Demand Uncertainty, Selection, and Trade *

Erick Sager †
Federal Reserve Board

Olga A. Timoshenko ‡
Temple University

February 9, 2026

Abstract

This paper examines the effect of idiosyncratic uncertainty on elasticities of trade flows with respect to variable trade costs in a canonical model of trade with monopolistic competition and heterogeneous firms. We identify two channels through which uncertainty impacts these elasticities. First, uncertainty lowers elasticities by lowering export participation *thresholds* (selection effect). Second, uncertainty alters the *distribution* and lowers the dispersion of export selection shocks (dispersion effect), which has an ambiguous effect on trade elasticities. We develop a novel methodology to quantify trade elasticities under uncertainty and apply it to Brazilian firm-level export data. Quantitatively, we find that the endogenous selection component of the trade elasticity is small on average, reflecting the large dispersion in exporter sizes observed in the data. Relative to a complete information framework, uncertainty amplifies trade elasticities on average by a modest amount, though the effect is heterogeneous across industries. The largest differences between the two information environments are observed among highly substitutable products or those with high uncertainty, suggesting that the choice of information structure matters most in these cases.

Keywords: Demand uncertainty, firm size distribution, extensive margin, selection, trade elasticities.

JEL: F12, F13.

---

*We thank Arevik Gnutzmann-Mkrtchyan, Mina Kim, Logan Lewis, Martin Lopez-Daneri, Andre Kurmann, Peter Morrow, Nuno Limao, Moritz Ritter, Andrés Rodrígues-Clare, Ina Simonovska, Ben Williams, and Yoto Yotov, as well as seminar participants at Drexel, the George Mason University, the University of Maryland, Temple University, Bank of Canada, NASM 2024, Asian Meeting of the Econometric Society in East and Southeast Asia 2023, Canadian Economic Associations Meetings 2018, BEROC International Economics Conference 2018, Southern Economic Association 88th Annual Meetings 2018, Washington Area International Trade Symposium 2018, 1st Mid-Atlantic Trade Workshop 2017, and Midwest International Trade Meeting 2017 for valuable comments and discussions. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Board of Governors or the Federal Reserve System. The work was supported in part by the facilities and staff of the George Washington University Colonial One High Performance Computing Initiative. The paper previously circulated under the title "Uncertainty and Trade Elasticities". First version: March 2017.

†2001 C Street NW, Washington DC, 20551, United States. E-mail: erick.r.sager@frb.gov

‡Corresponding author, Temple University, Department of Economics, 1115 Polett Walk, Philadelphia, PA 19122, United States. E-mail: olga.timoshenko@temple.edu

1 Introduction

Firms face information asymmetries in foreign markets, which subsequently shape their export behavior - whether to do so or not, how much to export, which prices to set, how many products to export and to which destinations.[1] Several recent firm-level surveys further provide evidence that firms form expectations about their profitability to guide their decisions.[2] Notably however, most of the literature focusing on estimating gains from trade and, specifically, a partial elasticity of trade flows with respect to variable trade costs (a key parameter in that exercise) does so in a framework that assumes complete information. In this paper, we examine the effect of idiosyncratic firm-level information asymmetries on trade elasticities, develop a method for estimating those in a model that incorporates such asymmetires, and quantify the bias in measuring trade elasticities arising from the assumption of complete information.

To do so, we model information asymmetries as unobserved firm-level idiosyncratic demand shocks as in Timoshenko (2015), and introduce this assumption into a canonical model of trade with monopolistic competition and heterogeneous firms à la Melitz (2003). In the model, firms decide whether to export or not, and how much to sell prior to observing realizations of demand shocks. They base their export decisions on known idiosyncratic productivity draws and on idiosyncratic expectations about demand shocks.

In the model, demand uncertainty affects the partial elasticity of trade with respect to variable trade costs by reducing export selection thresholds - the *selection effect* of uncertainty - and by altering the distribution of export selection shocks - the *dispersion effect* of uncertainty. On the one hand, a lower export selection threshold reduces the size of a marginal exporter, which generates smaller changes in trade flows as a results of changes in variable trade costs. On the other hand, the distribution of export selection shocks governs the mass of exporters at that threshold. Uncertainty reduces the dispersion of export selection shocks, which has an ambiguous effect on the mass of exporters at a given threshold. Therefore, the total effect of uncertainty on the partial trade elasticities is theoretically ambiguous and largely depends on the size and direction of the dispersion effect. This result alone underscores the importance of distributional assumptions that the literature makes

[1] Such asymmetries include aggregate economic uncertainty (Baker et al., 2016; Baley et al., 2020), trade policy uncertainty (Handley and Limao, 2015), idiosyncratic uncertainty (Jovanovic, 1982; Timoshenko, 2015).

[2] In the U.S., the Census Bureau's Management and Organizational Practices Survey collect data on a firm's five-point forecast of own future employment, output and inputs (Bloom et al., 2020). In Japan, Basic Survey on Overseas Business Activities collects data on a firm's forecast for next-year sales for its foreign affiliates (Chen et al., 2023). In Germany, IFO Business Climate Survey collects data on a firm's predictions of whether its production will increase, decrease or stay the same in the next three months (Bachmann et al., 2013).

when modeling firm-level heterogeneity.

We proceed by developing a novel approach to quantifying both trade elasticities in a model with uncertainty, and the magnitude of the selection and dispersion channels through which uncertainty impacts them. To do so, the estimation requires disentangling the export selection shocks, which consist of productivity and demand expectations and which determine firms' export decisions and therefore trade elasticities, from the unexpected component of demand shocks. Our methodology exploits the structure of residuals in the export sales and quantity data, purged of the aggregate effects common across firms, and uses the model to impose identifying structure on these residuals. In short, model assumptions on the timing of firms' decisions imply that the unexpected component of demand shocks is orthogonal to the quantity decisions of firms. This orthogonality restriction allows us to isolate the unexpected component of demand from sales residuals and subsequently recover firm-level export selection shocks.

Next, quantifying trade elasticities requires parameterizing the distribution of export selection shocks. Given that the dispersion effect of uncertainty is ambiguous and depends on the entire shape of the distribution, it is important to impose a distributional assumption with the most flexibility to capture asymmetries observed in the data. As shown in the work of Sager and Timoshenko (2019), the Double Exponentially Modified Gaussian (Double EMG) distribution provides such a flexible structure by combining features of the Normal and double Pareto distributions, therefore yielding a bell-shaped distribution that exhibits various degrees of fatness in the left- and right- tails. We parameterize the distribution of the firm-level export selection shocks with a Double EMG distribution, and estimate its parameters using the Generalized Methods of Moments procedure.

We apply the methodology to Brazilian firm-level export data for the period between 1997 and 2000. Notably, our sample period includes Brazil's 1999 exchange rate devaluation, which provides an opportunity to empirically motivate the relevance of demand uncertainty. While devaluations are typically interpreted as reductions in variable trade costs, our framework suggests an additional channel: macroeconomic turbulence may increase the uncertainty foreign buyers face when forming demand for Brazilian products. Consistent with this interpretation, we document that the dispersion of unexpected demand shocks increased significantly following the devaluation, while the dispersion of export selection shocks remained stable—suggestive evidence that demand uncertainty is an empirically relevant feature of our setting.

Turning to the quantification of trade elasticities, we find that, under uncertainty, the average endogenous selection effect of partial trade elasticity amounts to 0.02, albeit heterogeneous across products and is higher in products with a larger elasticity of substitution across varieties. The magnitude implies that entrants and exiters change trade flows by an additional 2% relative to the change in trade flows generated by incumbent firms. These estimates are comparable to those obtained in the Melitz-Pareto framework with a thick Pareto tail parameter.

When compared to the counterfactual estimates of trade elasticities obtained under the complete information assumption, we find that trade elasticities are higher under uncertainty relative to complete information in about eighty percent of observations and lower in twenty percent of observations. Notably, the more uncertain the demand is, the larger is the amplification effect of uncertainty. Furthermore, the amplification effect of uncertainty increases with the elasticity of substitution across varieties with dampening effect concentrated among inelastic products. This means the mass of exporters at the threshold, governed by the underlying distribution of export selection shocks, plays the dominant role in determining how trade flows respond to changes in trade costs among substitutable products, while the selection effect and the size of marginal exporter plays the dominant role among inelastic products.

Our work is related to several strands of the literature on international trade. A large body of literature looks into the structure of trade elasticity and its decomposition into the intensive and extensive margins pioneered by Chaney (2008). In particular, Melitz and Redding (2015) show that the extensive margin of the partial trade elasticity depends on the distributional assumptions with respect to the sources of firm-level heterogeneity. We contribute to this literature by demonstrating that uncertainty does not affect the intensive margin of the partial trade elasticity, but rather has an ambiguous effect on the extensive margin.

Several papers propose various methods to estimate partial trade elasticities. These methods rely on complete information environment and use aggregate trade flows and prices data (see Eaton and Kortum (2002) and Simonovska and Waugh (2014)), trade flows and tariff data (Caliendo and Parro, 2015), or firm-level data (Bas et al., 2017).[3] We extend Bas et al. (2017) methodology to an environment with information asymmetries.

A growing branch of the literature has demonstrated that models incorporating idiosyncratic uncertainty along the lines of Jovanovic (1982) are well suited to match salient patterns of empirically observed firm behavior such as firm growth as a function of age and size (Arkolakis, Papageorgiou, and Timoshenko (2018)), firm product switching behavior (Timoshenko, 2015), and firm input and output pricing behavior (Bastos, Dias, and Timoshenko,

[3]This literature further finds elasticities estimated from aggregate trade flows are smaller than those estimated from disaggregated industry-level data (Imbs and Mejean (2015)), and that there is substantial heterogeneity in bilateral trade elasticities due to heterogeneity in countries' industrial production (Imbs and Mejean (2017)).

2018). We contribute to this literature by further exploring implications of the idiosyncratic (demand) uncertainty framework for measuring partial trade elasticities.

A body of literature on information asymmetries in trade has focused on trade policy uncertainty. Handley and Limao (2015) find that trade policy uncertainty generates a wedge in the export participation thresholds between the two information environments and lowers entry into foreign markets by reducing the value of the export participation threshold. In contrast, we find the opposite result. The distinction arises from differences in the timing of when information is revealed to firms and the option value of waiting such timing may produce. In Handley and Limao (2015) firms first observe a realization of tariff policy and then make their decisions. The framework therefore features the option value of waiting: Firms can condition their entry decisions on a realization of a shock and only enter when the realization of a shock is high enough, a mechanism absent from our framework. In our framework, uncertainty is revealed only after entry and production decisions have been made. Therefore, waiting has no impact on a firm's decision relevant information.

Our analysis complements the literature estimating trade elasticities from price and trade flow data. Simonovska and Waugh (2014) use cross-destination price variation to estimate sectoral trade elasticities that aggregate over firm heterogeneity and adjustment margins. Our decomposition reveals that these aggregate elasticities reflect not only firm-level responses to trade costs but also an endogenous selection component that varies with the distribution of firm-level shocks and export thresholds. By quantifying this selection component separately, we provide a micro-founded explanation for cross-sectional heterogeneity in trade elasticities and show how uncertainty contributes to this heterogeneity through the selection and dispersion channels.

Our results are most closely related to Baley, Veldkamp, and Waugh (2020), who consider a framework that demonstrates a possibility that firms can export more when there is greater uncertainty about the terms of trade in bilateral trade relationships. In the authors' framework, the ambiguous effect of uncertainty on trade is driven by how preferences weigh differently changes in the mean versus variance in returns to exporting. In our framework the ambiguity stems from the dispersion effect of uncertainty and changes in the distribution of export selections shocks.[4]

[4]Other papers that consider the effects of information on trade include Bergin and Lin (2012) who show that the entry of new varieties increases at the time of the announcement of the future implementation of the European Monetary Union, suggesting that changes in the information available to firms have immediate consequences for firms' decisions. Lewis (2014) studies the effect of exchange rate uncertainty on trade; Allen (2014) shows that information frictions help to explain price variation across locations; Fillat and Garetto (2015) show that aggregate demand fluctuations can explain variation in stock market returns between multinational and non-multinational firms; Dickstein and Morales (2018) show that traditional estimates of the model parameters, such as fixed costs of exporting, depend on how firm's expectations are modeled.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 characterizes the effect of uncertainty on trade elasticities. Section 4 details our empirical methodology for quantifying trade elasticites in an environment with uncertainty. Section 5 describes our data and presents elasticity estimation results. Section 6 performs a counterfactual analysis of trade elasticities in an environment with complete information. Section 7 concludes. All proofs, derivations, and robustness checks are relegated to the Appendix.

2 Theoretical Framework

This section outlines our main theoretical framework which will serve as the structural benchmark for quantifying trade elasticities in a model with uncertainty. We consider an economic environment in which heterogeneous firms export products to monopolistically competitive markets. This environment is similar to that in Melitz (2003) with an added dimension of demand uncertainty according to Jovanovic (1982) as adapted to a heterogeneous firms framework by Timoshenko (2015) and Arkolakis et al. (2018). We assume exogenous entry as in Chaney (2008).[5]

2.1 Demand

There are N countries and K sectors in each country. Each country is indexed by j and each sector is indexed by k.

Each country is populated by a mass of L_j identical consumers. Each consumer within country j owns an equal share of domestic firms and is endowed with a unit of labor that is inelastically supplied to the labor market. The preferences of a representative consumer in country j are represented by a nested constant elasticity of substitution utility function

$$U_j = \prod_{k=1}^{K} \left[\left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{ijk}} \left(e^{z^p_{ijk}(\omega)} \right)^{\frac{1}{\epsilon_k}} c_{ijk}(\omega)^{\frac{\epsilon_k - 1}{\epsilon_k}} d\omega \right)^{\frac{\epsilon_k}{\epsilon_k - 1}} \right]^{\mu_k}, \tag{1}$$

where Ω_{ijk} is the set of varieties in sector k consumed in country j originating from country i, $c_{ijk}(\omega)$ is the consumption of variety $\omega \in \Omega_{ijk}$, ϵ_k is the elasticity of substitution across varieties within sector k, $z^p_{ijk}(\omega)$ is the demand shock for variety $\omega \in \Omega_{ijk}$, and μ_k is the Cobb-Douglas utility parameter for goods in sector k such that $\sum_{k=1}^{K} \mu_k = 1$.

Cost minimization yields a standard expression for the optimal demand for variety $\omega \in$

[5] All derivations are relegated to Appendix A.

Ω_{ijk}, given by

$$c_{ijk}(\omega) = e^{z^p_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}, \tag{2}$$

where $p_{ijk}(\omega)$ is the price of variety $\omega \in \Omega_{ijk}$, Y_{jk} is total expenditures in country j on varieties from sector k, and P_{jk} is the aggregate price index in country j in sector k.[6]

2.2 Supply

Each variety $\omega \in \Omega_{ijk}$ is supplied by a monopolistically competitive firm f that has access to a linear production technology that transforms labor into output, $q = \exp(z^a)\ell$. Upon entry, a firm f selling from country i to country j in sector k is endowed with an idiosyncratic labor productivity level z^a_{fijk} and a set of idiosyncratic destination-sector specific demand shocks, $\{z^p_{fijk}\}_{j=1,\ldots,N}$.[7] Each demand and supply shocks pair (z^p_{fijk}, z^a_{fijk}) is drawn from a joint distribution to be characterized later.

Firms from country i selling output in sector k to country j face fixed costs, f_{ijk}, and variable 'iceberg' trade costs, τ_{ijk}. Fixed and variable costs are denominated in units of labor, and w_j denotes the wage rate in country j.

Each firm can potentially supply one variety of a product from each sector. Firms decide which markets to export to (the extensive margin decision) and how much to export to each of the chosen markets (the intensive margin decision). Without loss of generality, we assume that firms choose a quantity to export. Prices are the result of market clearing given the exported quantity of the variety, and then export sales are realized along with prices.

2.3 Information Structure

We consider an environment with complete information and an environment with uncertainty.

In the environment with complete information, firms observe all idiosyncratic shocks before making decisions. Namely, firms observe their supply, z^a_{fijk}, and demand, z^p_{fijk}, shocks before deciding where to export and how much to export. Denote the firm's decision relevant *export selection* shock in the complete information environment by z^{CI}_{fijk}. As we demonstrate below, z^{CI}_{fijk} is given by

$$z^{CI}_{fijk} = (\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}. \tag{3}$$

[6] The assumed Cobb-Douglas utility specification over consumption bundles across sectors implies $Y_{jk} = \mu_k Y_j$, where Y_j is aggregate income in country j.

[7] The idiosyncratic demand shocks are realized by consumers, but are a payoff relevant state for the firms. Thus, when firms enter, they draw their realization of the idiosyncratic demand of consumers that determines their sales. Following Foster, Haltiwanger, and Syverson (2008), who document that idiosyncratic firm-level demand shocks, rather than productivity, account for a greater variation of sales across firms, we focus on the demand shocks that are firm specific.

In the environment with uncertainty, firms do not observe all idiosyncratic shocks before making export decisions. The timing of the information and firm's decisions follows Arkolakis et al. (2018) and is as follows.

1. First, firms observe their supply side shocks, z^a_{fijk}, and form expectations about demand shocks, $E(z^p_{fijk}|z^a_{fijk})$.

2. Next, firms decide whether and where to export, and how much to export to the chosen destinations.

3. Production takes place and all the quantities are shipped; prices clear in destination markets.

4. Lastly, firms observe their sales and infer their demand shocks, z^p_{fijk}, from the realized observations of prices and sales.

Denote the firm's decision relevant *export selection* shock in the environment with uncertainty by z^U_{fijk}. As we demonstrate below, z^U_{fijk} is given by

$$z^U_{fijk} = (\epsilon_k - 1)z^a_{fijk} + E(z^p_{fijk}|z^a_{fijk}). \tag{4}$$

Observe from equations (3) and (4), that export decision in the environment with uncertainty are based on partial information about the realization of demand shocks. This difference leads to different implications regarding the magnitude of the partial trade elasticities with respect to variable trade costs across information environments, and provides novel insights into which data are suited to structurally identify the partial trade elasticities in the environment with uncertainty.

2.4 Stochastic Structure and Distributional Assumptions

We impose the following structure on the distribution of shocks. The pair (z^a_{fijk}, z^p_{fijk}) is drawn from a joint distribution. Using the orthogonal projection of z^p_{fijk} on z^a_{fijk}, the demand shock can be expressed as

$$z^p_{fijk} = E(z^p_{fijk}|z^a_{fijk}) + \upsilon_{fijk}, \tag{5}$$

where υ_{fijk} are i.i.d and are orthogonal to z^a_{fijk} by construction. In the environment with uncertainty, we will refer to υ_{fijk} as the unexpected component of the demand shock.

To enable empirical quantification, we further assume that the conditional expectation of z^p_{fijk} is linear in z^a_{fijk}, namely

$$E(z^p_{fijk}|z^a_{fijk}) = \alpha_{jkt} z^a_{fijk}, \tag{6}$$

where $\alpha_{jkt} = \rho_{jkt}(V^{1/2}_{z^p_{fijk}}/V^{1/2}_{z^a_{fijk}})$, ρ_{jkt} is the correlation between z^p_{fijk} and z^a_{fijk}, and therefore $\upsilon_{fijk} \sim$ i.i.d. $N\left[0, (1-\rho^2_{jkt})V_{z^p_{jkt}}\right]$.

Notice, we assume that υ_{fijk} are i.i.d. with mean zero. This implies $E(e^{\upsilon_{fijk}}) = e^{V_{\upsilon_{fijk}}/2} > 1$, which differs from the convention in a branch of the uncertainty literature where $E(e^{\upsilon_{fijk}}) = 1$ is imposed through the normalization $\mu_{\upsilon_{fijk}} = -1/2V_{\upsilon_{fijk}}$ (Arellano et al., 2019; Fernández-Villaverde and Guerrón-Quintana, 2020). We show in Appendix C that our results are invariant to the choice of normalization.[8]

As noted in equations (3) and (4), firm export decisions depend on a composite *export selection* shock that combines productivity and (expected) demand. We parameterize the distribution of this composite shock using the Double Exponentially Modified Gaussian (Double EMG) distribution, $DEMG(\mu, \sigma^2, \lambda_L, \lambda_R)$, described by the following cumulative distribution function:

$$\begin{aligned} G(z) = \Phi\left(\frac{z-\mu}{\sigma}\right) \quad &- \quad \frac{\lambda_L}{\lambda_L+\lambda_R} e^{-\lambda_R(z-\mu)+\frac{\sigma^2}{2}\lambda_R^2}\Phi\left(\frac{z-\mu}{\sigma}-\lambda_R\sigma\right) \\ &+ \quad \frac{\lambda_R}{\lambda_L+\lambda_R} e^{\lambda_L(z-\mu)+\frac{\sigma^2}{2}\lambda_L^2}\Phi\left(-\frac{z-\mu}{\sigma}-\lambda_L\sigma\right), \end{aligned} \tag{7}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.

The Double EMG distribution provides a flexible generalization of common distributional assumptions used in the literature. From equation (7), for example, as $\sigma \to 0$ and $\lambda_L \to 0$, the Double EMG distribution converges to an Exponential (Pareto) distribution, as assumed in Chaney (2008). As $\lambda_L \to +\infty$ and $\lambda_R \to +\infty$, the Double EMG distribution converges to a Normal distribution, as assumed in Bas et al. (2017) and Fernandes et al. (2023). As $\sigma \to 0$, the Double EMG converges to a Double Exponential (Pareto) distribution.

2.5 Environment with Complete Information

In the complete information environment, a firm f's problem selling from country i to country j in sector k consists of maximizing profit

$$\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) = \max_{q_{fijk}} p_{fijk}q_{fijk} - \frac{w_i\tau_{ijk}}{e^{z^a_{fijk}}}q_{fijk} - w_i f_{ijk}, \tag{8}$$

[8] Specifically, the selection effect (the wedge between export participation thresholds under uncertainty and complete information) and the dispersion effect (the difference in the distributions of export selection shocks) depend only on the variance of υ_{fjkt}, not its mean. Consequently, all our qualitative and quantitative findings hold regardless of whether the mean of υ_{fjkt} is set to zero or normalized to ensure $E(e^{\upsilon_{fjkt}}) = 1$.

subject to the demand equation (2). A firm exports if its optimal profit from exporting is positive, $\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) \geq 0$, which yields the following export selection equation

$$e^{(\epsilon_k-1)z^a_{fijk}+z^p_{fijk}} \geq e^{z^{CI^*}_{ijk}}, \tag{9}$$

where a firm exports if inequality (9) is satisfied, and does not export otherwise. Variable $z^{CI^*}_{ijk}$ denotes the export selection threshold under complete information and is given by

$$z^{CI^*}_{ijk} = \log\left(\frac{\epsilon_k w_i f_{ijk}}{B_{ijk} f^\tau(\tau_{ijk})}\right), \tag{10}$$

where B_{ijk} is an origin-destination-sector fixed effect common across firms exclusive of the variable trade costs, and function $f^\tau(.)$ is a strictly monotonically decreasing function.[9]

A firm's export selection equation (9) implies that a firm's export decision is based on a joint realization of the supply and demand shocks, that together comprise a firm's export *selection shock*. Denote by z^{CI}_{fijk} a firm's export selection shock under complete information. From inequality (9), z^{CI}_{fijk} is defined as

$$z^{CI}_{fijk} = (\epsilon_k - 1)z^a_{fijk} + z^p_{fijk}. \tag{11}$$

The export selection equation (9) can therefore be written as

$$z^{CI}_{fijk} \geq z^{CI^*}_{ijk}. \tag{12}$$

2.6 Environment with Uncertainty

In an environment with uncertainty, a firm f from country i chooses the quantity it will export to country j in sector k in order to maximize its *expected* profit

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \max_{q_{fijk}} E_{z^p_{fijk}|z^a_{fijk}}\left(p_{fijk}q_{fijk} - \frac{w_i\tau_{ijk}}{e^{z^a_{fijk}}}q_{fijk}\right) - w_i f_{ijk} \tag{13}$$

subject to the demand equation (2). A firm exports if its optimal expected profit from exporting is positive, $E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] \geq 0$, which yields the following export selection equation

$$e^{(\epsilon_k-1)z^a_{fijk}}\left[E_{z^p_{fijk}|z^a_{fijk}}\left(e^{\frac{z^p_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k} \geq e^{z^{CI^*}_{ijk}}, \tag{14}$$

where a firm exports if inequality (14) is satisfied, and does not export otherwise. Using the orthogonal projection of z^p_{fijk} on z^a_{fijk} in equation (5), export selection equation (14) can be

[9] Specifically, $B_{ijk} = \left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}$, and $f^\tau(\tau_{ijk}) = \tau_{ijk}^{1-\epsilon_k}$.

written as

$$e^{(\epsilon_k-1)z^a_{fijk}+E(z^p_{fijk}|z^a_{fijk})}\left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k} \geq e^{z^{CI*}_{ijk}}. \tag{15}$$

Denote by z^U_{fijk} a firm's export selection shock under uncertainty, and by z^{U*}_{ijk} the export selection threshold under uncertainty. From inequality (15), z^U_{fijk} and z^{U*}_{ijk} are defined as

$$z^U_{fijk} = (\epsilon_k - 1)z^a_{fijk} + E(z^p_{fijk}|z^a_{fijk}) \tag{16}$$

and

$$z^{U*}_{ijk} = z^{CI*}_{ijk} - \log\left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k} \tag{17}$$

The export selection equation (15) can therefore be written as

$$z^U_{ijk} \geq z^{U*}_{ijk}. \tag{18}$$

3 Characterization of Trade Elasticities

In both information environments, the total trade flows from country i to country j in sector k, X_{ijk}, can be expressed as

$$X_{ijk} = J_i\left[1 - G_{ijk}(z^*_{ijk})\right]\int_{z^*_{ijk}}^{+\infty} B_{ijk}f^\tau(\tau_{ijk})e^z \frac{g_{ijk}(z)}{1 - G_{ijk}(z^*_{ijk})}dz,$$

where J_i is the exogenous mass of potential entrants in country i, z is the decisions relevant export selection shock as defined in equation (11) for the case of complete information and in equation (16) for the case of uncertainty, z^*_{ijk} is the export selection threshold as defined in equation (10) for the case of complete information and in equation (17) for the case of uncertainty, and $g_{ijk}(z)$ and $G_{ijk}(z)$ are the probability density and cumulative distribution functions of the decision-relevant export selection shock respectively.

The partial elasticity of trade with respect to the iceberg trade costs, τ_{ijk}, can then be written as

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}} = \underbrace{\frac{\partial \log f^\tau(\tau_{ijk})}{\partial \log \tau_{ijk}}}_{\substack{\text{firm-level} \\ \text{trade elasticity}}} \underbrace{\left[1 + \gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))\right]}_{\text{endogenous selection}}, \tag{19}$$

where $\gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))$ is a monotonically increasing hazard rate function associated with the random variable distributed according to the probability density function $h_{ijk}(z^*_{ijk}, g_{ijk}(z))$

given by[10]

$$h_{ijk}(z^*_{ijk}, g_{ijk}(z)) \;=\; \frac{e^{z^*_{ijk}} g(z^*_{ijk})}{\int_{-\infty}^{+\infty} e^z g_{ijk}(z) dz}.$$

The first component of the partial trade elasticity in equation (19) is referred to as the firm-level trade elasticity (Bas et al., 2017) and captures the response of incumbent exporters to changes in variable trade costs. It is determined by the elasticity of substitution across varieties, is given by $(1 - \epsilon_k)$, and does not depend on the information environment. The firm-level trade elasticity is subsequently augmented by the endogenous selection component that arises due to the presence of entry and exit mechanism in export markets. It is the endogenous selection component that is impacted by the information environment, as we elaborate below.

3.1 Canonical Cases

It is helpful to start the analysis by considering two canonical expressions of the partial trade elasticity. First, in the context of Krugman (1980) model, all firms are identical and there is no endogenous selection. In this case the endogenous selection component is zero, and the partial trade elasticity is fully determined by the elasticity of substitution across varieties:

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}}^{\text{Krugman (1980)}} = 1 - \epsilon_k.$$

Second, in the context of Chaney (2008), firms are heterogeneous in their idiosyncratic productivity level which is assumed to be drawn from a Pareto distribution. In this case, $g_{ijk}(z)$ follows a Pareto distribution with the shape parameter denoted by ξ_{ijk}, and the partial trade elasticity takes the following form:

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}}^{\text{Chaney (2008)}} = (1 - \epsilon_k) \cdot \Bigg[1 + \underbrace{\frac{\xi_{ijk}}{\epsilon_k - 1} - 1}_{\gamma_{ijk}} \Bigg]. \tag{20}$$

Notice that even though Chaney (2008) framework features endogenous selection, the partial trade elasticity is independent of the export selection threshold. In this case, the endogenous selection effect on the partial trade elasticity is determined by the shape parameter of the export sales distribution, $\xi_{ijk}/(\epsilon_k - 1)$.[11]

[10]The proof of monotonically increasing property of $\gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))$ is included in Appendix B.

[11]Using the notation developed in this paper, in Chaney (2008) the export sales are given by $r_{fijk}(z) = B_{ijk}\tau_{ijk}^{1-\epsilon_k} e^{(\epsilon_k - 1)z}$. When e^z follows a Pareto distribution with a shape parameter ξ_{ijk}, $r_{fijk}(z)$ follows a Pareto distribution with a shape parameter $\xi_{ijk}/(\epsilon_k - 1)$.

The shape parameter of firm size distribution has been estimated to lie in the range of 1.01 to 1.2, implying the values of γ_{ijk} in the range of 0.01 to 0.2.[12] Substituting the shape parameter estimates into equation (20) subsequently reveals that endogenous selection increases trade elasticities above the firm-level effect of incumbent firms by 1% to 20%. Notably, the fatter is the tail of the export sales distribution, i.e the closer is the shape parameter to unity, the smaller is the role of the endogenous selection in determining the partial elasticity of trade flows with respect to variable trade costs.

This range will serve us as a reference point against which we will compare our estimates of the endogenous component of the trade elasticity in the model with uncertainty and a generalized distribution of export selection shocks.

In the general case, the endogenous selection effect on the partial trade elasticity depends on the *selection effect* through the export selection threshold z^*_{ijk} and the *dispersion effect* through the distribution of the decision relevant selection shock $g_{ijk}(z)$. Both of these channels depend on the information environment as we now discuss.

3.2 The Effect of Uncertainty on Endogenous Selection

Equation (19) highlights two distinct ways in which uncertainty impacts the partial elasticity of trade with respect to variable trade costs. First, uncertainty impacts the elasticity through the export selection threshold, z^*_{ijk}. We will refer to this effect as the *selection effect of uncertainty*. Second, uncertainty impacts the elasticity through the distribution of the export selection shock, $g_{ijk}(z)$. We will refer to this effect as the *dispersion effect of uncertainty*.

3.2.1 The selection effect of uncertainty

Result 1: *Holding all else constant, more stringent selection increases the partial trade elasticity.*

Equations (10) and (17) define the export selection thresholds under complete information, $z^{CI^*}_{ijk}$, and uncertainty, $z^{U^*}_{ijk}$, respectively. Notice that the two thresholds are related as follows

$$z^{CI^*}_{ijk} \quad = \quad z^{U^*}_{ijk} + \log\left[E\left(e^{\frac{v_{fijk}}{\epsilon_k}}\right)\right]^{\epsilon_k}. \tag{21}$$

[12]The main benchmark for estimates of the shape parameter of firm size distribution is that of Axtell (2001) with the mean value of 1.06 in the context of U.S. employment firm size distribution. Kondo et al. (2023) provide a more recent analysis of estimates of the shape parameter of firm size distribution and find the estimates to lie in the range of 1.01 to 1.23, for a sufficiently large firm size threshold used to fit a Pareto distribution. Sager and Timoshenko (2019) estimate the shape parameter specifically in the context of export sales distribution for Brazilian exporters, and find the values to lie in the range of 1.08 to 1.42, depending on the firm size threshold.

Therefore, uncertainty introduces a wedge between entry thresholds in the two information environments. This result mirrors the one obtained in Handley and Limao (2015), who refer to the wedge as the "uncertainty factor".[13] The wedge captures expectations about realizations of the unknown uncertainty factor. While in our framework the uncertainty is with respect to an unexpected component of the idiosyncratic demand shock, υ_{fijk}, in Handley and Limao (2015) the uncertainty factor captures expectations about future tariff realizations and the frequency of the tariff regime change, τ_{ijk}.

Provided $\log \left[E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}} \right) \right]^{\epsilon_k}$ is positive, the selection threshold under complete information is larger than under uncertainty, implying a more stringent selection mechanism under complete information in our framework.[14] This result is opposite to the one in Handley and Limao (2015), who find that tariff uncertainty leads to a higher entry threshold under uncertainty and therefore lower entry in an environment where future tariffs are uncertain.

This distinction arises from differences in the timing of when information is revealed to firms, and the option value of waiting such timing may produce. In our framework, uncertainty is revealed *after* entry and production decisions have been made. Therefore, waiting has no impact on a firm's decision-relevant information. In contrast, in Handley and Limao (2015) firms first observe a realization of tariff policy and then make their decisions. Handley and Limao (2015) framework therefore features the option value of waiting. Firms can condition their entry decisions on a realization of a shock and only enter when the realization of a shock is high enough, a mechanism absent from our framework.

Given that $\gamma_{ijk}(z^*_{ijk}, g_{ijk}(z))$ is a hazard rate function that is monotonically increasing in z^*_{ijk} for a given distribution $g_{ijk}(z)$, a lower selection threshold under uncertainty implies that the selection effect of uncertainty reduces the partial trade elasticity, holding all else constant.

Panel A in Figure 1 demonstrates selection effect of uncertainty. Panel A plots $\gamma(z^*, g(z))$ as a function of z^* holding $g(.)$ fixed. Holding the distribution of export selection shocks fixed, a lower export selection threshold under uncertainty ($z^{U^*} < z^{CI^*}$) reduces γ, as indicated by the downward arrow. This effect unambiguously reduces the partial trade elasticity under uncertainty relative to complete information.

Intuitively, the result can be understood as follows. The endogenous selection effect on the partial trade elasticity captures changes in trade flows due to the entry (or exit) of exporters at the selection margin. Therefore, the size of the selection effect depends on the size of the marginal exporter as well as the mass of firms at the selection threshold. This

[13]Equations (9) and (10) in Handley and Limao (2015) and the discussion therein.

[14]The standard distributional assumptions made in the literature, e.g. the Normal, Exponential, and the Double Exponentially Modified Gaussian distributions, all meet this requirement (with appropriate restrictions on parameters).

can be seen when expressing the aggregate trade flows using (expected) export revenue as follows

$$X_{ijk} \propto \underbrace{\int_{r(z^*)}^{+\infty}}_{\text{the selection effect}} r(z) \underbrace{g(z)}_{\text{the dispersion effect}} dr(z), \tag{22}$$

where $r(z)$ is the (expected) export revenue. The (expected) size of the marginal exporter is given by $r(z^*)$. Therefore, a higher value of the selection threshold will result in a larger size of the marginal exporter, and therefore, larger changes in trade flows as a result of changes in the variable trade costs. Given that the export selection threshold is lower under uncertainty, uncertainty has a dampening effect on the partial trade elasticity through the selection effect.

3.2.2 The dispersion effect of uncertainty

Result 2: *Holding all else constant, the dispersion of a selection shock has an ambiguous effect on the partial trade elasticity.*

Equations (11) and (16) define the export selection shocks under complete information, z^{CI}_{fijk}, and uncertainty, z^{U}_{fijk}, respectively. Notice that the two shocks are related as follows

$$z^{CI}_{fijk} = z^{U}_{fijk} + \upsilon_{fijk}, \tag{23}$$

where υ_{fijk} are i.i.d. Therefore, the selection shock under uncertainty has a lower dispersion than under complete information. This dispersion effect of uncertainty on the partial trade elasticity is ambiguous.[15]

Panel B in Figure 1 demonstrates dispersion effect of uncertainty. Panel B plots $\gamma(z^*, g(z))$ as a function of z^* for two distributions. The solid line plots γ for the baseline distribution, while the dashed line plots γ for a mean-preserving spread with higher variance, representing the distribution under complete information. The arrows show that the effect of moving from complete information to uncertainty is ambiguous: at lower thresholds (left), γ decreases, while at higher thresholds (right), γ increases. This demonstrates that the dispersion effect can either amplify or dampen the partial trade elasticity depending on the location of the export selection threshold.

The intuition for this result can similarly be understood from equation (22). In addition to the size of the marginal exporter, aggregate trade flows depend on the mass of firms at any given value of the selection shock, $g(z)$, including the mass of firms at the margin given by $g(z^*)$. As shown above, the information environment impacts the distribution of the

[15] See Appendix B.

underlying selection shocks, and therefore the mass of firms at the margin. The overall effect of dispersion is non-linear and depends on how the curvature of the distribution changes and the value of the threshold where the density is evaluated.

Taken together, **Result 1** and **Result 2** imply that uncertainty has an ambiguous effect on the partial elasticity of trade flows with respect to variable trade costs. We therefore proceed by developing an estimation methodology to quantify the partial elasticity of trade flows with respect to variable trade costs in an environment with uncertainty, and compare those elasticities to counterfactual values obtained under the assumption of complete information.

3.3 Discussion

In this subsection we discuss how our results extend to a dynamic setting, how our framework is sufficiently general to accommodate other sources of uncertainty, and how our results are robust to an alternative timing and price setting mechanism.

3.3.1 Relationship to Dynamic Models

Although our analysis focuses on a static environment, a key insight is that the partial trade elasticity is still governed by the *selection* and *dispersion* effects in dynamic models, including those with sunk costs and option values (Dixit, 1989; Novy and Taylor, 2020), and learning (Kozeniauskas and Lyon, 2023; Fernandes and Tang, 2014).

To make this point precise, consider a stationary dynamic environment in which aggregate exports can be written as the sum of cohort-level trade flows,

$$X = \sum_{n=0}^{+\infty} X_n, \tag{24}$$

where $n = 0$ denotes entrants and higher n correspond to exporters of longer duration.[16] Differentiating with respect to variable trade costs τ yields

$$\frac{\partial \log X}{\partial \log \tau} = \sum_{n=0}^{+\infty} \underbrace{\frac{X_n}{X}}_{\substack{\text{cohort} \\ \text{trade share}}} \underbrace{\frac{\partial \log X_n}{\partial \log \tau}}_{\substack{\text{cohort partial} \\ \text{trade elasticity}}} . \tag{25}$$

Equation (25) shows that, in a dynamic setting, the partial trade elasticity is a weighted average of cohort-level elasticities, with weights given by cohort trade shares.

To characterize the cohort-level elasticities, we introduce notation general enough to encompass various dynamic models. Denote by $\mathbf{x}^s$ a vector of *state* variables that characterize

[16] We omit subscript t since we consider a stationary equilibrium. For clarity, we also omit country and product subscripts.

a firm's information set when making export decisions. Denote by $\mathbf{x}^r$ a vector of *transitory* profitability shocks that affect realized profits but may or may not be observed when decisions are made. Denote by $\mathbf{x}_n^{s*}(\tau, \mathbf{x}^s)$ a vector of export participation thresholds for exporters of age n, implicitly defined through the Bellman equations of the firm's dynamic problem.

The cohort-level trade flow can then be written as

$$X_n = \underbrace{\int_{\mathbf{x}^r, \mathbf{x}^s \geq \mathbf{x}_n^{s*}(\tau, \mathbf{x}^s)}}_{\text{selection effect}} \underbrace{r(\mathbf{x}^s, \mathbf{x}^r, \tau)}_{\substack{\text{firm-level}\\ \text{trade elasticity}}} \underbrace{m_n(\mathbf{x}^s, \mathbf{x}^r)}_{\text{dispersion effect}} \, d\mathbf{x}^r \, d\mathbf{x}^s, \tag{26}$$

where $r(\mathbf{x}^s, \mathbf{x}^r, \tau)$ is per-period export revenue and $m_n(\mathbf{x}^s, \mathbf{x}^r)$ is the measure of exporters in the respective state in cohort n. Equation (26) mirrors equation (22) in our static model and highlights the three drivers of the partial trade elasticity: the firm-level trade elasticity, the selection effect (operating through the thresholds), and the dispersion effect (operating through the distribution of firms across states).

Dynamic features such as sunk entry costs, option values of exit, and learning affect trade elasticities by reshaping these thresholds and distributions. They do not introduce additional elasticity channels beyond selection and dispersion.

In models with sunk costs and option values, such as Dixit (1989) and Novy and Taylor (2020), exporters face irreversible entry decisions and potentially costly exit. These features generate cohort-specific export thresholds and export hysteresis. In terms of equation (26), option values affect trade elasticities by shifting $\mathbf{x}_n^{s*}$ across cohorts and by redistributing firms across $m_n(\cdot)$. They do not introduce an additional elasticity channel beyond selection and dispersion.

In learning models such as Fernandes and Tang (2014) and Kozeniauskas and Lyon (2023), firms update beliefs about demand based on their export experience.[17] Learning enlarges the state space $\mathbf{x}^s$ and affects the evolution of the distribution $m_n(\mathbf{x}^s, \mathbf{x}^r)$ over time. Export thresholds $\mathbf{x}_n^{s*}$ become history-dependent, reflecting accumulated information. Nevertheless, the effect of uncertainty on trade elasticities continues to operate through selection and dispersion, now defined over a richer state space.

Dynamic considerations therefore alter the interpretation of selection and dispersion effects by making them cohort- and state-dependent, but they do not overturn the basic mechanisms identified in the static model. Quantitatively evaluating the role of specific dynamic mechanisms would require solving and estimating fully dynamic models and computing cohort-specific elasticities and trade shares. While such an exercise lies beyond the scope of this paper, our framework clarifies how existing dynamic models map into the

[17] In Fernandes and Tang (2014), firms can also update beliefs based on observed neighbors' export performance.

selection–dispersion decomposition. Appendix D provides a detailed discussion of how sunk costs operate under complete information versus uncertainty in our model.

3.3.2 Other Sources of Uncertainty

The model in the paper focuses on uncertainty in idiosyncratic demand. However, the analytical structure is naturally formulated in terms of a *decision-relevant export selection shock* and the variance of its unexpected component, rather than demand uncertainty per se. This allows the framework to accommodate other sources of uncertainty, including supply-side and policy-related shocks, without altering the core decomposition of trade elasticities. Appendix E provides details on the mapping of different sources of uncertainty into our framework.

3.3.3 Alternative Timing and Pricing Mechanism

The baseline model assumes that firms commit to export quantities (quantity-setting) before demand is realized, with prices adjusting to clear markets. This section examines the robustness of our results to an alternative timing in which firms set prices (price-setting) before demand is realized and quantities adjust.

Our timing assumption implies that firms produce and ship goods before receiving payment, a post-shipment payment arrangement commonly referred to as *exporter finance* in the trade finance literature. IMF (2009) reports that exporter finance accounts for 42 percent of global export transactions. In Latin America, Ahn (2015) finds that exporter finance accounts for 80 to 90 percent of import transaction value in Colombia and Chile. While we do not model export payment methods explicitly, this evidence suggests that the timing of decisions implied by our model—production and shipment before payment and demand realization—reflects a prevalent feature of international trade.

As we further demonstrate in Appendix F.1, the definition of the export selection shock and the partial trade elasticity formula and its decomposition into selection and dispersion margins under uncertainty are invariant to the timing assumption and continue to take the form in equations (16) and (19) respectively.

The key difference between timing assumptions lies in the relationship between export selection thresholds across information environments. Under price-setting, the wedge between thresholds becomes $\frac{1}{2}V_{\upsilon_{fjkt}}$ rather than $\frac{1}{2\epsilon_k}V_{\upsilon_{fjkt}}$ in our baseline model. Since $\epsilon_k > 1$, the threshold wedge is larger under price-setting, implying a stronger selection effect. This difference would affect quantitative estimates of trade elasticities but does not alter the qualitative mechanism: uncertainty continues to affect trade elasticities through selection and dispersion margins under either timing assumption.

Finally, we provide empirical evidence in support of the quantity-setting mechanism. The two timing assumptions generate different predictions for price variation. Under quantity-setting, prices reflect both productivity and demand shocks, implying substantial price variation even after controlling for firm identity. Under price-setting, prices depend only on productivity, so firm fixed effects should explain nearly all price variation. We exploit this distinction and provide suggestive evidence supporting the quantity-setting mechanism. The details are included in Appendix F.2.

4 Empirical Methodology

In this section we develop an empirical methodology to quantify partial elasticites of trade with respect to variable trade costs in an environment with uncertainty. In doing so we adapt the methodology of Berman et al. (2019) to our framework. Equation (19) informs us about what data are needed to structurally identify partial trade elasticities. First, notice that the overall level of the partial trade elasticity is determined by the direct effect of changes in variable trade costs on the sales of incumbent exporters, the firm-level trade elasticity $\partial \log f^{\tau}(\tau_{ijk})/\partial \log \tau_{ijk}$. This component does not depend on the information structure. In the model with CES preferences considered here, $\partial \log f^{\tau}(\tau_{ijk})/\partial \log \tau_{ijk} = (1-\epsilon_k)$, and hence is entirely determined by preferences, namely the elasticity of substitution across varieties, ϵ_k.

Second, the firm-level trade elasticity is then augmented by the endogenous selection component, which depends on the selection threshold, z^{U*}_{ijk}, and the distribution, $g^{U}_{ijk}(.)$, of the underlying idiosyncratic export selection shock, z^{U}_{fijk}. Hence, to structurally estimate the partial elasticity of trade with respect to variable trade costs, more specifically the endogenous selection component, one needs to recover the firm-level export selection shocks together with the distribution governing the export selection shocks, and quantify the respective export selection threshold. We recover all these objects from the data on export quantities and revenues, as we now explain in detail.

Finally, our estimation of the endogenous selection effect of the partial trade elasticity, and subsequent counterfactual analysis are performed separately for each product-destination-year triplet.

To be consistent with the level of observations in the datasets we use, from hereon we omit the origin subscript i and add a time subscript t where appropriate. The dataset is described in Section 5.1 below and includes an export firm-level panel data for single origin country — Brazil.

4.1 Firm-Level Shocks

From the firm's maximization problem (13), the optimal export quantity and realized export revenue for firm f exporting to country j product k in year t are given by

$$q_{fjkt}(z^a_{fjkt}, z^p_{fjkt}) = \left(\frac{\epsilon_k}{\epsilon_k - 1}\frac{w\tau_{jkt}}{E\left(e^{\frac{\upsilon_{fjkt}}{\epsilon_k}}\right)}\right)^{-\epsilon_k} Y_{jkt}P^{\epsilon_k - 1}_{jkt} e^{\epsilon_k z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt})} \tag{27}$$

$$r_{fjkt}(z^a_{fjkt}, z^p_{fjkt}) = \left(\frac{\epsilon_k}{\epsilon_k - 1}\frac{w\tau_{jkt}}{E\left(e^{\frac{\upsilon_{fjkt}}{\epsilon_k}}\right)}\right)^{1-\epsilon_k} Y_{jkt}P^{\epsilon_k - 1}_{jkt} e^{(\epsilon_k - 1) z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt}) + \frac{\upsilon_{fjkt}}{\epsilon_k}} \tag{28}$$

Notice that the export quantity in equation (27) and revenue in equation (28) depend on two main components: the aggregate market conditions common across all firms exporting product k to country j and idiosyncratic firm-level demand and supply side shocks. We will denote the logarithm of the aggregate market component by FE^q_{jkt} and FE^r_{jkt} respectively, and the weighted sums of firm-level idiosyncratic shocks by ζ^q_{fjkt} and ζ^r_{fjkt} respectively. Log-linearized export quantity and revenue can then be written as

$$\log q_{fjkt} = FE^q_{jkt} + \underbrace{\epsilon_k z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt})}_{\zeta^q_{fjkt}} \tag{29}$$

$$\log r_{fjkt} = FE^r_{jkt} + \underbrace{(\epsilon_k - 1) z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt}) + \frac{\upsilon_{fjkt}}{\epsilon_k}}_{\zeta^r_{fjkt}} \tag{30}$$

Estimating equations (29) and (30) allows to recover residuals $\hat{\zeta}^q_{fjkt}$ and $\hat{\zeta}^r_{fjkt}$ that are used to infer export selection shocks under uncertainty. Using equation (4), notice that the log-revenue residual is comprised of the sum of the export selection shock under uncertainty and the i.i.d. orthogonal component as follows

$$\zeta^r_{fjkt} = z^U_{fjkt} + u_{fjkt}, \tag{31}$$

where

$$u_{fjkt} = \upsilon_{fjkt}/\epsilon_k. \tag{32}$$

To separate the export selection shock from the unanticipated component of the demand shock, υ_{fjkt}, we will utilize the log-quantity residuals that do not encompass the i.i.d. shock. To do so, we use linear conditional expectation from equation (6) and substitute the log-quantity residual into equation (31) to obtain

$$\zeta^r_{fjkt} = \beta_{jkt}\zeta^q_{fjkt} + u_{fjkt}, \tag{33}$$

where $\beta_{jkt} = ((\epsilon_k - 1) + \alpha_{jkt})/(\epsilon_k + \alpha_{jkt})$. Estimating equation (33) by destination-year-product triplets allows us to recover the firm-level export selection shocks as follows

$$\hat{z}^U_{fjkt} = \hat{\beta}_{jkt}\hat{\zeta}^q_{fjkt}. \tag{34}$$

Finally, there are several caveats in estimating residuals in equations (29) and (30). First, consistently estimating the residuals requires that they be independent from the aggregate market conditions captured in the fixed effect terms such as the origin's wage, w, and destinations' aggregate conditions, the expenditure level Y_{jkt} and the price level P_{jkt}. This independence assumption implies that the underlying firm-level idiosyncratic labor productivity and demand shocks do not vary systematically with the origin's aggregate costs and destinations' aggregate characteristics. Second, the identifying assumption also rules out the possibility that productivity and demand shocks are correlated across markets. Studying the impact of such spillovers on the partial elasticity of trade flows with respect to variable trade costs lies outside the scope of this paper. The final caveat is that a potential presence of classical measurement error in the export quantity data could bias our estimates. We perform robustness checks to address this possibility in Appendix I.

4.2 The Distribution of Export Selection Shocks

4.2.1 Parameterizing Distributions

To proceed with estimating the partial trade elasticities we, first, need to parametrize the distribution, $g^U_{jkt}(.)$, of the export selection shocks, z^U_{fjkt}.

The majority of the trade literature has relied on either a Pareto distribution (Axtell, 2001; Chaney, 2008) or a log-Normal distribution (Bas et al., 2017; Fernandes et al., 2023) in modeling firm level heterogeneity.[18] However, the Brazilian data reject the assumption of a Pareto distribution and favors a more flexible distribution that can capture left-tail fattness as well as right-tail fatness, or the absence of fat tails at all in some markets. To this end, we parameterize the distributions using a Double Exponentially Modified Gaussian (DEMG) distribution that combines features of both the Normal and double Pareto distributions to obtain cases with left-tail fatness, right-tail fatness or at least one thin tail. Sager and Timoshenko (2019) have shown that a DEMG distribution provides a superior fit to the empirical distribution of the logarithm of export sales compared to an Exponential or a Normal alone (note that the logarithm of a Pareto distribution follows an exponential distribution and the logarithm of a log-Normal follows a Normal distribution).

We estimate parameters of the Double EMG distribution separately for each of the observations in our sample, where an observation is defined as a distribution of export selection

[18] A notable exception includes Nigai (2017) who assumes a mixture of log-Normal and Pareto distributions.

shocks in country j for product k at time t.

4.2.2 Distribution Estimation Method

We follow Sager and Timoshenko (2019) in estimating the parameters of the Double EMG distribution using a Generalized Method of Moments (GMM) procedure that minimizes the sum of squared residuals,

$$\min_{(\mu,\sigma^2,\lambda_L,\lambda_R)} \sum_{i=1}^{N_P} \left(x_i^{data} - x_i(\mu, \sigma^2, \lambda_L, \lambda_R)\right)^2,$$

where x_i^{data} is the i-th percentile of the empirical export selection socks distribution for a given product-destination-year, $x_i(\mu, \sigma^2, \lambda_L, \lambda_R)$ is the model implied i-th percentile for given parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$, and N_P is the number of percentiles used in estimation. We use the 1st through 99th percentiles of the empirical distribution to estimate parameters. In practice, this choice eases computational burden compared to using each data point, without significantly changing the parameter estimates we recover. Furthermore, note that choosing parameters to minimize the sum of squared residuals is equivalent to Head et al.'s (2014) method of recovering parameters from quantile regressions.

Hence, for each product-destination-year observation, we choose distribution parameters $(\mu, \sigma^2, \lambda_L, \lambda_R)$ so that the percentiles of the theoretical distribution of export selection shocks match the percentiles of the respective empirical distribution.

4.2.3 Correcting for Endogenous Selection

In fitting a distribution to the recovered export selection shocks, $\hat{z}^U_{fjkt}$, it is important to note that the model implies truncation in the data. Namely, selection shocks are observed only when $z^U_{fjkt} \geq z^{U*}_{jkt}$. To account for the endogenous selection into exporting, we follow the approach by Sager and Timoshenko (2019). Namely, we proceed by fitting a truncated probability distribution function $g^U_{jkt}(.)$ to the data and take the truncation point z^{U*}_{jkt} to be given by the zeroth percentile of the corresponding empirical distribution of the export selection shocks.

4.3 Selection Thresholds

We adapt the methodology of Bas et al. (2017) to recover the (scaled) export selection thresholds by matching the model-implied average-to-minim ratios of export quantity to those in the data. Using equation (27) and the definition of ζ^q_{fjkt} from equation (29) we can write

$$\frac{\tilde{q}_{jkt}}{q^{\min}_{fjkt}} = e^{-\zeta^{q^*}_{jkt}} \int_{\zeta^{q*}_{jkt}}^{+\infty} \frac{e^{\zeta^q} g^{\zeta^q}_{jkt}(\zeta^q)}{1 - G^{\zeta^q}_{jkt}(\zeta^{q^*}_{jkt})} d\zeta^q. \tag{35}$$

We solve equation (35) for $\zeta^{q^*}_{jkt}$ to recover the export selection threshold $z^{U*}_{jkt} = \hat{\beta}_{jkt}\zeta^{q^*}_{jkt}$. In solving equation (35), we measure the average-to-minimum ratio of quantity using the average-to-minimum ratio of the exponential of estimated quantity residuals, $\hat{\zeta}^q_{fjkt}$. From equation (34), the distribution $g^{\zeta^q}_{jkt}(.)$ follows the distribution $g^U_{jkt}(.)$ scaled by parameter $1/\beta_{jkt}$. We use the estimates of $1/\hat{\beta}_{jkt}$ and $\hat{g}^U_{jkt}(.)$ obtained in Sections 4.1 and 4.2 respectively.

5 Data and Estimation Results

In this section we use data across Brazilian exporters on the distribution of export quantities and sales by product-destination over time to quantify trade elasticities in an environment with uncertainty. A product is defined as a 6-digit HS code.

5.1 Data

The data come from the Brazilian customs declarations collected by SECEX (*Secretaria de Comercio Exterior*).[19] The data record export value and weight (in kilograms) of the shipments at the firm-product-destination-year level. A product is defined at the 6-digit Harmonized Tariff System (HS) level. We use the data for the period between 1997 and 2000, when both the sales and the weight data are available.

We proxy the theoretical notion of export quantity with an empirical measure of export weight.[20] Since the properties of export weight differ substantially across industries, we further conduct our analysis at the product-destination-year level.

We define an observation to be a distribution of export quantity or sales across firms for a given product-destination-year triplet, and focus on observations where at least 100 firms export in at least one of the four years for a given product-destination pair.[21] The final sample consists of 288 product-destination-year observations, and covers 14 destinations and 35 industries.[22] For each product-destination-year observation, we clean the data by

[19] For a detailed description of the dataset see Molinaz and Muendler (2013). The data have further been used in Flach (2016) and Flach and Janeba (2017).

[20] Export weight is used as a measure of export quantity in a number of studies including Manova and Zhang (2012); Bastos et al. (2018).

[21] The thresholds of 100 firms makes our results comparable to other papers in the literature (see Fernandes et al. (2023), Sager and Timoshenko (2019)) and ensures that an empirical distribution can be accurately described by percentiles. The qualitative features and basic quantitative results are not heavily dependent on the exact threshold we select within the neighborhood of 100 firms (results are available upon request).

[22] We note that there are 232,266 product-destination-year observations in the entire data-set. We focus on

dropping export sales and export quantity values that fall below the 1st or above the 99th percentiles. Table 1 provides summary statistics of log-export quantities and log-export sales distributions in our final sample.

5.2 The 1999 Devaluation and Demand Uncertainty

Our sample period of 1997–2000 spans a significant macroeconomic event: Brazil's abandonment of its crawling peg exchange rate regime in January 1999, which resulted in a sharp depreciation of the Real—approximately 40% against the U.S. dollar within the first few months. The standard interpretation of such a devaluation in the trade literature is a reduction in variable trade costs, τ, as Brazilian products become cheaper in foreign markets.

Our framework, however, suggests an additional interpretation. During periods of macroeconomic turbulence, foreign customers may face heightened uncertainty about the stability and reliability of Brazilian suppliers, exchange rate pass-through, and future economic conditions. This uncertainty would manifest in our model as an increase in the variance of the unexpected component of the demand shock, v_{fjkt}, rather than solely as a shift in the level of trade costs. Importantly, such demand-side uncertainty should not affect the dispersion of the export selection shock, z^U_{fjkt}, which in the model is determined by productivity and expected demand—factors that are less likely to be disrupted by exchange rate movements. In other words, devaluation does not alter production technology, but it may increase the uncertainty foreign buyers face when forming demand for Brazilian products.

We test this prediction by examining how the cross-firm dispersion of the estimated shocks evolves over our sample period. Specifically, for each product-destination-year cell (j, k, t), we compute the standard deviation across firms of the recovered export selection shocks, $\hat{z}^U_{fjkt}$, from equation (34), and the standard deviation of the unexpected demand component, $\hat{u}_{fjkt}$, from equation (33). We then regress these dispersions on year dummies, controlling for product-destination fixed effects:

$$\text{Std.Dev.}_{\cdot jkt} = \alpha_{jk} + \sum_{\tau=1998}^{2000} \beta_\tau \mathbf{1}[t = \tau] + \epsilon_{jkt}, \tag{36}$$

where the year 1997 serves as the omitted baseline category. The coefficients β_τ capture changes in the dispersion of shocks relative to the pre-devaluation period.

Figure 3 presents the results. Panel A plots the average standard deviation of the export

a sub-sample of 288 observations where at least 100 firms export in at least one of the four years for a given product-destination pair. Among the remaining 231,978 observations, the median and the average number of exporters is 1 and 3.3 respectively. Hence, these markets are unlikely to be characterized by a monopolistic competition environment, and the forces of endogenous market selection that we seek to identify in our paper would not apply.

selection shock, z^U_{fjkt}, by year, along with 95% confidence intervals. The dispersion remains remarkably stable throughout the sample period, fluctuating in a narrow range around 1.68. A joint F-test fails to reject the null hypothesis that the year effects are equal across all years ($F(3, 213) = 0.62$, $p = 0.605$). This stability is consistent with our interpretation: the devaluation did not fundamentally alter the technological environment or the predictable component of demand facing Brazilian exporters.

Panel B plots the average standard deviation of the unexpected demand component, υ_{fjkt}, by year. Here, we observe a markedly different pattern. The dispersion remains stable at approximately 0.87 during the pre-devaluation years of 1997–1998, but increases sharply to approximately 0.92 in the post-devaluation years of 1999–2000. This increase of roughly 5–6 percentage points represents a substantial rise relative to the baseline level. A test comparing the pre-devaluation period (1997–1998) to the post-devaluation period (1999–2000) strongly rejects equality.[23]

These findings provide suggestive empirical support for the model's distinction between the expected and unexpected components of demand shocks. While we cannot rule out alternative mechanisms—such as compositional changes in the exporter population, heterogeneous exchange rate pass-through or measurement issues—these would need to explain why only the unexpected component exhibits increased dispersion while the export selection shock remains stable. The 1999 devaluation appears to have increased the uncertainty Brazilian exporters faced in foreign markets, as captured by a rise in the dispersion of realized demand relative to expectations, while leaving the distribution of productivity and expected demand largely unaffected. This pattern is consistent with foreign customers facing greater difficulty in predicting their demand for Brazilian products during a period of macroeconomic instability, lending empirical credence to the relevance of demand uncertainty in shaping export behavior and, consequently, trade elasticities.

Having established empirical motivation for the role of demand uncertainty in our setting, we now turn to the estimation of the model's structural parameters.

5.3 Parameter and Threshold Estimates

In this section we present estimates of the distribution parameters of the export selection shocks, and the respective entry threshold estimates.

[23] We construct this test by regressing the standard deviation on a post-devaluation indicator (equal to one for years 1999 and 2000) with product-destination fixed effects. The coefficient on the post-devaluation indicator is 0.052 (s.e. = 0.011) with $t = 4.53$ and $p < 0.001$.

5.3.1 Parameter Estimates

Table 2 summarizes estimates of distribution parameters across 288 observations for the distributions of the export selection shocks by product-destination-year triplets. The average sample value of σ is 1.20, rejecting rejecting the Exponential and Double Exponential specifications ($\sigma = 0$) common in the literature. Furthermore, distributions exhibit substantial heterogeneity in the fatness of both tails. The value of the right tail parameter, λ_R varies between 0.72 and 76.57, with about 26 percent of observations exhibiting a fat right tail ($\lambda_R < 2$) and approximately 80 percent of observations exhibiting fat left tail ($\lambda_L < 2$). These estimates are consistent with the previous empirical research documenting fatness in the right tail of sales or employment distributions across firms.[24] [25]

Figure 2 plots the density of the export selection shock at the average parameter estimates reported in Table 2, alongside a Normal distribution with the same standard deviation for comparison. The y-axis is plotted on a logarithmic scale to highlight differences in tail behavior. The figure illustrates that the average distribution of export selection shocks is asymmetric and exhibits fatter tails than the Normal, particularly in the left tail, reflecting greater mass among low-value export selection shocks. These features highlight why allowing for flexible tail behavior is important for measuring the dispersion effect of uncertainty, which depends on the full shape of the selection-shock distribution.

5.3.2 Entry Thresholds

Figure 4 provides a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of log-export quantity residuals. Each dot in the Figure corresponds to a product-destination-year observation. Figure 4 demonstrates a negative relationship between the average-to-minimum ratio and the entry threshold. The larger is the average-to-minimum ratio, the smaller is the marginal exporter relative an average exporter. Hence, the respective entry threshold must be lower.

5.4 Estimates of Trade Elasticities

Given the estimated distribution parameters and entry thresholds presented in Section 5.3, we compute the partial trade elasticity, $\partial \log X_{jkt} / \partial \log \tau_{jkt}$, and the endogenous selection effect on trade elasticity, γ_{jkt}, according to equation (19). Note from equation (19) that the full endogenous selection effect on partial trade elasticity is captured by $(1 + \gamma_{jkt})$. For

[24] See Axtell (2001), di Giovanni et al. (2011), and Kondo et al. (2023).

[25] The values of parameter estimates are stable across time with only weak evidence of the right tail getting thinner over time. We do not observe strong systematic variation between parameter values and the elasticity of substitutions cross varieties. Additional details are available upon request.

presentation and clarity purposes, when presenting the quantitative results from hereon, we will omit adding unity to γ_{jkt}, and will refer to γ_{jkt} alone as the endogenous selection effect.

Result 3: *On average, the endogenous selection effect, γ_{jkt}, amounts to 0.02*

Table 3 presents the estimates of partial trade elasticities and the endogenous selection effects in a model with uncertainty. As shown in the first row of the table, the mean endogenous selection effect equals to 0.02. As discussed in Section 3.1, this magnitude is comparable to the one obtained from a standard trade model similar to that of Melitz (2003) where the distribution of export sales follows a Pareto distribution with a shape parameter 1.02, the value which is largely consistent with the shape parameter estimates obtained in the literature.[24]

A useful interpretation of the magnitude of the endogenous selection effect, γ_{jkt}, is the percent by which partial trade elasticity increases relative to a benchmark value without endogenous selection. Notice from equation (19) that in the absence of endogenous selection, the total partial trade elasticity is determined by the firm-level elasticity, $\partial \log f^{\tau}(\tau_{jkt})/\partial \log \tau_{jkt}$. Endogenous selection mechanism subsequently increase the firms-level trade elasticity by a factor of $(1 + \gamma_{jkt})$. The mean value of γ_{jkt} of 0.02, therefore, indicates that entrants and exitors change trade flows by an additional 2% relative to the change in trade flows generated by incumbent firms.

Result 4: *The endogenous selection effect, γ_{jkt}, is heterogeneous across products and is higher in products with a larger elasticity of substitution across varieties.*

Figure 5 depicts a relationship between the average selection effect for a given product and that product's elasticity of substitution across varieties.[26] The figure exhibits a weakly positive relationship indicating that selection effect is larger in products where varieties are more substitutable.

5.5 Understanding the Magnitude of the Selection Effect

Although the model allows for sizable endogenous selection effects in principle, the estimated magnitudes are small in the data. This finding does not imply that uncertainty is quantitatively unimportant; rather, it reflects a feature of the data that pins down export entry thresholds far in the left tail of the distribution of export selection shocks. In such regions, the hazard rate function governing γ approaches zero, implying that changes in trade costs induce only a limited response along the extensive margin. The key empirical moment driving this result is the large dispersion in exporter sizes observed within product-destination

[26]For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

markets: average exporters are orders of magnitude larger than the smallest exporters, with the sample mean of the average-to-minimum ratio of export quantity being 277.62. Because the model identifies export thresholds by matching this ratio, the large observed dispersion implies very low entry thresholds, which in turn compress the endogenous selection component of the trade elasticity toward zero. As a result, trade flows in this class of models are overwhelmingly driven by incumbent firms, even though uncertainty materially shapes the underlying distribution of exporters and the amplification patterns documented above. Appendix G formalizes this intuition by relating the estimated thresholds to properties of the hazard rate implied by the fitted shock distribution.

6 Counterfactuals

As discussed in Section 3.2, uncertainty impacts partial trade elasticity through selection and dispersion effects, with the total effect being ambiguous. We conduct the following three counterfactual experiments to disentangle the two effects and quantify the effect of uncertainty on partial trade elasticities.[27]

First, to isolate the selection effect of uncertainty, we compute the counterfactual values of the selection effect and partial trade elasticities by varying the selection thresholds from the baseline values of z_{jkt}^{U*} to their respective counterfactual values of z_{jkt}^{CI*}, while keeping the distribution of the export selection shocks at their baseline values estimated under uncertainty.

Second, to isolate the dispersion effect of uncertainty we compute the counterfactual values of the selection effect and partial trade elasticities by varying the distribution of the selection shocks from the baseline values of $g_{jkt}^{U}(.)$ to their respective counterfactual values, $g_{jkt}^{CI}(.)$, while keeping the entry threshold values at their baseline values estimated under uncertainty.

Finally, we compute the complete counterfactual values of the endogenous selection effects and partial trade elasticities under complete information and compare the obtained values to the baseline estimates under uncertainty.[28]

Result 5: *The selection effect of uncertainty reduces partial trade elasticities by an average*

[27] The details on quantifying counterfactual values are included in Appendix H.

[28] By keeping the underlying distribution of productivity and demand expectations fixed, we identify how much of the observed trade elasticity is attributable to the information structure. An alternative approach could separately estimate the model under complete information and under uncertainty. However, if uncertainty is the true data generating process, then estimating under complete information recovers a misspecified model. The resulting parameters would reflect both the true primitives and the bias from misspecification, making it difficult to interpret differences between the two sets of estimates as reflecting the economic effect of uncertainty. Details are available upon request.

of 8% relative to their counterfactual values under complete information.

Panel A in Table 4 presents counterfactual trade elasticities arising from varying the selection thresholds from the baseline values estimated under uncertainty to the respective counterfactual values computed under complete information, holding all else constant. Notice that the average counterfactual endogenous selection effect, γ_{jkt}, is 0.80. In this counterfactual scenario, entering and exiting exporters contribute an additional 80%, relative to incumbent exporters, to generating new trade flows from a decline in variable trade costs relative to a modest 2% in the baseline estimation. Therefore, uncertainty substantially dampens the selection effect on the partial trade elasticities. This is solely due to a more stringent selection under complete information. As shown in Section 3.2, the export selection thresholds are higher under complete information, which results in a marginal firm being larger. Therefore changes in trade costs will generate larger changes in trade volumes due to larger size of marginal firms in an environment with complete information relative to uncertainty.

We subsequently define the amplification effect of uncertainty as the ratio of partial trade elasticities computed in the baseline scenario of uncertainty relative to their counterfactual values computed under complete information. The second row in Panel A Table 4 indicates that the amplification effect on the partial trade elasticity due to selection is 0.92 on average. Hence, the total partial trade elasticities are on average 8% lower due to the selection effect of uncertainty in a model with uncertainty relative to a model with complete information.

Result 6: *The dispersion effect of uncertainty increases partial trade elasticities in about ninety seven percent of observations and decreases partial trade elasticities in the remaining three percent of observations. The magnitude of the dispersion effect is small.*

The dispersion effect of uncertainty captures the mass of firms at the market participation threshold, holding all else constant. This mass depends on how the distribution of export selection shocks changes between information environments. We back out the distributions of shocks from the microdata on export sales and quantities.

Panel B in Table 4 presents counterfactual trade elasticities arising from varying the distribution of the selection shocks from the baseline values estimated under uncertainty to the respective counterfactual values computed under complete information, holding all else constant.

The average amplification effect of dispersion is greater than unity. Notice from the second row in Panel B in Table 4 that the partial trade elasticities are on average 2% higher under uncertainty compared to counterfactual values under complete information.

This magnitude is rather small as evident from comparing the average endogenous selection effect: 0.02 in the baseline estimation relative to 0.0005 in the discussed counterfactual.

Result 7: *The total effect of uncertainty on the partial trade elasticity.*

(i) Uncertainty increases partial trade elasticities in about eighty percent of observations and decreases partial trade elasticities in the remaining twenty percent of observations.

(ii) The amplification effect of uncertainty increases with the variance of the unexpected component of the demand shocks.

(iii) The amplification effect of uncertainty increases with the elasticity of substitution across varieties with negative effects concentrated among inelastic products.

The total effect of uncertainty on the partial trade elasticities depends on the interaction of the selection and dispersion effects. Figures 6 provides a scatter plot of the estimates of endogenous selection effects, γ_{jkt}, obtained in the baseline estimation under uncertainty (x-axis) versus the respective counterfactual values under complete information (y-axis). We find that in the majority of observations (80%) the endogenous selection effect is larger under uncertainty. Comparing results in Table 3 and Panel C in Table 4, the average endogenous selection effect under uncertainty, 0.02, is higher than under complete information, 0.001, resulting in on average 1% higher partial trade elasticities under uncertainty. In a subset of observations where the amplification effect is below unity, i.e. uncertainty dampens trade elasticities relative to the complete information environment, the endogenous selection effect is about 23% lower under uncertainty resulting in an insignificant impact on total partial trade elasticities.[29]

The small magnitude of the total amplification effect is largely determined by the dispersion effect of uncertainty. Notice, from Panel A in Table 4 that in the absence of dispersion, the counterfactual trade elasticities are significantly larger: the mean of the endogenous selection component being 0.80 versus 0.02 under uncertainty. The large selection effect is dampened by the dispersion effect of uncertainty. As can be seen from Table 3 and Panel B in Table 4, the dispersion effect alone, reduces the average selection effect from 0.02 under uncertainty to 0.0005 under complete information (relative to 0.001 in the full counterfactual, Panel C in Table 4) resulting in total trade elasticities being on average higher by 2%, which is close to the overall amplification effect of uncertainty on partial trade elasticities noted in Panel C in Table 4 and amounting to 1%.

Panel A in Figure 7 further demonstrates the importance of the distribution of export selection shocks in determining the magnitude of trade elasticities. The figure depicts a

[29] In this subset of observations, the partial trade elasticity declines by an average of one hundredth of a percent.

relationship between the amplification effect of uncertainty and the standard deviation of the unexpected component of the demand shocks, υ_{fjkt}.[30] The figure demonstrates that the larger is the dispersion of the unexpected component of the demand shocks, the larger is the total amplification effect of uncertainty.

We further find that there exists substantial heterogeneity in the amplification effect of uncertainty across industries. Panel B in Figure 7 depicts a relationship between the amplification effect of uncertainty and the elasticity of substitution across products. Notably, in industries with low elasticity of substitution across varieties, the amplification effect is below unity, meaning that in those products trade elasticities are larger under complete information and that the selection effect plays a dominant role in determining the magnitude of trade elasticities. Hence, when products are less substitutable, the size of the marginal exporter matters more than the mass of firms at any given threshold in predicting how trade flows change in response to changes in trade costs.[31]

Our finding of a positive relationship between the elasticity of substitution across varieties and the amplification effect of uncertainty is complementary to the finding of Baley, Veldkamp, and Waugh (2020) that the elasticity of substitution between domestic and foreign varieties (the Armington elasticity) governs the relative strength of the mean-versus-variance effects of aggregate uncertainty on exports, and therefore determines whether higher uncertainty raises or lowers trade flows. In contrast to firm-level idiosyncratic demand uncertainty considered in this paper, Baley et al. (2020) analyze aggregate cross-country uncertainty associated with countries' endowment shocks, which affects the expected mean and the variance of future terms of trade. Higher variance discourages exports through a precautionary substitution motive, while higher expected mean of the terms of trade encourages exports through a convexity (mean-preserving-spread) effect. The Armington elasticity determines which force dominates, so that with highly substitutable goods uncertainty reduces exports, whereas with more complementary goods it can increase them.

7 Conclusion

In this paper, we developed a model that introduces firm-level uncertainty about idiosyncratic demand in foreign markets into a canonical model of trade (c.f. Melitz (2003)), and used the model to study the effect of uncertainty on the partial elasticity of trade with respect to variable costs.

[30] All values in Panel A in Figure 7 have been normalized by their respective industry averages, where an industry is defined as a 6-digit HS code.

[31] The partial trade elasticities estimated in this paper are local measures. Exploring non-local responses to large changes in trade costs — and how these differ across information environments — is an interesting avenue that we leave for future work.

The model predicts that while uncertainty does not change the functional form of the partial trade elasticity relative to an economy with complete information, it changes the forces governing selection into exporting. In particular, we identified two channels through which uncertainty impacts trade – through export participation thresholds (the selection effect) and through the distribution of shocks governing export selection (the dispersion effect) – and showed that although the model predicts a lower partial trade elasticity in a model with uncertainty due to the selection effect, the dispersion effect is ambiguous. The total effect of uncertainty on trade elasticities is therefore theoretically ambiguous.

Using the structure of the model, we developed a new empirical methodology to quantify partial elasticities of trade with respect to variable trade costs in an environment with uncertainty using firm-level data. We applied the methodology to the Brazilian firm-level customs data and found that, on average, uncertainty amplifies partial trade elasticities relative to an environment with complete information. This indicates that the dispersion effect of idiosyncratic firm-level shocks has the dominant effect on the partial trade elasticities, although there is heterogeneity in the effect across industries.

References

Ahn, J. (2015): "Understanding Trade Finance: Theory and Evidence from Transaction-Level Data," International Monetary Fund, mimeo.

Allen, T. (2014): "Information Frictions in Trade," *Econometrica*, 82, 2041–2083.

Arellano, C., Y. Bai, and P. J. Kehoe (2019): "Financial Frictions and Fluctuations in Volatility," *Journal of Political Economy*, 127, 2049–2103.

Arkolakis, C., T. Papageorgiou, and O. Timoshenko (2018): "Firm Learning and Growth," *Review of Economic Dynamics*, 27, 146–168.

Axtell, R. L. (2001): "Zipf Distribution of U.S. Firm Sizes," *Science*, 293, 1818–1820.

Bachmann, R., S. Elstner, and E. R. Sims (2013): "Uncertainty and Economic Activity: Evidence from Business Survey Data," *American Economic Journal: Macroeconomics*, 5.

Baker, S. R., N. Bloom, and S. J. Davis (2016): "Measuring Economic Policy Uncertainty," *The Quarterly Journal of Economics*, 131, 1593–1636.

Baley, I., L. Veldkamp, and M. Waugh (2020): "Can global uncertainty promote international trade?" *Journal of International Economics*, 126, 103347.

Bas, M., T. Mayer, and M. Thoenig (2017): "From Micro to Macro: Demand, Supply, and Heterogeneity in the Trade Elasticity," *Journal of International Economics*, 108, 1–19.

Bastos, P., D. Dias, and O. Timoshenko (2018): "Learning, Prices, and Firm Dynamics," *Canadian Journal of Economics*, 51, 1257–1311.

Bergin, P. R. and C.-Y. Lin (2012): "The dynamic effects of a currency union on trade," *Journal of International Economics*, 87, 191–204.

Berman, N., V. Rebeyrol, and V. Vicard (2019): "Demand learning and firm dynamics: evidence from exporters," *Review of Economics and Statistics*, 101, 91–106.

Bloom, N., S. J. Davis, L. Foster, B. Lucking, S. Ohlmacher, and I. Saporta-Eksten (2020): "Business-Level Expectations and Uncertainty," NBER Working Paper No. 28259.

Broda, C. and D. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121, 541–585.

Caliendo, L. and F. Parro (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82, 1–44.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98, 1707–21.

Chen, C., T. Senga, C. Sun, and H. Zhang (2023): "Uncertainty, imperfect information, and expectation formation over the firm's life cycle," *Journal of Monetary Economics*, 140, 60–77.

di Giovanni, J., A. A. Levchenko, and R. Rancière (2011): "Power Laws in Firm Size and Openness to Trade: Measurement and Implications," *Journal of International Economics*, 58, 42–52.

Dickstein, M. J. and E. Morales (2018): "What do Exporters Know?" *The Quarterly Journal of Economics*, 133, 1753–1801.

Dixit, A. (1989): "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, 97, 620–638.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Feenstra, R. C. (1994): "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84, 157–77.

Fernandes, A. M., P. J. Klenow, S. Meleshchuk, M. D. Pierola, and A. Rodríguez-Clare (2023): "The Intensive Margin in Trade: How Big and How Important?" *American Economic Journal: Macroeconomics*, 15, 320–54.

Fernandes, A. P. and H. Tang (2014): "Learning to export from neighbors," *Journal of International Economics*, 94, 67–84.

Fernández-Villaverde, J. and P. A. Guerrón-Quintana (2020): "Uncertainty shocks and business cycle research," *Review of Economic Dynamics*, 37, S118–S146.

Fillat, J. L. and S. Garetto (2015): "Risk, Returns, and Multinational Production," *The Quarterly Journal of Economics*, 130, 2027–2073.

Flach, L. (2016): "Quality upgrading and price heterogeneity: Evidence from Brazilian exporters," *Journal of International Economics*, 102, 282–290.

Flach, L. and E. Janeba (2017): "Income inequality and export prices across countries," *Canadian Journal of Economics*, 50, 162–200.

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98, 394–425.

Handley, K. and N. Limao (2015): "Trade and Investment under Policy Uncertainty: Theory and Firm Evidence," *American Economic Journal: Economic Policy*, 7, 189–222.

Head, K., T. Mayer, and M. Thoenig (2014): "Welfare and Trade without Pareto," *American Economic Review*, 104, 310–16.

Imbs, J. and I. Mejean (2015): "Elasticity Optimism," *American Economic Journal: Macroeconomics*, 7, 43–83.

——— (2017): "Trade Elasticties," *Review of International Economics*, 25, 383–402.

IMF (2009): *World Economic Outlook, October 2009, Sustaining the Recovery*, International Monetary Fund.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50, 649–670.

Kondo, I., L. Lewis, and A. Stella (2023): "Heavy Tailed, but not Zipf: Firm and Establishment Size in the United States," *Journal of Applied Econometrics*, 38, 767–785.

Kozeniauskas, N. and S. Lyon (2023): "Demand Learning, Customer Capital, and Exporter Dynamics," Working Paper.

Krugman, P. (1980): "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70, 950–959.

Lewis, L. T. (2014): "Exports Versus Multinational Production Under Nominal Uncertainty," *Journal of International Economics*, 94, 371–386.

Manova, K. and Z. Zhang (2012): "Export Prices Across Firms and Destinations," *The Quarterly Journal of Economics*, 127, 379–436.

Melitz, M. and S. Redding (2015): "New Trade Models, New Welfare Implications," *American Economic Review*, 105, 1105–46.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.

Molinaz, D. and M.-A. Muendler (2013): "Preparing to Export," NBER Working Paper No. 18962.

Nigai, S. (2017): "A tale of two tails: Productivity distribution and the gains from trade," *Journal of International Economics*, 104, 44–62.

Novy, D. and A. M. Taylor (2020): "Trade and Uncertainty," *The Review of Economics and Statistics*, 102, 749–765.

Rinne, H. (2014): *The Hazard rate : Theory and inference (with supplementary MATLAB-Programs)*, Justus-Liebig-Universitat.

Sager, E. and O. A. Timoshenko (2019): "The Double EMG Distribution and Trade Elasticities," *Canadian Journal of Economics*, 52, 1523–1557.

Simonovska, I. and M. E. Waugh (2014): "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 92, 34–50.

Soderbery, A. (2015): "Estimating Import Supply and Demand Elasticities: Analysis and Implications," *Journal of International Economics*, 96, 1–17.

Timoshenko, O. A. (2015): "Product Switching in a Model of Learning," *Journal of International Economics*, 95, 233–249.

Figures and Tables

Table 1: Properties of the log-export quantity and log-export sales distributions across product-destination-year observations over 1997-2000.

Statistic	Mean	Std. Dev.	Min	Max
Panel A: Properties of log-quantity				
Standard Deviation	2.21	0.50	1.06	3.26
Skewness	0.03	0.33	-1.08	0.81
Interquartile Range	3.23	0.81	1.36	5.52
Kelly Skew	0.01	0.14	-0.39	0.55
Panel B: Properties of log-sales				
Standard Deviation	1.94	0.37	0.92	2.75
Skewness	-0.10	0.28	-0.85	1.00
Interquartile Range	2.75	0.57	1.13	4.18
Kelly Skew	-0.02	0.12	-0.30	0.45

Note: the summary statistics are reported across 288 product-destination-year observations. A product is defined as a 6-digit HS code. Export quantity is measured as export weight in kilograms.

Table 2: Double EMG distribution parameter estimates of the distributions of export selection shocks.

Parameter	Mean	Std. Dev.
σ	1.20	0.65
λ_L	4.04	8.39
λ_R	12.64	12.86

Notes: the summary statistics are reported across 288 product-destination-year observations. A product is defined as a 6-digit HS code.

Table 3: Trade elasticity estimates under uncertainty.

Measure	Mean	Std. Dev.
Endogenous selection, γ_{jkt}	0.02	0.10
Total partial trade elasticity, $\partial \log X_{jkt}/\partial \log \tau_{jkt}$	3.44	3.67

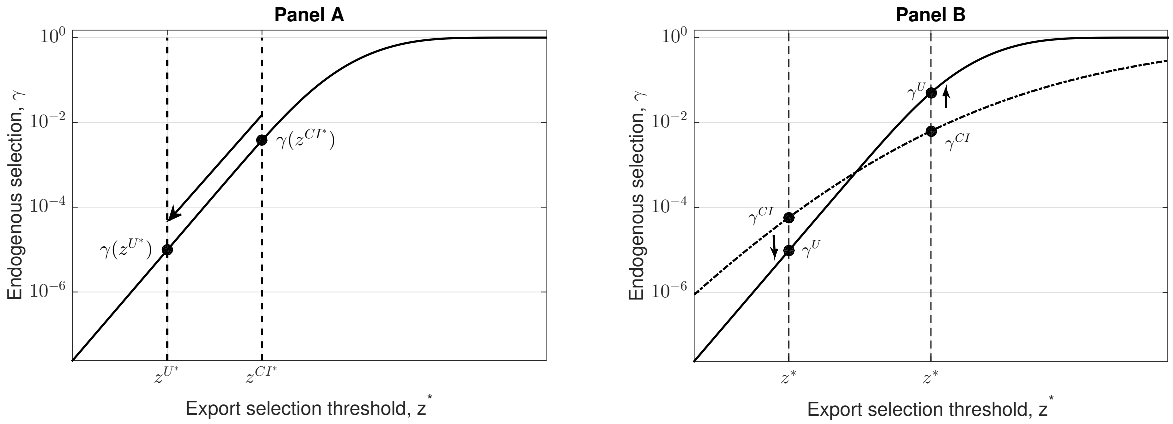
Notes: the summary statistics are reported across 274 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. A product is defined as a 6-digit HS code.

Table 4: Counterfactual trade elasticity estimates under complete information.

	Endogenous Selection γ_{jkt}		Partial Trade Elasticity, $\partial \log X_{jkt}/\partial \log \tau_{jkt}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: selection effect of uncertainty				
Selection effect	0.80	5.31	7.50	20.09
Amplification due to selection	0.32	0.25	0.92	0.21
Panel B: dispersion effect of uncertainty				
Dispersion effect	0.0005	0.002	3.38	3.64
Amplification due to dispersion	$3.7 \cdot 10^{62}$	$6.2 \cdot 10^{63}$	1.02	0.10
Panel C: total effect of uncertainty				
Total effect	0.001	0.003	3.38	3.64
Total amplification effect	$1.2 \cdot 10^{57}$	$2.0 \cdot 10^{58}$	1.01	0.10

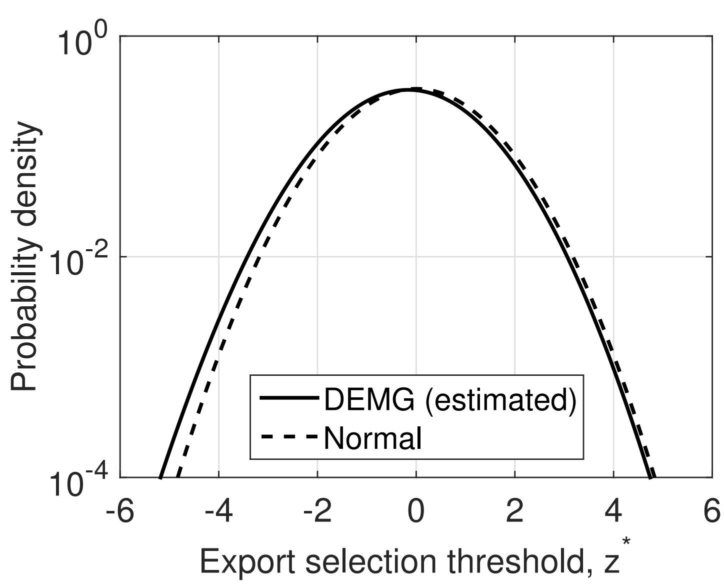
Notes: all summary statistics are reported across 274 destination-year-hs6 observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. The amplification effect is computed as the ratio of the baseline estimate of trade elasticity under uncertainty relative to its counterfactual value under complete information for the indicated counterfactual scenario. In Panel A, the counterfactual values are obtained by varying the selection thresholds from the baseline values of $z_{jkt}^{U^*}$ to their respective counterfactual values of $z_{jkt}^{CI^*}$, while keeping the distribution of the export selection shocks at their baseline values estimated under uncertainty. In Panel B, the counterfactual values are obtained by varying the distribution of the selection shocks from the baseline values of g_{jkt}^{U} to their respective counterfactual values, g_{jkt}^{CI}, while keeping the entry threshold values at their baseline values estimated under uncertainty. Panel C computes complete counterfactual values by varying both, the selection thresholds and distributions of selection shocks to their counterfactual values.

Figure 1: Selection and dispersion effects of uncertainty.



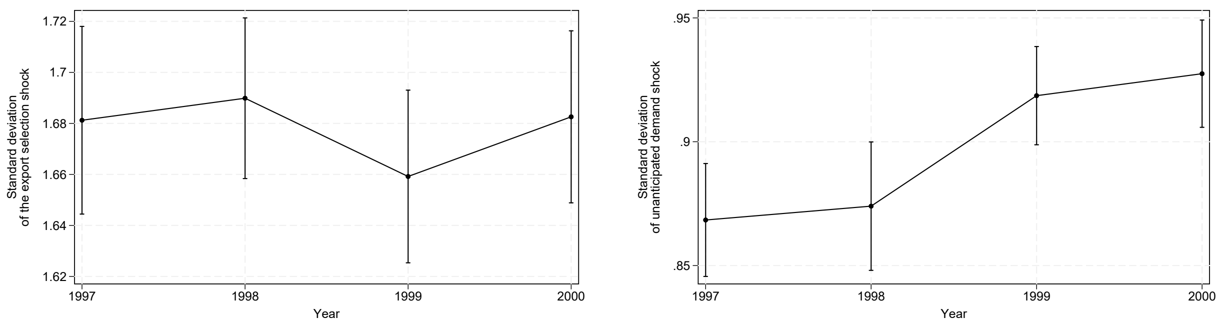
Notes: the figure plots $\gamma(z^*, g(z))$ as a function of z^* for a symmetric DEMG distribution with parameters $(\mu, \sigma^2, \lambda_L, \lambda_R) = (0, 0.5, 2, 2)$. The y-axis is plotted on a logarithmic scale. Panel A illustrates the selection effect of uncertainty: holding the distribution fixed, a lower export selection threshold under uncertainty $(z^{U^*} < z^{CI^*})$ reduces γ. Panel B illustrates the dispersion effect of uncertainty. The solid line corresponds to the baseline distribution, while the dashed represents a mean-preserving spread with higher variance $(\sigma^2 = 4)$, corresponding to complete information. The dispersion effect is ambiguous: at lower thresholds γ decreases, while at higher thresholds it increases.

Figure 2: Estimated Distribution of Export Selection Shocks.



Notes: the figure plots the probability density function of the export selection shock under uncertainty using the Double EMG distribution with average parameter estimates from Table 2: $(\mu, \sigma, \lambda_L, \lambda_R) = (0, 1.20, 4.04, 12.64)$. The dashed line depicts a Normal distribution with the same standard deviation $(\sigma = 1.20)$ for comparison. The y-axis is on a logarithmic scale. The figure illustrates the role of asymmetric tail behavior in shaping the dispersion of export selection shocks.

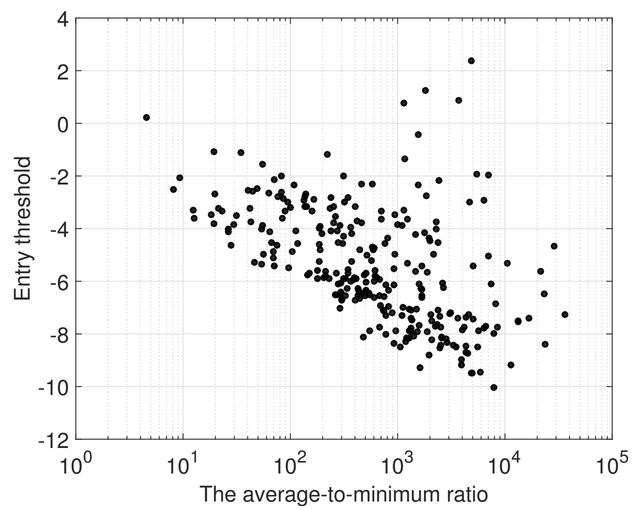
Figure 3: Dispersion of Shocks Around the 1999 Devaluation



(a) Export Selection Shock (z^U_{fjkt}) (b) Unexpected Demand (υ_{fjkt})

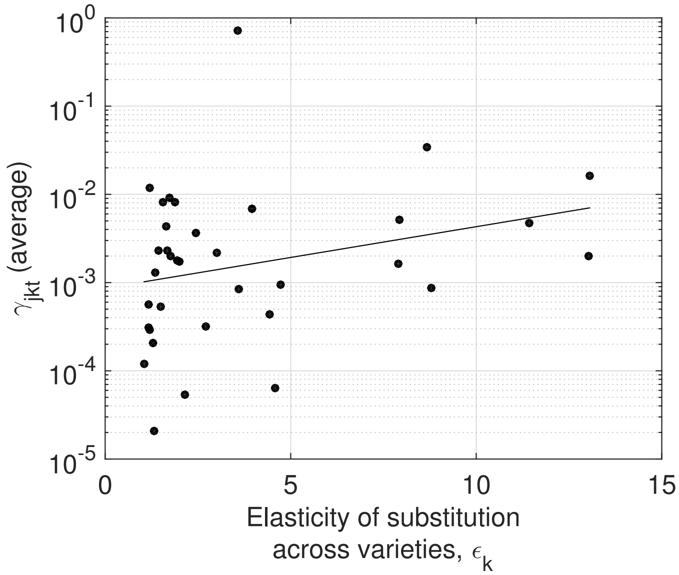
Notes: The figure plots the average standard deviation of firm-level shocks within each product-destination-year cell, by year, controlling for product-destination fixed effects. Error bars represent 95% confidence intervals based on robust standard errors. Panel (a) shows that the dispersion of export selection shocks (reflecting productivity and expected demand) remained stable across the sample period; a joint F-test cannot reject equality across years ($p = 0.605$). Panel (b) shows that the dispersion of unexpected demand shocks increased significantly following Brazil's exchange rate devaluation in January 1999; the difference between pre-devaluation (1997–1998) and post-devaluation (1999–2000) periods is statistically significant ($p < 0.001$).

Figure 4: The entry thresholds and average-to-minimum ratios.



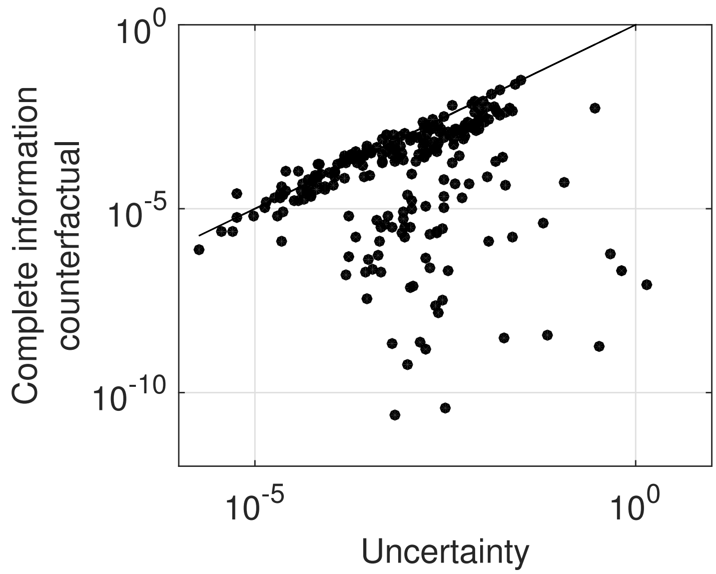
Notes: The figure depicts a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios for observations with an estimate of the Double EMG tail parameter $\lambda_R > 1$. The threshold is not defined for $\lambda_R \leq 1$. Each dot corresponds to a product-destination-year observation. Values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution.

Figure 5: Heterogeneity in endogenous selection effect, γ_{jkt}, across products.



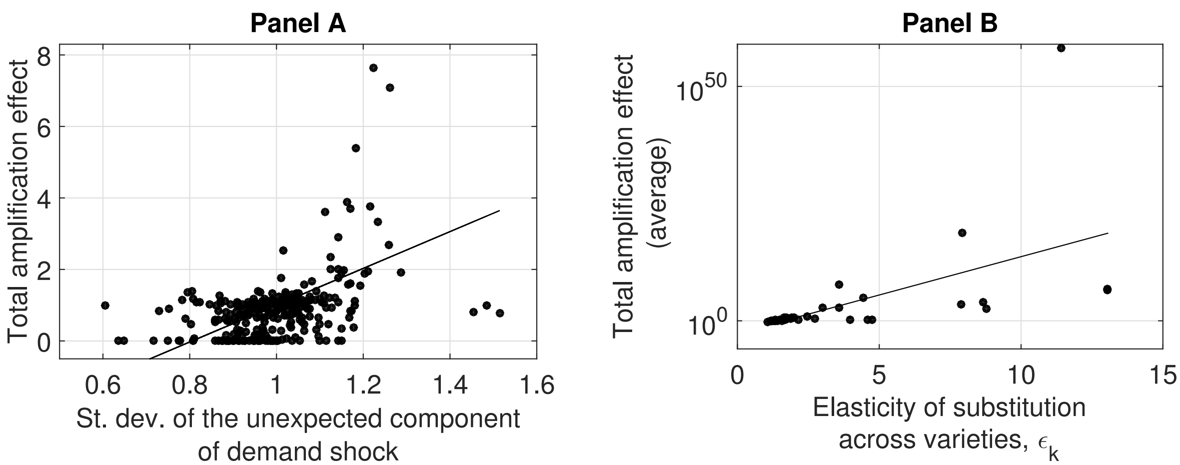
Notes: each dot computes the average across destination-year observations endogenous selection effect, γ_{jkt}, for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

Figure 6: Estimates of the endogenous selection effect, γ_{jkt}.



Notes: For the ease of visual presentation this graph omits depicting counterfactual values that are below 10^{-12}. There are 30 of such observations. The solid line is the 45-degree line.

Figure 7: Total amplification effect.



Notes: In Panel A, for the ease of visual presentation this graph omits depicting counterfactual values that are above 8. There are two such observations. The solid line is the OLS best fit line. All values are normalized by the respective product averages, a product is a 6-digit HS code. Each dot corresponds to a product-destination-year observation. In Panel B, each dot computes the average across destination-year observations amplification effect for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code the elasticity of substitution across varieties are obtained from Soderbery (2015).

A Theoretical Appendix

In this section we provide derivations for the theoretical results in Section 2.

A.1 Environment with Complete Information

The problem of firm f selling from country i to country j in sector k consists of maximizing profit subject to the demand equation (2):

$$\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) = \max_{q_{fijk}} p_{fijk} q_{fijk} - \frac{w_i \tau_{ij}}{e^{z^a_{fijk}}} q_{fijk} - w_i f_{ijk}. \tag{37}$$

The first order conditions with respect to quantity yield the optimal quantity given by

$$q_{fijk}(z^a_{fijk}, z^p_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{\epsilon_k z^a_{fijk} + z^p_{fijk}}. \tag{38}$$

Using equations (2) and (38), a firm's optimal revenue is further given by

$$r_{fijk}(z^a_{fijk}, z^p_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}}. \tag{39}$$

Substituting equations (39) and (38) into equation (37) yields optimal profit given by

$$\pi_{fijk}(z^a_{fijk}, z^p_{fijk}) = \frac{1}{\epsilon_k} \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} - w_i f_{ijk}. \tag{40}$$

A firm exports if its profit from exporting is positive:

$$\begin{aligned} \pi_{fijk}(z^a_{fijk}, z^p_{fijk}) &\geq 0 \\ e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} &\geq \frac{w_i f_{ijk}}{\frac{1}{\epsilon_k}\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k}} \end{aligned} \tag{41}$$

Denote by $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}$ and $f^\tau(\tau_{ij}) = \tau_{ij}^{1-\epsilon_k}$. Then, inequality (41) can be written as

$$e^{(\epsilon_k - 1) z^a_{fijk} + z^p_{fijk}} \geq \frac{\epsilon_k w_i f_{ijk}}{B_{ijk} \tau_{ij}^{1-\epsilon_k}}. \tag{42}$$

Denote by

$$z^{CI^*}_{ijk} = \log\left(\frac{\epsilon_k w_i f_{ijk}}{B_{ijk} f^\tau(\tau_{ij})}\right) \tag{43}$$

and substitute into inequality (42) to obtain export selection equation (9).

Trade Elasticity: Given the endogenous selection into exporting that is based on the realization of profitability shocks, the total trade flows from country i to country j in sector k, X_{ijk}, are defined as

$$X_{ijk} = J_i(1-G_{ijk}(z_{ijk}^{CI^*}))\int_{z_{ijk}^{CI^*}}^{+\infty} r_{fijk}(z_{fijk}^{CI})\frac{g_{ijk}(z_{fijk}^{CI})}{1-G_{ijk}(z_{ijk}^{CI^*})}dz_{fijk}^{CI},$$

where z_{fijk}^{CI} is the decision relevant export selection shocks defined as $(\epsilon_k-1)z_{fijk}^{a}+z_{fijk}^{p}$, and $g_{ijk}(.)$ and $G_{ijk}(.)$ are the respective probability and cumulative density functions of z_{fijk}^{CI}. J_i is the exogenous mass of entrants in country i. Substituting equation (39) for the revenue and omitting subscripts and superscripts on z_{fijk}^{CI} to ease notation yields

$$X_{ijk} = J_i(1-G_{ijk}(z_{ijk}^{CI^*}))\int_{z_{ijk}^{CI^*}}^{+\infty} B_{ijk}f^{\tau}(\tau_{ij})e^{z}\frac{g_{ijk}(z)}{1-G_{ijk}(z_{fijk}^{CI^*})}dz,$$

where $B_{ijk}=\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k-1}Y_{jk}P_{jk}^{\epsilon_k-1}w_i^{1-\epsilon_k}$, $f^{\tau}(\tau_{ij})=\tau_{ij}^{1-\epsilon_k}$. Differentiating with respect to τ_{ij} yields:

$$\frac{\partial X_{ijk}}{\partial\tau_{ij}} = \frac{\partial f^{\tau}(\tau_{ij})}{\partial\tau_{ij}}J_i\int_{z_{ijk}^{CI^*}}^{+\infty}B_{ijk}e^{z}g_{ijk}(z)dz - J_i\frac{\partial z_{ijk}^{CI^*}}{\partial\tau_{ij}}B_{ijk}f^{\tau}(\tau_{ij})e^{z_{ijk}^{CI^*}}g_{ijk}(z_{ijk}^{CI^*}). \quad (44)$$

Differentiate equation (43) with respect to τ_{ij} to obtain

$$\frac{\partial z_{ijk}^{CI^*}}{\partial\tau_{ij}} = -\frac{\partial\log f^{\tau}(\tau_{ij})}{\partial\tau_{ij}}. \quad (45)$$

Substituting equation (45) into equation (44) yields

$$\begin{aligned}\frac{\partial X_{ijk}}{\partial\tau_{ij}} &= \frac{\partial f^{\tau}(\tau_{ij})}{\partial\tau_{ij}}J_i\int_{z_{ijk}^{CI^*}}^{+\infty}B_{ijk}e^{z}g_{ijk}(z)dz + J_i\frac{\partial\log f^{\tau}(\tau_{ij})}{\partial\tau_{ij}}B_{ijk}f^{\tau}(\tau_{ij})e^{z_{ijk}^{CI^*}}g_{ijk}(z_{ijk}^{CI^*}) = \\ &= \frac{\partial\log f^{\tau}(\tau_{ij})}{\partial\log\tau_{ij}}\left(\tau_{ijk}J_i\int_{z_{ijk}^{CI^*}}^{+\infty}B_{ijk}e^{z}f^{\tau}(\tau_{ijk})g_{ijk}(z)dz + \tau_{ijk}J_iB_{ijk}f^{\tau}(\tau_{ij})e^{z_{ijk}^{CI^*}}g_{ijk}(z_{ijk}^{CI^*})\right) = \\ &= \frac{\partial\log f^{\tau}(\tau_{ij})}{\partial\log\tau_{ij}}\left(\tau_{ijk}^{-1}X_{ijk} + \tau_{ijk}^{-1}X_{ijk}\frac{e^{z_{ijk}^{CI^*}}g_{ijk}(z_{ijk}^{CI^*})}{\int_{z_{ijk}^{CI^*}}^{+\infty}e^{z}g_{ijk}(z)dz}\right).\end{aligned}$$

Hence,

$$\frac{\partial\log X_{ijk}}{\partial\log\tau_{ij}} = \frac{\partial\log f^{\tau}(\tau_{ij})}{\partial\log\tau_{ij}}\left(1+\frac{e^{z_{ijk}^{CI^*}}g_{ijk}(z_{ijk}^{CI^*})}{\int_{z_{ijk}^{CI^*}}^{+\infty}e^{z}g_{ijk}(z)dz}\right).$$

A.2 Environment with Uncertainty

The problem of firm f selling from country i to country j in sector k consists of maximizing the *expected* profit subject to the demand equation (2):

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \max_{q_{fijk}} E_{z^p_{fijk}|z^a_{fijk}} \left(p_{fijk} q_{fijk} - \frac{w_i \tau_{ij}}{e^{z^a_{fijk}}} q_{fijk} \right) - w_i f_{ijk}. \tag{46}$$

The first order conditions with respect to quantity yield the optimal quantity given by

$$q_{fijk}(z^a_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} w_i^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{-\epsilon_k} e^{\epsilon_k z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k}. \tag{47}$$

Using equations (2) and (47), a firm's realized revenue is further given by

$$r_{fijk}(z^a_{fijk}, z^p_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} e^{\frac{z^p_{fijk}}{\epsilon_k} + (\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k - 1}. \tag{48}$$

Substituting equations (48) and (47) into equation (46) yields optimal expected profit given by

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} - w_i f_{ijk}. \tag{49}$$

A firm exports if its expected profit from exporting is non-negative:

$$\begin{aligned} E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] &\geq 0 \\ e^{(\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} &\geq \frac{w_i f_{ijk}}{\frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k}}. \end{aligned} \tag{50}$$

Substituting equation (43) into inequality (50) yields

$$e^{(\epsilon_k - 1) z^a_{fijk}} \left(E_{z^p_{fijk}|z^a_{fijk}} \left(e^{\frac{z^p_{fijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} \geq e^{z^{CI*}_{ijk}}.$$

Trade Elasticity: Using the orthogonal projection of z^p_{fijk} on z^a_{fijk} in equation (5), export revenue (48) can be written as

$$r_{fijk}(z^a_{fijk}, \upsilon_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} \left(E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right)^{\epsilon_k - 1} e^{E(z^p_{fijk}|z^a_{fijk}) + (\epsilon_k - 1) z^a_{fijk} + \frac{\upsilon_{fijk}}{\epsilon_k}}. \tag{51}$$

Using equations (16) and (17), the total trade flows from country i to country j in sector k, X_{ijk}, can be written as

$$\begin{aligned} X_{ijk} &= J_i(1 - G_{ijk}(z^{U*}_{ijk})) \int_{z^{U*}_{ijk}}^{+\infty} \left[\int_{-\infty}^{+\infty} r_{fijk}(z^U_{fijk}, \upsilon_{fijk}) g^{\upsilon}_{ijk}(\upsilon_{fijk}) d\upsilon_{fijk}\right] \frac{g_{ijk}(z^U_{fijk})}{1 - G_{ijk}(z^{U*}_{ijk})} dz^U_{fijk} = \\ &= J_i(1 - G_{ijk}(z^{U*}_{ijk})) \int_{z^{U*}_{ijk}}^{+\infty} B_{ijk} f^{\tau}(\tau_{ij}) e^{z^U_{fijk}} \frac{g_{ijk}(z^U_{fijk})}{1 - G_{ijk}(z^{U*}_{ijk})} dz^U_{fijk}, \end{aligned} \tag{52}$$

where z^U_{fijk} is the decision relevant export selection shocks defined as $E(z^p_{fijk}|z^a_{fijk}) + (\epsilon_k - 1)z^a_{fijk}$, and $g_{ijk}(.)$ and $G_{ijk}(.)$ are the respective probability and cumulative density functions of z^U_{fijk}, z^{U*}_{fijk} is the export selection threshold, $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \left(E\left(e^{\frac{\upsilon_{fijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}$, $f^{\tau}(\tau_{ij}) = \tau_{ij}^{1-\epsilon_k}$.

Differentiation equation (52) with respect to τ_{ij} and omitting subscripts and superscripts on z^U_{fijk} to ease notation yields:

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = \frac{\partial f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z^{U*}_{ijk}}^{+\infty} B_{ijk} e^z g_{ijk}(z) dz - J_i \frac{\partial z^{U*}_{ijk}}{\partial \tau_{ij}} B_{ijk} f^{\tau}(\tau_{ij}) e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk}). \tag{53}$$

Differentiate equation (17) with respect to τ_{ij} to obtain

$$\frac{\partial z^{U*}_{ijk}}{\partial \tau_{ij}} = -\frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \tau_{ij}}. \tag{54}$$

Substituting equation (54) into equation (53) yields

$$\begin{aligned} \frac{\partial X_{ijk}}{\partial \tau_{ij}} &= \frac{\partial f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z^{U*}_{ijk}}^{+\infty} B_{ijk} e^z g_{ijk}(z) dz + J_i \frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} B_{ijk} f^{\tau}(\tau_{ij}) e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk}) = \\ &= \frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \tau_{ij}} \left(\tau_{ij}^{-1} X_{ijk} + \tau_{ij}^{-1} X_{ijk} \frac{e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk})}{\int_{z^{U*}_{ijk}}^{+\infty} e^z g_{ijk}(z) dz}\right) \end{aligned}$$

Hence,

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} = \frac{\partial \log f^{\tau}(\tau_{ij})}{\partial \log \tau_{ij}} \left(1 + \frac{e^{z^{U*}_{ijk}} g_{ijk}(z^{U*}_{ijk})}{\int_{z^{U*}_{ijk}}^{+\infty} e^z g_{ijk}(z) dz}\right).$$

B Properties of γ

Consider function $\gamma(x)$ defined as

$$\gamma(x) = \frac{e^x g(x)}{\int_x^{+\infty} e^u g(u) du},$$

where $g(x)$ is a probability density function defined on $x \in \mathbb{R}$. $\gamma(x)$ can further be expressed as a hazard rate

$$\gamma(x) = \frac{h(x)}{1 - H(x)}, \tag{55}$$

where the probability density function $h(x)$ is defined as

$$h(x) \equiv \frac{e^x g(x)}{\int_{-\infty}^{+\infty} e^u g(u) du}, \tag{56}$$

and the corresponding cumulative distribution function is defined as

$$H(x) = \frac{\int_{-\infty}^{x} e^u g(u) du}{\int_{-\infty}^{+\infty} e^u g(u) du}. \tag{57}$$

Assume $g(x)$ satisfies Assumption 1 below.

Assumption 1 (A1) *The probability density function $g(x)$ has the following properties:*

(i) $E(e^x) \equiv \int_{-\infty}^{+\infty} e^x g(x) dx$ exists and is finite, and

(ii) the function $\log\left(\int_x^{+\infty} e^u g(u) du\right)$ is concave in z.

Assumption (i) ensures that the probability and the cumulative distribution functions $h(.)$ and $H(.)$ are well defined. Assumption (ii) ensures that function $\gamma(x)$ is a monotonically increasing function of x, as we show below. Assumption (ii) states that the log of the conditional expectation of an exponential function is a concave function of the threshold value. Intuitively, this assumption requires that the upper tail of the distribution $g(x)$ does not have too much mass.[32] Without such a restriction, total sales of marginal firms relative to average sales could become very small as the threshold increases, and the extensive margin elasticity, $\gamma(x)$, might not be monotonically increasing in x. The standard distributional assumptions made in the literature all meet this requirement.[33]

Proposition 1 below establishes two properties of function $\gamma(x)$ underlying Result 1 and Result 2.

[32] Heavy-tailed distributions, e.g. distributions that violate assumption (i), are sometimes said to have the property of log-convexity.

[33] For example, the Normal distribution, Exponential distribution (with an appropriate restriction on the scale parameter) and the Double Exponentially Modified Gaussian distribution all satisfy this requirement.

Proposition 1 *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

(i) $\gamma(x) \equiv [e^x g(x)]/\int_x^{+\infty} e^u g(u)du$ *is an increasing function of* x.

(ii) Let $\tilde{g}(x)$ *be a mean preserving spread of* $g(x)$*, with an respectively defined* $\tilde{\gamma}(x)$*. Then* $\gamma(x)$ *and* $\tilde{\gamma}(x)$ *satisfy the single crossing property. That is, there exists* x^* *such that* $\tilde{\gamma}(x) \leq \gamma(x)$ *for all* $x \geq x^*$*, and* $\tilde{\gamma}(x) \geq \gamma(x)$ *for all* $x \leq x^*$.

Proof of Proposition 1

Part (i) First, define $h(x) = (e^x g(x))/E$, where $E = \int_{-\infty}^{+\infty} e^u g(u)du$. Notice that $h(x)$ is positive for all x and that $\int_{-\infty}^{+\infty} h(x)dx = 1$. Hence, $h(x)$ is a probability density function. The corresponding cumulative density function is given by $H(x) = \int_{-\infty}^{x} e^u g(u)du/E$. The corresponding survival function is given by $1 - H(x) = \int_x^{+\infty} e^u g(u)du/E$.

Next, function $\gamma(x)$ can then be written as

$$\gamma(x) = \frac{e^x g(x)}{\int_x^{+\infty} e^u g(u)du} = \frac{h(x)}{1 - H(x)}.$$

Hence, $\gamma(x)$ is a hazard rate associated with the distribution $H(x)$. By Theorem 10 in Rinne (2014), the hazard rate $\gamma(x)$ is monotonically increasing in x if and only if its logarithmic survival function, $\log(1 - H(x))$, is concave. Notice that by part (ii) of A1, $\log(1 - H(x))$ is a concave function of x. Hence, $\gamma(x)$ is increasing in x. For completeness, we reproduce the proof of this result below.

Notice that

$$\gamma(x) = -\frac{d \log(1 - H(x))}{dx}.$$

Hence,

$$\frac{d\gamma(x)}{dx} = -\frac{d^2 \log(1 - H(x))}{dx^2}.$$

Since $\log(1 - H(x))$ is a concave function of x, $d^2 \log(1 - H(x))/dx^2 < 0$. Therefore, $d\gamma(x)/dx > 0$.

Part (ii) Function $\tilde{\gamma}(x)$ is given by

$$\tilde{\gamma}(x) = \frac{e^x \tilde{g}(x)}{\int_x^{+\infty} e^u \tilde{g}(u)du} = \frac{\tilde{h}(x)}{1 - \tilde{H}(x)},$$

where $\tilde{g}(.)$ is a mean preserving spread of $g(.)$, $\tilde{h}(x) = [e^x \tilde{g}(x)]/\int_{-\infty}^{+\infty} e^u \tilde{g}(u)du$, and $\tilde{H}(x)$ is the corresponding cumulative distribution function.

$\gamma(x) > \tilde{\gamma}(x)$ if and only if $H(x) > \tilde{H}(x)$ as follows for the following set of equivalent inequalities:

$$\begin{aligned}
\gamma(x) = -\frac{d\log(1-H(x))}{dx} &> -\frac{d\log(1-\tilde{H}(x))}{dx} = \tilde{\gamma}(x) \\
d\log(1-H(x)) &< d\log(1-\tilde{H}(x)) \\
\int d\log(1-H(x)) &< \int d\log(1-\tilde{H}(x)) \\
\log(1-H(x)) &< \log(1-\tilde{H}(x)) \\
H(x) &> \tilde{H}(x).
\end{aligned}$$

We will now show in three steps that $H(x)$ crosses $\tilde{H}(x)$ once from below, and therefore there exists x^* such that $H(x) > \tilde{H}(x)$ holds for $x > x^*$, and therefore (ii) holds.

Step 1: Denote by X and $\tilde{X}$ random variables distributed according to $g(x)$ and $\tilde{g}(x)$ respectively. Since $\tilde{g}(x)$ is a mean preserving spread of $g(x)$, it holds that $\tilde{X} = X + \hat{X}$, where $\hat{X}$ is distributed according to $\hat{g}(x)$ with mean zero, and $\hat{X}$ is independent from X. Hence, $\tilde{g}(.)$ is a convolution of $g(.)$ and $\hat{g}(.)$ and can be written as

$$\tilde{g}(x) = \int_{-\infty}^{+\infty} g(x-u)\hat{g}(u)du.$$

Step 2: Denote by X^h, $\tilde{X}^h$, $\hat{X}^h$ random variables distributed according to $h(x)$, $\tilde{h}(x)$, and $\hat{h}(x)$ respectively, where $\hat{h}(x) = [e^x\hat{g}(x)] / \int_{-\infty}^{+\infty} e^x\hat{g}(x)dx$. Similarly, it can be show that $\tilde{h}(.)$ is a convolution of $h(.)$ and $\hat{h}(.)$:

$$\begin{aligned}
\int_{-\infty}^{+\infty} h(x-u)\hat{h}(u)du &= \frac{\int_{-\infty}^{+\infty} e^{x-u}g(x-u)e^u\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{\int_{-\infty}^{+\infty} e^x g(x-u)\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{e^x\tilde{g}(x)}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \tilde{h}(x).
\end{aligned}$$

Thus, it hold that $\tilde{X}^h = X^h + \hat{X}^h$, where $\tilde{X}^h$ and $\hat{X}^h$ are independent.

Step 3: Consider a random variable $\bar{X} = X^h + \hat{X}^h - E(\hat{X}^h)$ with the cumulative distribution function denoted by $\bar{H}(x)$. $\bar{X}$ is a mean preserving spread of X^h and therefore the two corresponding cumulative distribution functions satisfy the single-crossing property whereby $H(x) = \bar{H}(x)$ if $x = E(X^h)$; $H(x) < \bar{H}(x)$ for $x < E(X^h)$, and $H(x) > \bar{H}(x)$ for $x > E(X^h)$.

Next, notice that $\tilde{X}^h = \bar{X} + E(\hat{X}^h)$. Therefore the cumulative distribution function of

$\tilde{X}^h$ is a shift of the cumulative distribution function of $\bar{X}$ along the x-axis, namely $\tilde{H}(x) = \bar{H}(x - E(\hat{X}^h))$. Hence $\tilde{H}(x)$ preserves the same single-crossing property with respect to $H(x)$. Namely $\exists x^*$ that that $H(x) = \tilde{H}(x)$ if $x = x^*$; $H(x) < \tilde{H}(x)$ for $x < x^*$, and $H(x) > \tilde{H}(x)$ for $x > x^*$. ■

C Invariance to Mean Normalization

This appendix shows that the selection and dispersion effects of uncertainty are invariant to the normalization of the mean of the unexpected demand component υ_{fjkt}. For simplicity, we drop $fjkt$ subscripts.

The export selection equations in the environment with uncertainty (U) and complete information (CI) are given by

$$\text{U:} \quad (\epsilon - 1)z^a + E(z^p|z^a) + \epsilon \log \left[E\left(e^{\frac{\upsilon}{\epsilon}} \right) \right] \geq \hat{z}^* \tag{58}$$

$$\text{CI:} \quad (\epsilon - 1)z^a + E(z^p|z^a) + \upsilon \geq \hat{z}^*, \tag{59}$$

where $\hat{z}^*$ encompasses the general equilibrium terms and is the same across information environments.

Using the assumption of a linear conditional expectation in (6) and keeping the mean of υ as μ_υ, (58) and (59) can be written as

$$\text{U:} \quad \underbrace{((\epsilon - 1) + \alpha)(z^a - \mu_a)}_{z^U \sim (0; V_U)} \geq \underbrace{\hat{z}^* - ((\epsilon - 1) + \alpha)\mu_a - \mu_\upsilon - \frac{1}{2}\frac{1}{\epsilon}V_\upsilon}_{z^{U*}} \tag{60}$$

$$\text{CI:} \quad \underbrace{((\epsilon - 1) + \alpha)(z^a - \mu_a) + (\upsilon - \mu_\upsilon)}_{z^{CI} \sim (0; V_U + V_\upsilon)} \geq \underbrace{\hat{z}^* - ((\epsilon - 1) + \alpha)\mu_a - \mu_\upsilon}_{z^{CI*}}, \tag{61}$$

where μ_a denotes the mean of z^a, and V_U denotes the variance of z^U.

With the re-normalization of shocks and thresholds in (60) and (61), the dispersion and selection effects of uncertainty can be most clearly seen.

The left-hand side of the inequalities define the export selection shocks. Notice that both shocks are normalized to have zero mean, and their respective distributions differ by the added variance of the unexpected component of the demand shock. This is the dispersion effect of uncertainty.

The right-hand side of the inequalities defines the export selection thresholds. Notice that regardless of the mean of υ, μ_υ, the two thresholds differ by $1/2 \times 1/\epsilon \times V_\upsilon$. This is the selection effect of uncertainty and it is independent of the mean of υ. Hence, without the loss of generality, the paper proceeds by assuming $\mu_\upsilon = 0$.

D Sunk Costs Under Complete Information Versus Uncertainty

This appendix provides an application of the general dynamic setting developed in Section 3.3.1 to the Dixit (1989) sunk cost extension of our model. The mechanism through which sunk costs affect export participation in Dixit (1989) operates through option value: when firms can observe profitability shocks before making entry decisions, they have an incentive to wait for favorable realizations before incurring irreversible costs. This generates hysteresis—a wedge between entry and exit thresholds. The key requirement for this mechanism is that the decision-relevant state varies over time and is observable before decisions are made.

Our framework features two information environments that differ precisely in whether this requirement is satisfied. To make the comparison concrete, we extend the baseline model by assuming that productivity z^a is drawn once upon firm creation and remains fixed, while demand shocks z^p are drawn each period from a known distribution. This timing assumption implies that variation in profitability over time comes from demand shocks, not productivity.

D.1 Complete Information

Under complete information, a firm observes both its fixed productivity z^a and its current-period demand shock z^p before making export decisions. The export selection shock $z^{CI} = (\epsilon-1)z^a+z^p$ therefore varies over time as new demand shocks are realized. This creates option value: a firm can wait for a sufficiently favorable realization of z^{CI} before incurring sunk entry costs, and an incumbent can remain in the market during temporarily unfavorable periods to avoid re-incurring those costs upon re-entry. Export participation is therefore characterized by an entry threshold z_H^{CI*} and an exit threshold z_L^{CI*}, with $z_H^{CI*} > z_L^{CI*}$, generating hysteresis as in Dixit (1989).

The partial trade elasticity in equation (25) can then be written as a weighted average of cohort-level elasticities, with weights given by cohort trade shares:

$$\frac{\partial \log X}{\partial \log \tau} = \frac{X_{\text{entrants}}}{X}\frac{\partial \log X_{\text{entrants}}}{\partial \log \tau} + \frac{X_{\text{incumbents}}}{X}\frac{\partial \log X_{\text{incumbents}}}{\partial \log \tau}, \tag{62}$$

where cohort-level trade flow in equation (26) takes the form

$$X_n = \underbrace{\int_{z^{CI}>z_n^{CI*}}}_{\text{selection effect}} \underbrace{r(z^{CI},\tau)}_{\substack{\text{firm-level}\\ \text{trade elasticity}}} \underbrace{m_n(z^{CI})}_{\text{dispersion effect}} \, dz^{CI}, \tag{63}$$

with $n \in \{\text{entrant, incumbent}\}$ and $m_n(z^{CI})$ is the mass of firm of cohort n. Sunk costs, therefore, do not introduce a new channel affecting trade elasticities. Conditional on cohort membership, cohort-level elasticities continue to be governed by selection and dispersion effects. Sunk costs affect partial trade elasticities only through the distribution of firms across cohorts and the location of cohort-specific thresholds.

D.2 Uncertainty

Under uncertainty, a firm observes its fixed productivity z^a but does not observe the current-period demand shock z^p before making export decisions. Instead, export decisions are based on $z^U = (\epsilon - 1)z^a + E(z^p|z^a)$, which combines productivity and the conditional expectation of demand. Crucially, for a given firm, z^U does not vary over time: productivity z^a is drawn once at entry and the conditional expectation $E(z^p|z^a)$ depends only on z^a. The time-varying demand shocks manifest only through their unexpected component, $\upsilon = z^p - E(z^p|z^a)$, which is revealed only *after* export and production decisions have been made.

This timing eliminates the option value mechanism. Firms cannot condition entry decisions on realizations of υ because these are observed only after committing to export. Consequently, introducing sunk costs does not generate differential entry and exit thresholds. Export participation is governed by a single threshold, determined by the present discounted value of expected profits exceeding sunk costs. The partial trade elasticity retains the same structure as in our static benchmark and does not depend on cohort.

D.3 Implications

This analysis clarifies that sunk costs do not offset or introduce channels beyond the selection and dispersion effects we identify. Under complete information framework in our setting, sunk costs generate hysteresis and redistribute selection across cohorts, but cohort-level elasticities remain governed by selection and dispersion as in equation (63). Under uncertainty, sunk costs do not generate hysteresis, and the selection–dispersion structure of trade elasticities is unchanged from the static model.

Quantitatively assessing how hysteresis under complete information affects aggregate trade elasticities would require solving and estimating a fully dynamic model to compute cohort-specific elasticities and trade shares. Such an exercise lies beyond the scope of this paper but represents a promising direction for future work.

E Other Sources of Uncertainty

This section demonstrates that the model presented in this paper is general enough to accommodate other sources of uncertainty, namely supply-side or policy uncertainty

E.1 Supply-Side Uncertainty

If a supply-side shock (e.g. productivity) is not fully observed at the time of export decisions, the selection shock under uncertainty would take the form

$$z^U_{fijk} = (\varepsilon_k - 1)E\big(z^a_{fijk}\big) + E\big(z^p_{fijk} \mid z^a_{fijk}\big)\,, \tag{64}$$

where the realization of a supply-side shock, z^a_{fijk}, is replaced by its expectation at the time of the export decision. Relative to the baseline specification, this modification affects both the export participation threshold and the distribution of the export selection shock.

First, replacing realized supply conditions with their expectation alters expected profitability and therefore the export participation threshold under uncertainty. As a result, the wedge between the export thresholds under complete information and uncertainty continues to capture the effect of uncertainty on selection, with its magnitude now reflecting both demand- and supply-side risk.

Second, supply uncertainty changes the variance of the unexpected component of the export selection shock, $(\varepsilon_k - 1)\left(z^a_{fijk} - E(z^a_{fijk})\right) + \left(z^p_{fijk} - E(z^p_{fijk} \mid z^a_{fijk})\right)$, thereby modifying the cross-sectional distribution of firms around the export cutoff. This directly affects the dispersion term that governs the mass of firms at the margin.

Consequently, the selection and dispersion effects of uncertainty continue to take the form described in equations (21) and (22), with the interpretation of both the threshold wedge and the dispersion term broadened to reflect the presence of supply-side uncertainty. The net effect on trade elasticities remains theoretically ambiguous and depends on the relative strength of these two channels.

E.2 Policy Uncertainty

Policy uncertainty can be incorporated in the framework through uncertainty in iceberg trade costs, τ_{ijk}. The implications for selection and dispersion effects depend on whether this uncertainty is common across firms or firm specific.

If policy uncertainty is common across firms within a given exporter–importer–product triplet, uncertainty in τ_{ijk} affects expected trade costs but does not generate cross-sectional heterogeneity at the firm level. In this case, the export participation threshold under uncertainty is given by

$$z^{U*}_{ijk} = \log(\varepsilon_k w_i f_{ijk}) - \log B_{ijk} - \log f^{\tau}(E(\tau_{ijk}))\,, \tag{65}$$

while the distribution of the export selection shock across firms remains unchanged. As a result, common policy uncertainty operates exclusively through the selection effect by shifting the export participation threshold, without affecting the dispersion term that governs the mass of firms at the margin.

By contrast, if firms face idiosyncratic uncertainty in trade costs, realizations of τ_{fijk} enter the unexpected component of the export selection shock. In this case, policy uncertainty affects both the export participation threshold through expected trade costs and the dispersion of the export selection shock through firm-level variation in realized trade costs. Mechanically, this case is equivalent to firm-specific demand or supply-side uncertainty discussed above, and policy uncertainty influences trade elasticities through both selection and dispersion effects.

In both cases, the partial trade elasticity retains the same analytical structure as in the baseline model, with policy uncertainty affecting trade elasticities exclusively through its impact on export participation thresholds and, when policy uncertainty is firm specific, through the dispersion of selection shocks around the cutoff.

This discussion highlights that the mechanisms emphasized in the paper are not specific to demand uncertainty but apply to a broader class of payoff-relevant uncertainties faced by exporting firms.

F Trade Elasticities Under Price-Setting

F.1 Theoretical Robustness

This appendix derives the export selection equation and trade elasticity formula under an alternative timing assumption in which firms set prices before demand is realized and quantities adjust to clear markets. The complete information case is unchanged from the baseline model and follows the derivations in Appendix A.1. We focus here on the environment with uncertainty.

Under uncertainty, firms observe productivity z^a_{fijk} but not the demand shock z^p_{fijk} before setting prices. Firms set prices to maximize expected profit:

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \max_{p_{fijk}} \; E_{z^p_{fijk}|z^a_{fijk}} \left(p_{fijk}\, q_{fijk} - \frac{w_i \tau_{ij}}{e^{z^a_{fijk}}} q_{fijk} \right) - w_i f_{ijk},$$

subject to demand $q_{fijk} = e^{z^p_{fijk}} p_{fijk}^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}$. The first-order condition yields

$$p_{fijk} = \frac{\epsilon_k}{\epsilon_k - 1} \frac{w_i \tau_{ij}}{e^{z^a_{fijk}}}. \tag{66}$$

Given the optimal price and realized demand shock, quantity is given by

$$q_{fijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k} (w_i \tau_{ij})^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{\epsilon_k z^a_{fijk} + z^p_{fijk}}. \tag{67}$$

Substituting equations (66) and (67) into (F.1) yields optimal expected profit given by

$$E_{z^p_{fijk}|z^a_{fijk}}[\pi_{fijk}(z^a_{fijk}, z^p_{fijk})] = \frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}}(w_i\tau_{ij})^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} e^{(\epsilon_k - 1)z^a_{fijk}} E_{z^p_{fijk}|z^a_{fijk}}(e^{z^p_{fijk}}) - w_i f_{ijk}. \tag{68}$$

The firm exports if expected profit is non-negative yielding the export selection equation given by

$$e^{(\epsilon_k-1)z^a_{fijk}} E_{z^p_{fijk}|z^a_{fijk}}\left(e^{z^p_{fijk}}\right) \geq \frac{w_i f_{ijk}}{\frac{(\epsilon_k-1)^{\epsilon_k-1}}{\epsilon_k^{\epsilon_k}} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1} \tau_{ij}^{1-\epsilon_k}}.$$

Substituting the orthogonal projection of the demand shock and taking the logarithm of both sides yields

$$\underbrace{(\epsilon_k - 1)z^a_{fijk} + E(z^p_{fijk}|z^a_{fijk})}_{z^U_{fijk}} \geq \underbrace{\log\left(\frac{w_i f_{ijk}}{\frac{(\epsilon_k-1)^{\epsilon_k-1}}{\epsilon_k^{\epsilon_k}} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1} \tau_{ij}^{1-\epsilon_k}}\right) - \log\left[E\left(e^{\upsilon_{fijk}}\right)\right]}_{z^{U*}_{ijk}},$$

were $z^{U^*}_{ijk} \equiv z^{CI^*}_{ijk} - \log\left[E\left(e^{\upsilon_{fijk}}\right)\right]$.

Notably, the thresholds across information environments satisfy

$$z^{CI^*}_{ijk} - z^{U^*}_{ijk} = \frac{1}{2} V_{\upsilon_{fijk}}. \tag{69}$$

Compare this to the quantity-setting case where the wedge is $\frac{1}{2\epsilon_k} V_\upsilon$. The selection effect is present under both timing assumptions, differing only in magnitude.

Trade Elasticity: Using equations (66) and (67) and the orthogonal projection, export revenue can be written as

$$r_{fijk}(z^a_{fijk}, \upsilon_{fijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} (w\tau_{jkt})^{1-\epsilon_k} Y_{jkt} P_{jkt}^{\epsilon_k - 1} e^{(\epsilon_k - 1)z^a_{fjkt} + E(z^p_{fjkt}|z^a_{fjkt}) + \upsilon_{fijk}}.$$

Total exports from origin i to destination j in product k can be expressed as

$$X_{ijk} = J_i B_{ijk} \cdot \tau_{ijk}^{1-\epsilon_k} \int_{z^{U*}_{ijk}}^{\infty} e^z g_{ijk}(z) dz, \tag{70}$$

were $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} E\left(e^{\upsilon_{fijk}}\right)$.

Differentiating with respect to $\log \tau_{ijk}$ and using $\frac{\partial z^{U*}_{ijk}}{\partial \log \tau_{ijk}} = -(1 - \epsilon_k)$, we obtain

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ijk}} = (1 - \epsilon_k)\left[1 + \frac{e^{z^{U*}_{ijk}} g(z^{U*}_{ijk})}{\int_{z^{U*}_{ijk}}^{\infty} e^z g_{ijk}(z)\, dz}\right] = (1 - \epsilon_k)\left[1 + \gamma(z^{U*}_{ijk}, g_{ijk}(z))\right]. \tag{71}$$

This is identical in form to the trade elasticity under quantity-setting. The function $\gamma(z_{ijk}^{U*}, g_{ijk}(z))$ captures the selection and dispersion margins and depends only on the threshold z_{ijk}^{U*} and the distribution $g_{ijk}(\cdot)$ of the selection shock—objects that take the same functional form under either timing assumption.

F.2 Empirical Evidence

The two timing assumptions generate different predictions for price variation. Under quantity-setting, firms choose quantities before the demand shock is realized, and prices adjust to clear the market. This implies

$$\zeta_{fjkt}^{q} = (\epsilon_k + \alpha) z_f^a \tag{72}$$

$$\zeta_{fjkt}^{p} = -z_f^a + \upsilon_{fjkt}/\epsilon_k, \tag{73}$$

where ζ_{fjkt}^{q} and ζ_{fjkt}^{p} denote the residuals of log quantity and log price after removing destination-product-year fixed effects (see equations (29) and (30)). Quantity residuals depend only on firm productivity, while price residuals contain both productivity and the demand shock.

Under price-setting, firms commit to prices before demand is realized, and quantities adjust. This implies

$$\zeta_{fjkt}^{q} = (\epsilon_k + \alpha) z_f^a + \upsilon_{fjkt} \tag{74}$$

$$\zeta_{fjkt}^{p} = -z_f^a. \tag{75}$$

Price residuals depend only on firm productivity, while quantity residuals contain the demand shock (see equations (67) and (66)).

The critical distinction concerns what explains price variation. Under price-setting, prices depend solely on firm productivity, so after controlling for firm identity, there should be no residual price variation. Under quantity-setting, prices also reflect demand shocks, which vary across products and destinations even for the same firm.[34]

Table F1 reports the R^2 from regressions of price residuals on progressively finer fixed effects. If price-setting holds, firm fixed effects should explain nearly all variation in price residuals. Column (1) shows that firm fixed effects explain only 55 percent of price residual variation, far below the near-unity prediction of price-setting. Column (2) uses firm-product

[34] These tests assume that productivity is firm-specific rather than varying at the firm-product-destination-year level as permitted in the general model. If productivity varies freely across all dimensions, firm fixed effects would not fully explain price variation under either timing assumption, and the test would lose discriminatory power. We proceed nonetheless, recognizing that these tests provide suggestive rather than definitive evidence on the pricing mechanism.

fixed effects, raising the R^2 to 0.76. Column (3) uses firm-product-destination fixed effects, achieving $R^2 = 0.83$. Even this stringent specification leaves 17 percent of price variation unexplained. Column (4) adds year fixed effects; the within-cell residual standard deviation is 0.47 log points. Under price-setting, this residual should be approximately zero.

Table F2 summarizes the evidence. Both tests yield results consistent with the quantity-setting and inconsistent with the price-setting mechanism.

We acknowledge limitations of this analysis. Measurement error in unit values could generate spurious price variation. If reported quantities contain error, computed unit values will be noisy even if true prices are constant. Unobserved quality variation within firm-product-destination cells could also contribute to price dispersion. Despite these caveats, the pattern of results is difficult to reconcile with price-setting: the R^2 of 0.55 from firm fixed effects is far below unity, and the residual standard deviation of 0.47 within firm-product-destination cells is economically substantial.

Table F1: Variation in Price Residuals Explained by Fixed Effects.

	(1)	(2)	(3)	(4)
Fixed Effects	Firm	Firm-Product	Firm-Product -Destination	Firm-Product -Dest., Year
R^2	0.551	0.762	0.833	0.833
Residual Std. Dev.	—	—	—	0.472
Observations	29,041	25,622	22,876	22,876

Notes: Dependent variable is the log price residual after removing destination-product-year fixed effects. Under price-setting, firm fixed effects should fully explain price variation ($R^2 \approx 1$) since prices depend only on firm productivity. The residual standard deviation in column (4) represents within-firm-product-destination price variation after controlling for year effects; under price-setting, this should be approximately zero.

Table F2: Summary of Pricing Mechanism Tests.

Test	Statistic	Result	QS Pred.	PS Pred.
Price variation explained by firm FE	R^2	0.551	< 1	≈ 1
Within firm-prod.-dest. price variation	Residual Std. Dev.	0.472	> 0	≈ 0

Notes: QS = quantity-setting; PS = price-setting. Under price-setting, prices depend only on firm productivity, so firm fixed effects should explain nearly all price variation and within-cell variation should be near zero..

G Determinants of the Magnitude of the Selection Effect

This Appendix shows that the notably small estimated magnitude of the endogenous selection effect, γ, is driven both by the structure of the model and a feature of the data.

First, note that $\gamma(z^*, g(.))$ a monotonically increasing in z^* hazard rate function. Figure G1 depicts an example of such a function for a symmetric DEMG distribution $g(.)$ with zero mean and unit variance.[35] To highlight the behavior in the left tail, the y-axis is plotted on a logarithmic scale.

Conditional on $g(.)$, γ has asymptotic limits given by

$$\lim_{z^* \to -\infty} \gamma(z^*, g(.)) = 0 \text{ and } \lim_{z^* \to +\infty} \gamma(z^*, g(.)) = \lambda_R - 1,$$

where λ_R is the right tail parameter of the DEMG distribution, $g(.)$. Hence the value of the endogenous selection effect, γ, is bounded above by the right tail fatness of the distribution of the export selection shocks. As shown in Table 2, the mean estimated value of λ_R is 12.64, suggesting a possibility of a potentially large magnitude of the endogenous selection effect.

Figure G2 plots the relationship between the estimated values of the endogenous selection effect, γ, and the respective right tail parameter, λ_R. An can be seen from the figure, on average, there is a strong positive relationship between the two variables, suggesting the right-tail fatness plays an important role in determining the magnitude of the endogenous selection effect.

Recall, however, that conditional on distribution $g(.)$, and specifically on the value of its right tail parameter, λ_R, the value of γ depends on the value of the export selection threshold, z^*. The lower is the value of the threshold, the closer is the value of γ to zero, as demonstrated in Figure G1 and generally holds due to the function's asymptotic behavior in the left tail.

Therefore, an empirical moment identifying the export threshold plays an important role in pinning down the magnitude of the endogenous selection effect. In this paper we adapt the methodology of Bas et al. (2017) and identify the threshold by matching the average-to-minimum ratio of export quantity, equation (35). Notice, from equation (35) that the average-to-minimum ratio and the value of the export threshold are inversely related.

As an example, consider the distribution, $g(z)$, corresponding to the γ depicted in Figure G1. Suppose the average firm is four times larger than the smallest firm, implying the average-to-minimum ratio being equal to 4. The corresponding export threshold, z^*, is then

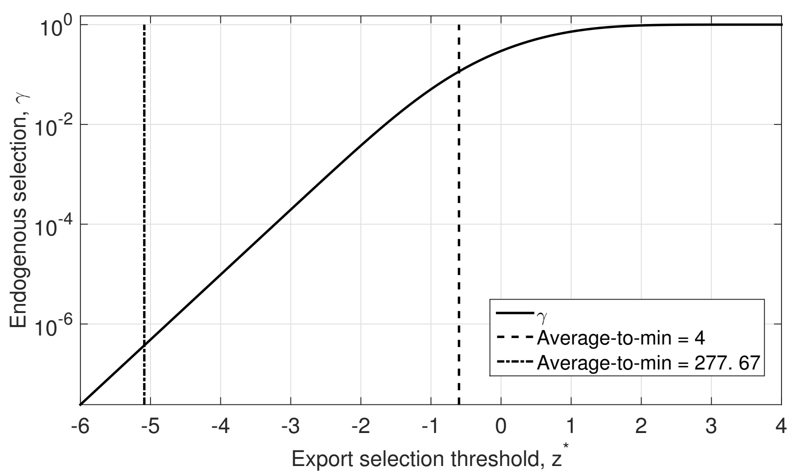
[35] The parameters of the DEMG distribution corresponding to the plot in Figure G1 are given by $(\mu, \sigma^2, \lambda_L, \lambda_R) = (0, 0.5, 2, 2)$.

equal to -0.60, and the resulting value of γ is 0.12.[36] The sample average of the average-to-minimum ratio of export quantity in our data, however, is 277.67, which would result in $z^* = -5.09$ and γ being of the order of 10^{-7} in this example.

Figure 4 plots the relationship between the estimated values of the export entry thresholds, z^*, and the respective values of the average-to-minimum ratios. The larger is the dispersion in the size of exporters in a given product-destination-year market, the larger is the average-to-minimum ratio of export quantities, the smaller is the value of the respective export threshold, which subsequently pushes the estimate of the partial trade elasticities towards a small magnitude.

Hence the small estimated magnitudes of the endogenous selection effects found in this paper are due to the presence of large dispersion in the size distribution of exporters, which empirically identifies trade elasticities given by the hazard rate associated with those distributions.

Figure G1: Endogenous selection effect, γ.



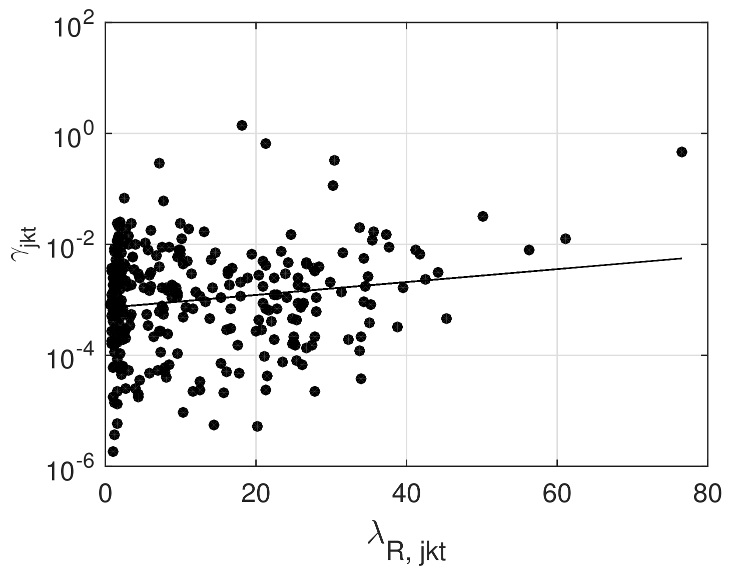
Notes: the figure plots $\gamma(z^*, g(z))$ as a function of z^* for a symmetric DEMG distribution $g(.)$ with parameters equal to $(\mu, \sigma^2, \lambda_L, \lambda_R) = (0, 0.5, 2, 2)$. The y-axis is plotted on a logarithmic scale. The vertical lines correspond to the values of export thresholds determined by the indicated average-to-minimum ratios.

H Counterfactual Analysis

In Section 6 we use the structure of the model to simulate counterfactual trade elasticities under complete information and compare counterfactual estimates to the baseline estimates

[36] To obtain the value of z^*, we solve equation (35) assuming, for the clarity of the argument, the scale parameter $\beta = 1$.

Figure G2: Endogenous selection effect, γ_{jkt}, and the right tail parameter, $\lambda_{R,jkt}$



Notes: the figure plots the relationship between the estimated values of the endogenous selection effect, γ_{jkt}, and the respective right tail parameter, $\lambda_{R,jkt}$. The solid line is the OLS best fit line.

to learn about how uncertainty impacts trade elasticities. Here we describe how we obtain counterfactual values of export selection thresholds and counterfactual values of the distribution of export selection shocks.

H.1 Counterfactual Export Selection Thresholds

Equation (17) establishes a relationship between export selection thresholds in the two information environments. Applying the assumption that υ_{fjkt} are i.i.d. $N[0, V(\upsilon_{fjkt})]$ yields

$$z_{jkt}^{CI^*} = z_{jkt}^{U^*} + \frac{1}{2}\frac{V(\upsilon_{fjkt})}{\epsilon_k}. \tag{76}$$

Notice from equation (32) that $V(\upsilon_{fjkt}) = \epsilon_k^2 V(u_{fjkt})$. Therefore, we recover the variance of the unexpected component of the demand shocks, $V(\upsilon_{fjkt})$, from the variance of the residual, u_{fjkt}, in equation (33).

Notice that our quantification method requires assuming values for the elasticities of substitution across varieties, ϵ_k. We proceed by using the values of the elasticities of substitution across varieties from Soderbery (2015), which refines estimates in Feenstra (1994) and Broda and Weinstein (2006).[37] In principle, these estimates for the elasticity of sub-

[37] Soderbery (2015) estimates the elasticity of substitution values at the HS-10 digit level using the U.S. import data. To use Soderbery (2015) estimates, we aggregate the elasticities to the HS-6 digit level

stitution across varieties are estimated under the assumption of complete information and could require an alternative identification assumption to accommodate incomplete information. However, in order to facilitate the comparison between environments with complete and incomplete information, we choose to hold the elasticities of substitution constant at their complete information values. This allows us to cleanly quantify differences in the trade elasticities across information environments that arise directly from the differences in the economic mechanism of selection.

H.2 Counterfactual Distribution of Export Selection Shocks

Equation (23) establishes a relationship between export selection thresholds in the two information environments. The export selection shock under complete information, z^{CI}_{fjkt} is a mean preserving spread of the selection shock under uncertainty, z^{U}_{fjkt}, where the unexpected component of the demand shock, υ_{fjkt}, is i.i.d. $N[0, V(\upsilon_{fjkt})]$ Therefore, z^{CI}_{fjkt} follows a Double EMG distribution of z^{U}_{fjkt}, with the mean of the Normal component increased by $V(\upsilon_{fjkt})$. As discussed in Section H.1, $V(\upsilon_{fjkt})$ is recovered from the variance of the residual, u_{fjkt}, in equation (33).

I Robustness

A potential concern in our analysis is a measurement error in the quantity data. A classical measurement error would have an ambiguous effect on our baseline and counterfactual results due to an ambiguous effect of the dispersion of export selection shocks on partial trade elasticities.

First, recall that in the baseline calculations, we recover the distribution of the export selection shocks from the distribution of $\hat{z}^{U}_{fjkt} = \hat{\beta}_{jkt}\hat{\zeta}^{q}_{fjkt}$, where $\hat{\zeta}^{q}_{fjkt}$ is the residual from the log-quantity regression

$$\log q_{fjkt} = FE^{q}_{jkt} + \underbrace{\epsilon_k z^{a}_{fjkt} + E(z^{p}_{fjkt}|z^{a}_{fjkt})}_{\zeta^{q}_{fjkt}} \tag{77}$$

A measurement error in the quantity data will increase the dispersion of the error, and therefore the dispersion of the recovered export selection shocks $\hat{z}^{U}_{fjkt}$. As stated in Result 2, the dispersion of a selection shock has an ambiguous effect on the partial trade elasticity.

Further, to calculate counterfactual trade elasticities, we recover the variance of the

equally weighing corresponding HS-10 sub-categories for each HS-6 category.

unexpected component of the demand shock as the variance of the residual in

$$\zeta^r_{fjkt} = \beta_{jkt}\zeta^q_{fjkt} + u_{fjkt}. \tag{78}$$

A measurement error in quantity data which amplifies the variance of ζ^q_{fjkt} will therefore simultaneously attenuate the variance of the unexpected component of the demand shock, which similarly has an ambiguous effect on the counterfactual trade elasticities.

To address concerns with the measurement error, we perform a number of robustness checks. First, all our analysis is conducted after removing severe outliers from the data. Namely in each product-destination-year observation, we drop firm-product-destination-year export sales or export quantity values when they fall below the first or above the 99th percentile of their respective distributions.[38] Removing those helps to reduces the dispersion in the data arising from a severe measurement error among extreme observations.

Second, including an extensive set of fixed effects in a regression of the type presented in equation (29) helps to purge variation in the data most likely to be impacted by a measurement error. In our baseline estimation we include product-destination-year fixed effects that help to account for differences among goods shipped to different destinations in a given year.

Additionally, we perform a robustness check by including an extra set of *firm-product level fixed effects* in the log-quantity regression (29). This helps to alleviate concerns arising from firms potentially shipping different varieties of goods belonging to a given product category. Tables I1 and I2, and Figures I1, I2 and I3 below replicate Tables 3 and 4, and Figures 5, 6 and 7 for this robustness check. The qualitative and quantitative results remain largely unchanged. The average values of the endogenous component of the partial trade elasticity changes from the baseline value of 0.02 to 0.01, and the average partial trade elasticity changes from 3.44 to 3.69. (Comparisons are based on Table 3 and Table I1 below.) The average amplification effect remains unchanged at the value of 1.01. (Comparisons are based on Panel C in Table 4 and Panel C in Table I2 below.) Uncertainty increases trade elasticities in about eighty percent of observation in our baseline calculations and in about seventy four percent of observation in this robustness check, with amplification effects below unity similarly concentrated among industries with low elasticities of substitution. (Comparisons are based on panel B in Figure 7 and panel B in Figure I3 below.)

Finally, we focus analysis on products which are less likely to be subjected to a measurement error. Recall that we conduct our analysis at the product-destination-year level, where a product corresponds to a 6-digit HS code. The original data are available at a finer level of disaggregation, 8-digits, where the last two digits are a country specific addition to a

[38] See Manova and Zhang (2012).

standard 6-digit HS code introduced to allow for greater product differentiation where such is needed. For each product-destination-year observation, we therefore look at the number of 8-digit sub-codes within the given hs-6 digit code. The fewer sub-codes there are, the more likely it is that the given exported products are more comparable to each other, and therefore such data will be less likely subjected to a measurement error in the quantity data. Out of 288 product-destination-year observations in our baseline sample, 174 observations have a single 8-digit code corresponding to the given 6-digit HS code. We reproduce our results using the sample of these 174 observations. Tables I3 and I4, and Figures I4, I5 and I6 below replicate Tables 3 and 4, and Figures 5, 6 and 7 respectively for this robustness check. Qualitative and quantitative results remain robust. The average values of the endogenous component of the partial trade elasticity remains unchanged at the value of 0.02, and the average partial trade elasticity changes from 3.44 to 2.67. (Comparisons are based on Table 3 and Table I3 below.) The average amplification effect changes from the baseline value of 1.01 to 1.02. (Comparisons are based on Panel C in Table 4 and Panel C in Table I4 below.) Uncertainty increases trade elasticities in about eighty percent of observation in our baseline calculations and in about seventy three percent of observation in this robustness check, with amplification effects below unity similarly concentrated among industries with low elasticities of substitution. (Comparisons are based on panel B in Figure 7 and panel B in Figure I3 below.)

Table I1: Trade elasticity estimates under uncertainty (firm-product fixed effects included).

Measure	Mean	Std. Dev.
Endogenous selection, γ_{jkt}	0.01	0.04
Total partial trade elasticity, $\partial \log X_{jkt}/\partial \log \tau_{jkt}$	3.39	3.69

Notes: the summary statistics are reported across 281 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. A product is defined as a 6-digit HS code.

Table I2: Counterfactual trade elasticity estimates under complete information (firm-product fixed effects included).

	Endogenous Selection γ_{jkt}		Partial Trade Elasticity, $\partial \log X_{jkt}/\partial \log \tau_{jkt}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: selection effect of uncertainty				
Selection effect	3.07	22.84	22.08	174.57
Amplification due to selection	0.02	0.06	0.63	0.29
Panel B: dispersion effect of uncertainty				
Dispersion effect	0.0006	0.002	3.35	3.64
Amplification due to dispersion	$1.9 \cdot 10^{117}$	$3.1 \cdot 10^{118}$	1.01	0.04
Panel C: total effect of uncertainty				
Total effect	0.003	0.008	3.35	3.64
Total amplification effect	$1.0 \cdot 10^{107}$	$1.7 \cdot 10^{108}$	1.01	0.04

Notes: all summary statistics are reported across 281 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. The amplification effect is computed as the ratio of the baseline estimate of trade elasticity under uncerinaty realtive to its counterfactual value under complete information for the indicated counterfactual scenario.

Table I3: Trade elasticity estimates under uncertainty (single 8-digit subcode).

Measure	Mean	Std. Dev.
Endogenous selection, γ_{jkt}	0.02	0.13
Total partial trade elasticity, $\partial \log X_{jkt}/\partial \log \tau_{jkt}$	2.67	3.08

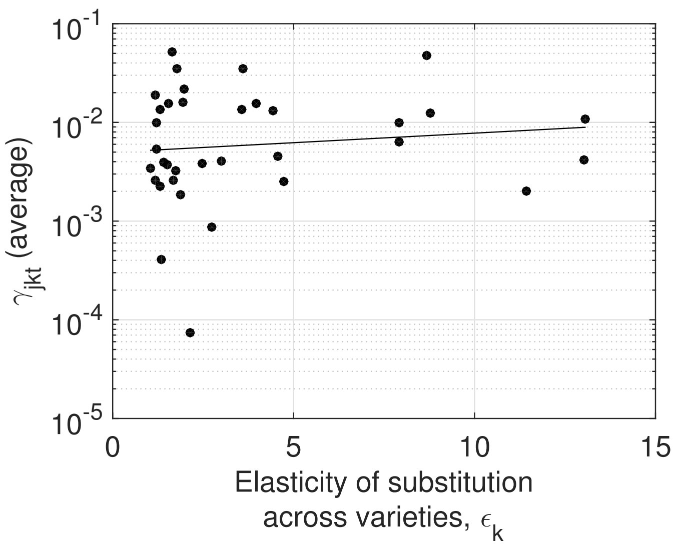
Notes: the summary statistics are reported across 174 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. A product is defined as a 6-digit HS code.

Table I4: Counterfactual trade elasticity estimates under complete information (single 8-digit subcode).

	Endogenous Selection γ_{jkt}		Partial Trade Elasticity, $\partial \log X_{jkt} / \partial \log \tau_{jkt}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: selection effect of uncertainty				
Selection effect	0.85	6.49	4.81	17.00
Amplification due to selection	0.38	0.25	0.96	0.14
Panel B: dispersion effect of uncertainty				
Dispersion effect	0.0006	0.002	2.60	3.00
Amplification due to dispersion	$3.5 \cdot 10^8$	$4.4 \cdot 10^9$	1.02	0.13
Panel C: total effect of uncertainty				
Total	0.001	0.004	2.60	3.00
Total amplification effect	$1.4 \cdot 10^6$	$1.4 \cdot 10^7$	1.02	0.13

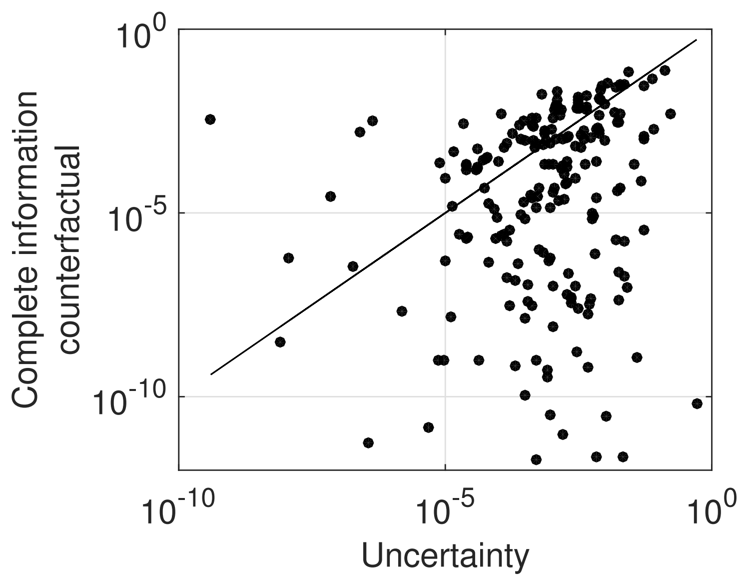
Notes: all summary statistics are reported across 174 product-destination-year observations for which estimates of the Double EMG right tail parameter are greater than unity. The elasticities are not defined otherwise. The amplification effect is computed as the ratio of the baseline estimate of trade elasticity under uncerinaty realtive to its counterfactual value under complete information for the indicated counterfactual scenario.

Figure I1: Heterogeneity in endogenous selection effect, γ_{jkt}, across products (firm-product fixed effects included).



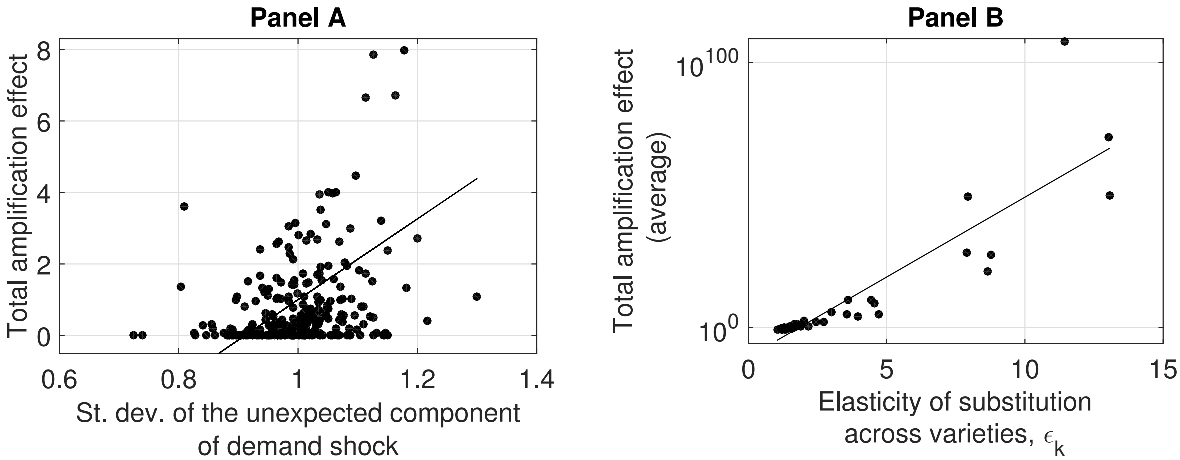
Notes: each dot computes the average across destination-year observations endogenous selection effect, γ_{jkt}, for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

Figure I2: Estimates of the endogenous selection effect, γ_{jkt} (firm-product fixed effects included).



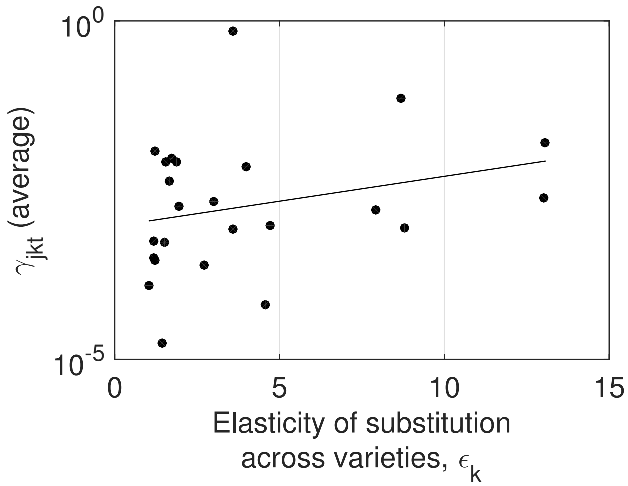
Notes: For the ease of visual presentation this graph omits depicting counterfactual values that are below 10^{-12}. There are 77 of such observations. The solid line is the 45-degree line.

Figure I3: Total amplification effect (firm-product fixed effects included).



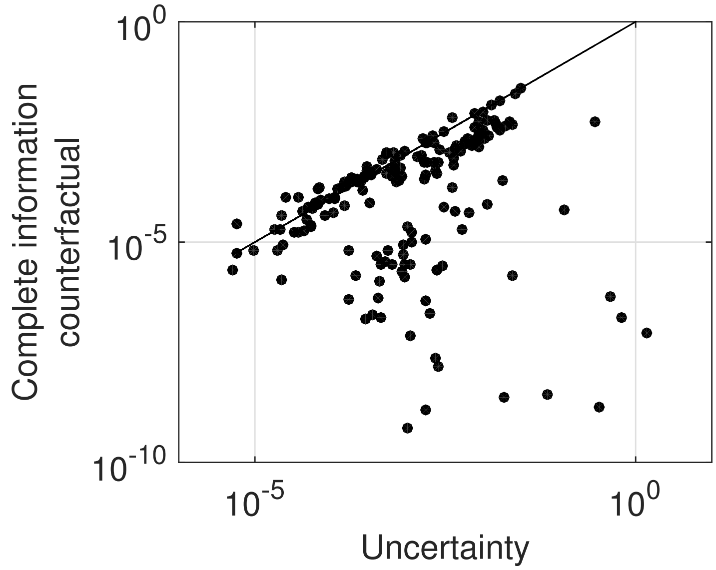
Notes: In Panel A, for the ease of visual presentation this graph omits depicting counterfactual values that are above 8. There are five such observations. The solid line is the OLS best fit line. All values are normalized by the respective product averages, a product is a 6-digit HS code. Each dot corresponds to a product-destination-year observation. In Panel B, each dot computes the average across destination-year observations amplification effect for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code the elasticity of substitution across varieties are obtained from Soderbery (2015).

Figure I4: Heterogeneity in endogenous selection effect, γ_{jkt}, across products (single 8-digit subcode).



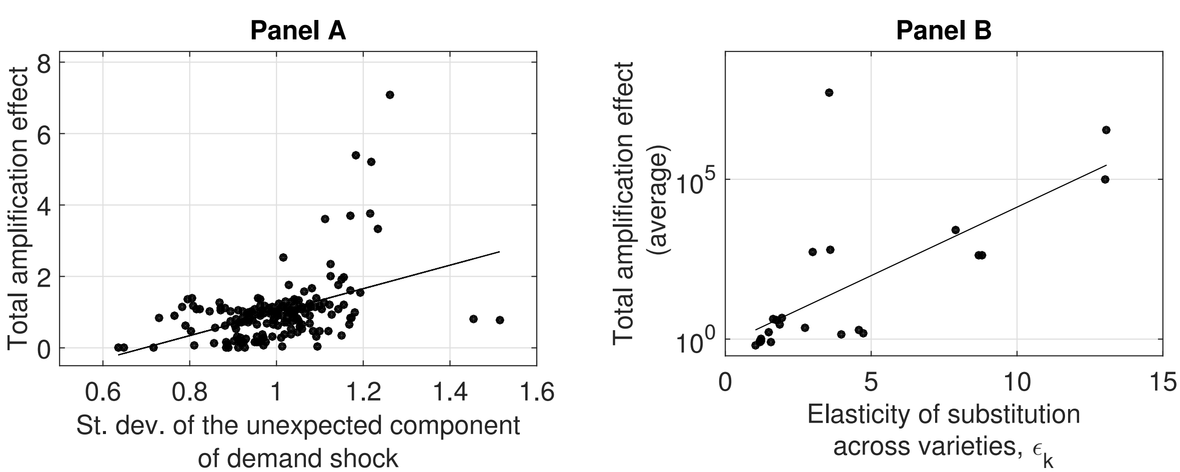
Notes: each dot computes the average across destination-year observations endogenous selection effect, γ_{jkt}, for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code, the elasticity of substitution across varieties is obtained from Soderbery (2015).

Figure I5: Estimates of the endogenous selection effect, γ_{jkt} (single 8-digit subcode).



Notes: The solid line is the 45-degree line.

Figure I6: Total amplification effect (single 8-digit subcode).



Notes: In Panel A, the solid line is the OLS best fit line. All values are normalized by the respective product averages, a product is a 6-digit HS code. Each dot corresponds to a product-destination-year observation. In Panel B, each dot computes the average across destination-year observations amplification effect for a given product defined as a 6-digit HS code. The solid line is the OLS best fit line. For each 6-digit HS code the elasticity of substitution across varieties are obtained from Soderbery (2015).