s}) p_1(\hat{s}) \right) \cdot \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi} var \left(\frac{1}{p_1(s)} \right) + cov \left(\frac{1}{p_1(s)}, \frac{z_1(s)}{p_1(s)} \right) \right.$$

$$> 0$$

Note that whether an increase in ξ generates the desired increase in investment growth variance is a quantitative question: the answer depends on the choice of model parameters and properties of the stochastic processes. ■

C.4 Proof for Proposition 7 (Productivity Distortions)

Consider the "simplified environment" of Appendix C.3. I will now additionally consider endogenous capital prices. To do so I will assume that at $t = 1$ the investment sector receives a productivity shock, given by $z_1^x(s)$, that has mean equal to one. Total productivity at $t = 1$ in the investment sector is then $A_x z_1^x(s)$. For simplicity I assume that the consumption sector's productivity is deterministic and equal to its mean value, $A_c \neq A_x$, at each date. The simplifying assumption follows Castro et al. (2009), who provide evidence for larger variance of productivity in investment goods producing firms.

For simplicity assume that each firm's initial net worth is equal, $n_0^c = n_0^x \equiv n_0$.

I write the consumption sector firm's problem as in Appendix C.3 and write the investment sector firm's problem as:

$$v_0^x(n_0, p_0) = \max_{d_t, k_{t+1}, b_{t+1}} d_0 + \sum_s \pi(s) \left[\beta d_1(s) + \beta^2 d_2(s) \right]$$

subject to

$$d_0 + p_0 k_1 - R^{-1} \sum_s \pi(s) b_1(s) \leq n_0$$

$$d_1(s) + p_1(s) k_2(s) - R^{-1} b_2 \leq p_1(s) A_x z_1(s) k_1 + (1-\delta) p_1(s) k_1 - b_1(s)$$

$$d_2(s) = A_x k_2(s) + (1-\delta) p_2 k_2(s) - b_2$$

$$b_1(s) \leq \max\{0, \theta(1-\delta)p_1(s)k_1 - \xi\}$$

$$b_2 \leq \max\{0, \theta(1-\delta)p_2 k_2(s) - \xi\}$$

$$d_t \geq 0, \quad k_t \geq 0$$

where p_0 is exogenously given. As before, I assume that productivity at $t = 2$ is non-stochastic. Because capital cannot be converted into the consumption good and because productivity is zero for all $t > 2$, the price of capital is zero at $t = 2$, thus $p_2 = 0$.

Following the proofs in Appendix C.3, the optimal allocation of capital and net worth is given by:

$$k_1^c = k_1^x = \frac{n_0 - R^{-1}\xi}{p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)}$$

$$k_2^i(s) = \frac{n_1^i(s) - R^{-1}\xi}{p_1(s)}$$

$$n_1^i(s) = r_1^i(s)(n_0 - R^{-1}\xi) + \xi$$

for $i \in \{c, x\}$ and where the return functions, $r_1^i(s)$, are given by:

$$r_1^c(s) = \frac{A_c z_1^c(s) + (1-\theta)(1-\delta)p_1(s)}{p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)}$$

$$r_1^x(s) = \frac{p_1(s)A_x z_1^x(s) + (1-\theta)(1-\delta)p_1(s)}{p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)}$$

Next I solve for the endogenous capital price $p_1(s)$. To do so, I substitute the optimal capital decisions into the capital market clearing condition:

$$\sum_{i \in \{c,x\}} k_1^i(s) = (1-\delta) \sum_{i \in \{c,x\}} k_0^i + A_x k_0^x$$

$$\frac{1}{p_1(s)} \sum_{i \in \{c,x\}} \left[r_1^i(s)(n_0 - R^{-1}\xi) + (1 - R^{-1})\xi \right] = [A_x + 2(1-\delta)] \frac{(n_0 - R^{-1}\xi)}{p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)}$$

Solving for $p_1(s)$:

$$p_1(s) = \frac{A_c + 2\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\left(p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)\right)}{A_x(1-z_1^x(s)) + 2\theta(1-\delta)}$$

To solve for the expectations term in the numerator, take expectations of the entire equation and rearrange:

$$\sum_s \pi(s)p_1(s) = \frac{\sum_s \pi(s)\frac{A_c + 2p_0 \cdot \frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}}{A_x(1-z_1^x(s)) + 2\theta(1-\delta)}}{1 + R^{-1}\theta(1-\delta)\sum_s \pi(s)\frac{1}{A_x(1-z_1^x(s)) + 2\theta(1-\delta)} \cdot \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}}$$

For notational convenience, I will define the down payment as a function of A_c:

$$D(A_c) \equiv p_0 - R^{-1}\theta(1-\delta)\sum_s \pi(s)p_1(s)$$

Substituting the expectations term, which is now notationally nested in $D(A_c)$, into $p_1(s)$ yields the equilibrium price as a function of model parameters:[16]

$$p_1(s) = \frac{A_c + 2\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}D(A_c)}{A_x(1-z_1^x(s)) + 2\theta(1-\delta)}$$

Given the market clearing price I will now construct aggregate productivity and its growth rate. Total output at $t = 0$ is given by $A_c k_0^c + A_x k_0^x$, and at $t = 1$ by $A_c k_1^c(s) + A_x z_x^1(s) k_1^x(s)$. Define aggregate productivity at $t = 1$ as $Z_1(s)$, given by the following equation:

$$Z_1(s) \cdot (k_1^c(s) + k_1^x(s)) \equiv \left(A_c \cdot \frac{k_1^c(s)}{k_1^c(s) + k_1^x(s)} + A_x z_x^1(s) \cdot \frac{k_1^x(s)}{k_1^c(s) + k_1^x(s)}\right) \cdot (k_1^c(s) + k_1^x(s))$$

[16] Similarly to Castro, Clementi, and MacDonald (2009), the presence of the financial frictions (here, due to $\theta < 1$ and $\xi > 0$) distorts the price by inducing a wedge that is additive with productivity.

and similarly, the construction of aggregate productivity at $t = 0$ is defined as Z_0:

$$Z_0 \cdot (k_0^c + k_0^x) \equiv \left(A_c \cdot \frac{k_0^c}{k_0^c + k_0^x} + A_x \cdot \frac{k_0^x}{k_0^c + k_0^x} \right) \cdot (k_0^c + k_0^x)$$

By substituting optimal capital decisions into $(Z_0, Z_1(s))$, productivity growth is expressed as:

$$\frac{Z_1(s)}{Z_0} = \frac{\frac{A_c}{A_c+A_x}\left(r_1^c(s) + \frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\right) + \frac{A_x}{A_c+A_x} \cdot z_1^x(s)\left(r_1^x(s) + \frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\right)}{\left(r_1^c(s) + \frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\right) + \left(r_1^x(s) + \frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\right)}$$

Lastly, by substituting in the expressions for returns, $r_1^i(s)$, and substituting the market clearing price, the growth rate is given by:

$$\begin{aligned}\frac{Z_1(s)}{Z_0} = \frac{A_c}{A_c + A_x} &\left[\frac{\left(A_x(1 - z_1^x(s)) + (1+\theta)(1-\delta)\right) + \left(A_x(1 - z_1^x(s)) + 2(1-\delta)\right)\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi} \cdot \frac{D(A_c)}{A_c}}{(A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi} \cdot \frac{D(A_c)}{A_c}} \right] \\ + \frac{A_x}{A_c + A_x} z_1^x(s) &\left[\frac{\left(A_x z_1^x(s) + (1-\theta)(1-\delta)\right) + \left(A_x(1 + z_1^x(s)) + 2(1-\delta)\right)\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi} \cdot \frac{D(A_c)}{A_c}}{(A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi} \cdot \frac{D(A_c)}{A_c}} \right]\end{aligned}$$

Taking the variance of productivity growth, we obtain:

$$\begin{aligned} var\left(\frac{Z_1(s)}{Z_0}\right) = &\frac{1 + \left(\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)^2}{\left((A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)^2} \cdot var\left(\left(\frac{A_x}{A_c + A_x} z_1^x(s) - \frac{A_c}{A_c + A_x}\right) A_x z_1^x(s)\right) \\ &+ \frac{1 + \left(\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)^2 \left(\frac{2}{1-\theta}\right)^2}{\left((A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)^2} \cdot (1-\theta)^2(1-\delta)^2 var\left(\frac{A_x}{A_c + A_x} z_1^x(s)\right) \\ &+ \frac{\left(1 + \left(\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)\right)\left(1 + \left(\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)\left(\frac{2}{1-\theta}\right)\right)}{\left((A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0-R^{-1}\xi}\ \frac{D(A_c)}{A_c}\right)^2} \times \\ &\times (1-\theta)(1-\delta) cov\left(\frac{A_x}{A_c + A_x} z_1^x(s), \left(\frac{A_x}{A_c + A_x} z_1^x(s) - \frac{A_c}{A_c + A_x}\right) A_x z_1^x(s)\right)\end{aligned}$$

Note that the covariance term on the last line is positive.[17] Also notice that even if $\xi = 0$,

[17] This covariance is akin to $cov[x, x(x-1)] = \mathbb{E}[x^2(x-1)] - \mathbb{E}[x]\mathbb{E}[x(x-1)] = \mathbb{E}[(x-1)^2 x] > 0$, for $x \in \mathbb{R}_{++}$.

a decrease in A_c leads to an increase in the variance. This is because a decrease in A_c increases the productivity of investment firms relative to consumption firms. Because investment firms' exhibit more volatile productivity shocks, the decrease in A_c induces a reallocation of capital into the more volatile sector.

When $\xi > 0$, changes in A_c additionally affect the down payment on capital, $D(A_c)$. This effect reflects the fact that the fixed cost of financial intermediation (ξ) is larger in poor countries than rich countries as a fraction of total output. I now show that there are conditions on parameters such that down payments amplify investment sector productivity shocks and generate larger amplification in low productivity (e.g. low A_c) than in high productivity economies.

LEMMA: If model parameters satisfy

(a) the expected price at $t = 1$ is large relative to p_0: $D(A_c) < 0$,

(b) the expected price increase is sufficiently large: $\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi} \cdot |D(A_c)| > \dfrac{A_x + 2(1-\delta)}{A_x + 4(1-\delta)}$,

(c) the down payment increases with productivity: $\frac{\partial}{\partial A_c}\left(\frac{D(A_c)}{A_c}\right) > 0$, and

(d) initial wealth is sufficiently large: $n_0 > R^{-1}\xi$;

then $var(Z_1(s)/Z_0)$ is decreasing in A_c.

Proof: Showing that $var(Z_1(s)/Z_0)$ is decreasing in A_c reduces to finding conditions on $D(A_c)$ such that the coefficient on the variance terms are decreasing in A_c.

$$\frac{\partial}{\partial A_c}\left[\frac{1 + \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\frac{D(A_c)}{A_c}}{(A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\frac{D(A_c)}{A_c}}\right] = \frac{\left[(A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\frac{D(A_c)}{A_c}\right]}{\left[(A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\frac{D(A_c)}{A_c}\right]^2}$$

$$+ \frac{\left[(A_x + 2(1-\delta)) - (A_x + 4(1-\delta))\right]\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi} \cdot \frac{\partial}{\partial A_c}\frac{D(A_c)}{A_c}}{\left[(A_x + 2(1-\delta)) + (A_x + 4(1-\delta))\frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}\frac{D(A_c)}{A_c}\right]^2}$$

Condition (c) ensures that the first term is positive. Since $(A_x + 2(1-\delta)) - (A_x + 4(1-\delta)) < 0$, we want to find parameter restrictions that imply $\frac{\partial[D(A_c)/A_c]}{\partial A_c} > 0$. Since we can write $D(A_c)/A_c$ as:

$$\frac{D(A_c)}{A_c} = \left(\frac{1 - R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right] \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}}{1 + R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right] \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}} \right) \frac{p_0}{A_c}$$
$$- \frac{R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right]}{1 + R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right] \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}}$$

we must have:

$$R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right] \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi} < 1$$

which is satisfied by a sufficiently large n_0. To ensure $D(A_c)/A_c < 0$, choose p_0 small enough so that:

$$\frac{p_0}{A_c} < \frac{R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right]}{1 - R^{-1}\theta(1-\delta)\mathbb{E}\left[\frac{1}{A_x(1-z_x(s))+2\theta(1-\delta)}\right] \frac{(1-R^{-1})\xi}{n_0 - R^{-1}\xi}}$$

■

C.5 Proof for Proposition 8 (Inefficiency)

The proof follows from a straightforward comparison of Euler equations for the price taking competitive firm and the social planner. Take a competitive equilibrium allocation and suppose it is constrained efficient. Then subtracting the competitive equilibrium consumption sector Euler equation from the social planner's Euler equation, and rearranging yields:

$$(1-\delta)\sum_{\mathbf{z}'} \pi(\mathbf{z},\mathbf{z}') \left[(1-\theta)m(\mathbf{z},\mathbf{z}') \frac{v_b(k', b'(\mathbf{z}'), \mathbf{z}')}{v_b(k, b, \mathbf{z})} + R^{-1}\theta \right] \psi(k', b'(\mathbf{z}'), \mathbf{z}') p(\mathbf{z}') - \psi(k, b, \mathbf{z}) p = 0$$

Unless $p(\mathbf{z}') = \bar{p} \in \mathbb{R}_+$ for all $(k', b'(\mathbf{z}'), \mathbf{z}') \in \mathbb{R}_+ \times \mathbb{R}_+ \times \mathcal{Z}^2$, there exists $(k', b'(\mathbf{z}'), \mathbf{z}')$ such that either $p_k(\mathbf{z}') \neq 0$ or $p_b(\mathbf{z}') \neq 0$ or both. Then $\psi(k', b'(\mathbf{z}'), \mathbf{z}') \neq 0$. But this violates the above condition, yielding a contradiction. $\rightarrow\leftarrow$

C.6 Proof for Proposition 9 (Decentralization)

Suppose that consumption sector firms take prices and taxes as given. Consumption sector firms face investment taxes and lump sum rebates, denoted (τ^c, T^c). Therefore the consumption sector firm's budget constraint becomes:

$$d_c + (1+\tau^c)p\left(k_c' - (1-\delta)k_c\right) - R^{-1}\sum_{\mathbf{z}'}\pi(\mathbf{z},\mathbf{z}')b_c'(\mathbf{z}') \leq F(k_c, z_c) - b_c + T^c$$

Because capital purchases are now taxed, the debt constraint must reflect the tax-adjusted value of capital:

$$b_c'(\mathbf{z}') \leq \theta(1-\delta)(1+\tau^c(\mathbf{z}'))p(\mathbf{z}')k_c' - \xi$$

Showing that the debt constraint is now written with a tax on collateralized capital requires a very straightforward extension of Proposition 1. The only additional assumption needed is that taxes are paid after the default decision. This means that lenders discount collateral values by the amount of investment taxes they will pay upon default and seizure.

For any given a state vector, define the policy as:

$$\tau^c(k,b,\mathbf{z}) = \psi_c(k,b,\mathbf{z})$$

$$T^c(k,b,\mathbf{z}) = \psi_c(k,b,\mathbf{z})p(k_c'(k,b,\mathbf{z}) - (1-\delta)k)$$

With this budget constraint and the defined tax and lump sum rebate, the competitive consumption sector firm's Euler equation and constraint set are equivalent to those in the social planner's problem.

Now take investment sector firms. Suppose investment sector firms face investment taxes, income taxes and lump sum rebates, denoted (τ^x, ω^x, T^x). The investment sector firm's budget constraint becomes:

$$d_x + (1+\tau^x)p\left(k_x' - (1-\delta)k_x\right) - R^{-1}\sum_{\mathbf{z}'}\pi(\mathbf{z},\mathbf{z}')b_x'(\mathbf{z}') \leq (1-\omega^x)pF(k_x, z_x) - b_x + T^x$$

Because capital purchases are now taxed, the debt constraint must reflect the tax-adjusted value of capital:

$$b_x'(\mathbf{z}') \leq \theta(1-\delta)(1+\tau^x(\mathbf{z}'))p(\mathbf{z}')k_x' - \xi$$

For any given a state vector, define the policy as:

$$\tau^x(k,b,\mathbf{z}) = \psi_x(k,b,\mathbf{z})$$
$$\omega^x(k,b,\mathbf{z}) = -\Psi(k,b,\mathbf{z})$$
$$T^x(k,b,\mathbf{z}) = -\Psi(k,b,\mathbf{z})pF(k,z) + \psi_x(k,b,\mathbf{z})p(k'_x(k,b,\mathbf{z}) - (1-\delta)k)$$

With this budget constraint and the defined tax and lump sum rebate, the competitive investment firm's Euler equation and constraint set are equivalent to those in the social planner's problem.

Because the optimality conditions and constraints for firms in both sectors are equivalent to the corresponding conditions and constraints in the social planner's problem, competitive equilibrium allocations featuring the defined tax policy are equivalent to constrained efficient allocations. ■

D Algorithm

Overview: This algorithm uses a variant of time iteration (c.f. Coleman (1990)) and value function iteration. Because firms can purchase Arrow securities subject to debt constraints, firms face a static portfolio allocation decision. I nest this static portfolio allocation problem within a time iteration loop to solve dynamic conditions. That is, given the optimal portfolio, I use time iteration to update the value function. First I use the budget constraint to compute the value of net worth that an agent choosing the given portfolio must have carried into the period. Then I use the Euler equation to find the value of the value function that is consistent with the continuation value generated by the optimal portfolio. I interpolate today's value function as a function of the endogenous grid on net worth back onto the net worth grid to recover the updated value function.

The following algorithm solves the firm's problem in a baseline case. This solution details how to solve a version Rampini and Viswanathan (2012) model, which does not include prices (it is a one-sector model) and without the consumer's stochastic discount factor. Let $F(k,z) = z^{1-\alpha}k^{\alpha}$. The firm solves:

$$v(n,z) = \max_{d,k,n(z')} d + \beta \sum_{z'} \pi(z,z')v\left(n'(z'),z'\right)$$

$$\text{s.t.} \quad d + pk' - R^{-1} \sum_{z'} \pi(z,z')b(z') \leq n$$
$$n'(z') = F(k',z') + (1-\delta)p(z')k' - b(z')$$
$$b(z') \leq \theta(1-\delta)p(z')k' - \xi$$
$$d \geq 0$$

with optimality conditions:

$$v_n(n,z) \geq 1 \qquad \text{[Envelope condition]}$$
$$\mu(z') = v_n(n,z) - \beta R v_n(n'(z'),z') \qquad \text{[FOC wrt } n'(z')\text{]}$$
$$1 = \beta \sum_{z'} \frac{v_n(n'(z'),z')}{v_n(n,z)} \cdot \frac{F_k(k',z') + (1-\delta)}{1 - R^{-1}\theta(1-\delta)} \qquad \text{[Euler equation]}$$

where $\mu(z')$ is the multiplier on the debt constraint.

Heuristic Algorithm:

1. Construct grids for capital investment ($k' \in G_k$) and net worth ($n \in G_n$). Use Rowenhourst's method (c.f. **?**) to discretize the stochastic process for z and obtain a grid, G_z.

   - Denote grid size by N_i for $i \in \{k,n,z\}$
2. Guess an initial value function over $(n,z) \in G_n \times G_z$
3. Step 1: Static Portfolio Allocation Problem
   - Guess and verify that $\mu(z') = 0$ for all $z' \in G_z$
   - Use the Euler equation and FOC wrt $n'(z')$ to compute the N_z dimensional fixed point for $n'(z')$ for each $(n,z) \in G_n \times G_z$
   - If the solution for $n'(z')$ violates the debt constraint, set $n'(z')$ to the value implied by a binding debt constraint.
4. Step 2: Time Iteration on Value Function
   - Use $n'(z')$ to compute today's net worth from the budget constraint
   - Use the Euler equation and values for $n'(z')$ to compute today's value function
   - Interpolate to recover the value function on grid $G_n \times G_z$

5. Check for convergence on the value function. If no convergence, update value function and return to Step 1.

Definition: Define the function $\Phi(x, y|\hat{x})$ as an "interpolating function." This function takes values of $y(x)$ on the grid for x and interpolates to find the values of $y(\hat{x})$ for $\hat{x}$ that are not on the grid for x. I use a shape preserving interpolant (linear interpolation) in my implementation of the algorithm.

Implementation of Algorithm:

1. Construct grids: k', z and n

2. Guess: $v_n^i(n,z)$ for all $(n,z) \in G_n \times G_z$

3. Compute the optimal portfolio $\{n'(z')\}$ for all $z' \in G_z$
   - Construct a grid for the expectation term in the Euler equation $V \in G_V$ such that values of $v \in G_V$ satisfy the envelope condition $(V \geq 1)$. A generic element of V corresponds to:[18]
   $$V \equiv v_n(n,z)$$
   - For each $(k',z) \in G_k \times G_z$, find $n'(z')$ such that the FOC wrt $n'(z')$ holds with $\mu(z') = 0$, e.g. $\beta R v_n(n'(z'),z') = V$, and impose the debt constraint, which implies $V > \beta R v_n(n'(z'),z')$:
   $$n'(z',V) = \max\left\{\Phi(v_n(n,z), n \in G_n \mid V \in G_V), F(k',z') - (1-\theta)(1-\delta)k'\right\}$$
   - Find $\bar{V}$ for each (k',z) such that the Euler equation holds:
   $$\bar{V} = \beta \sum_{z' \in G_z} \pi(z,z') v_n(n'(z',\bar{V}),z') \frac{F_k(k',z') - \theta(1-\delta)}{1 - R^{-1}\theta(1-\delta)}$$
   by interpolating to find the related value function and imposing $v_n(n,z) \geq 1$:
   $$v_n(n'(z',V),z') = \max\left\{1, \Phi\left(n \in G_n, v_n(n,z) \mid n'(z',V)\right)\right\}$$

[18] Alternatively, I could use a root-finder to solve for the value of $V \geq 1$ that solves the following fixed point. I found in trials that an intelligent construction of G_V performed faster than implementing a root-finder.

then finding the fixed point:

$$v_n^e(k',z) \equiv \Phi\left(V - \beta \sum_{z'\in G_z} \pi(z,z') v_n(n'(z',V),z') \frac{F_k(k',z') - \theta(1-\delta)}{1 - R^{-1}\theta(1-\delta)}, V \,\Big|\, 0\right)$$

where superscript "e" denotes the value of a function on the "endogenous grid."

- Compute the optimal portfolio:

$$n'(k',z,z') = \Phi\left(v_n(n,z), n \in G_n \mid v_n^e(k',z)\right)$$

4. Update the value function using the endogenous grid

- Compute the endogenous grid points

$$n^e(k',z) = d(k',z) + k' - R^{-1} \sum_{z'\in G_z} \pi(z,z') \left[F(k',z') + (1-\delta)k' - n(k',s,z')\right]$$

where I set $d(k',z) = 0$ since the firm pays no dividends while $n < \bar{n}(z)$. If $n^e(k',z) \geq \bar{n}(z)$, then $d(k',z) = n^e(k',z) - \bar{n}(z)$ and set $n^e = \bar{n}(z)$.

- Update the value function using the endogenous grid

$$v_n^{i+1}(n,z) = \max\left\{1, \Phi\left(n^e(k',z), v_n^e(k',z) \mid n \in G_n\right)\right\}$$

5. Check convergence of the value function

- Compute convergence statistic:

$$\kappa_i^v \equiv \max_{n,s} \left| v^{i+1}(n,z) - v^i(n,z) \right|$$

- Stop iterations if functions converge, $\kappa_i^v < \varepsilon$.
- Otherwise set $i = i + 1$ and go back to step 3.