Lecture 4

(1) Impatience

(2) Marginal Propensity to Consume

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February 20, 2018

Econ 606: Adv. Topics in Macroeconomics
Johns Hopkins University, Spring 2018

House Keeping

- **Due next time (2/27): Problem Set**
  - You are welcome to work together, but submit separate writeups
- Joseph starts in 2 weeks
- Plan for final two lectures:
  - Finish Precautionary Savings (Patience)
  - Study Friedman / Buffer Stock Model
  - Next Week:
    - General Equilibrium
    - Computation or Government Debt?

Last Time (2/13/18)

- **Buffer Stock Savings Model**
  - Borrowing Constraints
  - Prudence
  - Patience (started)

Review: Borrowing Constraints

- Consider "Strict" Permanent Income Hypothesis
- Add *No Borrowing Constraint*: $a_{t+1} \geq 0$
- Optimal Consumption:

$$\begin{aligned} c_t &= \min\{y_t + a_t, \mathbb{E}_t[c_{t+1}]\} \\ &= \min\{y_t + a_t, \mathbb{E}_t[\min\{y_{t+1} + a_{t+1}, \mathbb{E}_{t+1}[c_{t+2}]\}]\} \\ &= \dots \end{aligned}$$

- Future borrowing constraints affect current consumption:

$$\text{if } a_{t+1} > 0, \quad c_t = \mathbb{E}_t[\min\{y_{t+1} + a_{t+1}, \mathbb{E}_{t+1}[c_{t+2}]\}]$$

Last Time (2/13/18)

- **Buffer Stock Savings Model**
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Precautionary Savings

Prudence (Kimball, 1990)

- Consider: $u : \mathbb{R}_+ \to \mathbb{R}$, $u'(c) > 0$, $u''(c) < 0$
- Arrow-Pratt measure of of absolute risk aversion:

$$A(c) \equiv -\frac{u''(c)}{u'(c)}$$

- Decreasing absolute risk aversion (DARA) implies:

$$A'(c) = -\frac{u'''(c)}{u'(c)} + \left(\frac{u''(c)}{u'(c)}\right)^2 < 0$$

- Condition on the third derivative of the utility function:

$$u'''(c) > \frac{u''(c)^2}{u'(c)} > 0$$

- *Prudence*: Convexity of the marginal utility function, e.g. $u'''(c) > 0$

Precautionary Savings

Prudence

- Prudence implies:
  - Uncertainty increases
  - Consumption today decreases
  - Savings for tomorrow increases

Last Time (2/13/18)

- **Buffer Stock Savings Model**
  - Borrowing Constraints
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Patience

- Recall canonical consumption-savings problem:

$$v(a_t, y_t) = \max_{c_t, a_{t+1}} u(c_t) + \beta \mathbb{E}_t \left[v(a_{t+1}, y_{t+1}) \right]$$

$$\text{s.t.} \quad c_t + a_{t+1} \leq y_t + (1+r)a_t$$
$$a_{t+1} \geq \underline{a}$$

- Euler equation:

$$u'(c_t) \geq \beta(1+r) \mathbb{E}_t[u'(c_{t+1})] \qquad \text{w.e. if } a_{t+1} > \underline{a}$$

- $\beta(1+r)$ determines incentives to intertemporally substitute consumption

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  - Patient: $\beta(1+r) \geq 1$

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- $\beta(1+r)$ determines incentives to intertemporally substitute consumption
  - Patient: $\beta(1+r) \geq 1$
  - Impatient: $\beta(1+r) < 1$

Patience

Theoretical Characterization

- Deterministic Income
  - $\beta(1+r) > 1$

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- Stochastic Income
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  - $\beta(1+r) = 1$
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Taking Stock

Deterministic $y_t = \bar{y}$

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- If $\beta(1+r) < 1$, then $c_t = \bar{y}$ and $a_{t+1} = 0$ for all t sufficiently large

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$$v'(x_t) = \beta(1+r)v'(x_{t+1})$$

◂ Cash-In-Hand

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Next Steps

Theoretical Characterization

- Showed: Deterministic Income
  - $\beta(1+r) > 1 \quad \rightarrow \quad \{c_t, a_{t+1}\}$ diverge
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Patience (New)

Stochastic Case, $y_t \sim F(y_t)$**,** $\beta(1+r) > 1$

- As $t \to \infty$, $[\beta(1+r)]^t \to \infty$

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- Since $u''(c) < 0$, c_t must be increasing:

$$\lim_{t \to \infty} c_t = \infty$$

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- Savings must also diverge,
  to finance indefinite postponement of consumption

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- Assume utility function exhibits prudence, $u'''(c) > 0$

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- On average, consumption increases over time:

$$\lim_{t \to \infty} c_t = \infty$$

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- Suppose $y \overset{iid}{\sim} F([\underline{y}, \bar{y}])$ with $\bar{y} > \underline{y}$
- Define $\bar{x}_{t+1} \equiv (1+r)a_{t+1} + \bar{y}$

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- Will show that there exists an endogenous target cash-in-hand x^* s.t.
    - If $x_t > x^*$, then $\bar{x}_{t+1} < x^*$

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    1. Consumption increases with cash-in-hand
    2. Consumption is concave w.r.t. cash-in-hand

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- First: prove a series of Lemmas
  1. Consumption increases with cash-in-hand
  2. Consumption is concave w.r.t. cash-in-hand
  3. If $u(\cdot)$ in DARA class, then $\lim_{x \to \infty} A(c(x)) = 0$

Patience (Stochastic)

Lemma 1

- If $u''(c) < 0$ then $\partial c_t / \partial x_t > 0$.

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Proof:

- Envelope condition: $v'(x_t) = u'(c_t)$

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Proof:

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- Since $u'' < 0$ then $v'' < 0$

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- If $u''(c) < 0$ then $\partial c_t / \partial x_t > 0$.

Proof:

- Envelope condition: $v'(x_t) = u'(c_t)$
- Since $u'' < 0$ then $v'' < 0$
- Taking a derivative of $v'(x)$:

$$v''(x_t) = u''(c_t) \frac{\partial c_t}{\partial x_t}$$

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- By the concavity of both u and v:

$$\frac{\partial c_t}{\partial x_t} = \frac{v''(x_t)}{u''(c_t)} > 0 \qquad \blacksquare$$

Patience (Stochastic)

Lemma 2

- Fix a parameter, $\kappa > 1$. If $u(c)$ is in the DARA class, such that:

$$\frac{u'''(c)}{u''(c)} = \kappa \frac{u''(c)}{u'(c)}$$

and $v(x)$ satisfies:

$$\frac{v'''(x)}{v''(x)} \leq \kappa \frac{v''(x)}{v'(x)}$$

then the consumption policy function $c(x)$ is concave.

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Proof (Sketch):

- If $s(c) \equiv x(c) - c$ and $x(c)$ is convex, then $c(x)$ is concave

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Proof (Sketch):

- If $s(c) \equiv x(c) - c$ and $x(c)$ is convex, then $c(x)$ is concave
- If $x(c)$ is convex, so is savings $s''(c) = x''(c) \geq 0$

Patience (Stochastic)

Lemma 2

- Fix a parameter, $\kappa > 1$. If $u(c)$ is in the DARA class, such that:

$$\frac{u'''(c)}{u''(c)} = \kappa \frac{u''(c)}{u'(c)}$$

and $v(x)$ satisfies:

$$\frac{v'''(x)}{v''(x)} \leq \kappa \frac{v''(x)}{v'(x)}$$

then the consumption policy function $c(x)$ is concave.

Proof (Sketch):

- If $s(c) \equiv x(c) - c$ and $x(c)$ is convex, then $c(x)$ is concave
- If $x(c)$ is convex, so is savings $s''(c) = x''(c) \geq 0$
- Show $s(c)$ is convex using the Euler equation

Patience (Stochastic)

Lemma 3

- If $u(c)$ is in the DARA class, then:

$$\lim_{x\to\infty} \frac{u'\big(c(x_{t+1})\big)}{u'\big(c(\bar{x}_{t+1})\big)} = 1$$

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$$u'(c(x')) \approx u'(c(\bar{x}')) + u''(c(\bar{x}'))c'(\bar{x}')(x' - \bar{x}')$$

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(Carroll and Kimball, 1996)

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Patience (Stochastic)

Stochastic Case, $y \sim F(y)$, $\beta(1+r) < 1$

- Suppose $y \overset{iid}{\sim} F([\underline{y}, \bar{y}])$ with $\bar{y} > \underline{y}$
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- Convergence, $lim_{t\to\infty} x_t < \infty$ ■

Next Steps

Theoretical Characterization

- Showed: Deterministic Income
  - $\beta(1+r) > 1 \quad \rightarrow \quad \{c_t, a_{t+1}\}$ diverge
  - $\beta(1+r) = 1 \quad \rightarrow \quad \{c_t, a_{t+1}\}$ converge
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Further Characterization: $\beta(1+r) < 1$

1. Showed: $\mathbb{E}_t[x_{t+1}(x)] < x$ for large x

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$$\lim_{x\to 0} \frac{x_{t+1}(x)}{x} > \lim_{x\to 0} \frac{\underline{y}}{x} > 1$$

- Therefore: $x_{t+1}(x) > x$ for x sufficiently small

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Further Characterization

1. Showed: $\mathbb{E}_t[x_{t+1}(x)] < x$ for large x

2. Showed: $x_{t+1}(x) > x$ for small x

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    - By continuity!

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- Define x^* by $\mathbb{E}_t[x_{t+1}(x^*)] = x^*$
- Then expected budget constraint:

$$\mathbb{E}_t[x_{t+1}(x^*)] = (1+r)(x^* - c(x^*)) + 1 \qquad \text{s.t. } \mathbb{E}_t[y_{t+1}] = 1$$

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$$(1+r)c(x^*) = 1 + rx^*$$

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- Then expected budget constraint:

$$\mathbb{E}_t[x_{t+1}(x^*)] = (1+r)(x^* - c(x^*)) + 1 \qquad \text{s.t. } \mathbb{E}_t[y_{t+1}] = 1$$

$$x^* = (1+r)(x^* - c(x^*)) + 1$$

$$(1+r)c(x^*) = 1 + rx^*$$

- To show **at board**:

$$\mathbb{E}_t\left[\frac{\partial}{\partial x}\frac{x_{t+1}(x)}{x}\Big| x = x^*\right] < 0$$

Buffer Stock Target

Further Characterization

1. Showed: $\mathbb{E}_t[x_{t+1}(x)] < x$ for large x

2. Showed: $x_{t+1}(x) > x$ for small x

3. Showed: By continuity, $\exists\ x^*$ such that $\mathbb{E}_t[x_{t+1}(x^*)] = x^*$

4. Showed: x^* is stable,

$$\text{for } \epsilon > 0, \quad \text{if } x \in (x^*, x^* + \epsilon) \quad \text{then } x_{t+1}(x) < x$$

Next Steps

- Studied dynamics in $c_t(x), a_{t+1}(x)$ and $x_{t+1}(x)$

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    - Out of transitory shocks
    - Out of permanent shocks
    - Out of wealth

Next Steps

- Studied dynamics in $c_t(x), a_{t+1}(x)$ and $x_{t+1}(x)$
- Assumed simple iid income process
- What about: Marginal Propensity to Consume
  - Out of transitory shocks
  - Out of permanent shocks
  - Out of wealth
- Must develop a quantitatively relevant model

Income Process

Permanent and Transitory Shocks

- Income:

$$Y_t = \Gamma_t z_t$$

- Permanent:

$$\Gamma_t = \Gamma_{t-1} g_t$$

$$\log(g_{t+1}) = (1-\rho_g)\log(1+\gamma) + \rho_g \log(g_t) + \eta_{t+1}$$

$$\eta \sim \mathcal{N}(-\sigma_\eta^2/2, \sigma_\eta^2)$$

- Transitory:

$$\log(z_{t+1}) = (1-\rho_z)\mu + \rho_z \log(z_t) + \varepsilon_{t+1}$$

$$\varepsilon \sim \mathcal{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$$

Income Process

Permanent and Transitory Shocks

- Income:

$$Y_t = \Gamma_t z_t$$

- Permanent: assume $\rho_g = 0$

$$\Gamma_t = \Gamma_{t-1} g_t$$

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Interpreting Permanent Shocks

- Permanent shocks capture a stochastic growth rate

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Income Process

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Income Process

Interpreting Permanent Shocks

- Permanent shocks capture a stochastic growth rate
- γ is a deterministic trend growth rate
- η are iid shocks to the trend
- Growth rate shocks are: $g_t = (1+\gamma)e^{\eta_t} \approx 1 + (\gamma + \eta_t)$
- By iteration, can rewrite as:

$$\Gamma_t = \Gamma_{t-1} g_t \quad \Longrightarrow \quad \Gamma_t = \prod_{j=0}^{t} e^{g_t} = (1+\gamma)^t \prod_{j=0}^{t} e^{\eta_j}$$

Consumption-Savings Problem

$$V_t(A_t, \Gamma_t, z_t, g_t) = \max_{C_t, A_{t+1}} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(A_{t+1}, \Gamma_{t+1}, z_{t+1}, g_{t+1}) \right]$$

$$\begin{aligned} \text{s.t.} \quad C_t + A_{t+1} &\leq Y_t + (1+r)A_t \\ Y_t &= \Gamma_t z_t \\ \Gamma_{t+1} &= \Gamma_t g_{t+1} \end{aligned}$$

Consumption-Savings Problem

- Since ε and η are iid, using cash-in-hand gives:

$$V_t(X_t, \Gamma_t) = \max_{C_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}(X_{t+1}, \Gamma_{t+1}) \right]$$

$$\text{s.t.} \quad X_{t+1} = (1+r)(X_t - C_t) + Y_{t+1}$$

$$\Gamma_{t+1} = \Gamma_t g_{t+1}$$

Consumption-Savings Problem

- Homogeneous utility and linear constraints imply separability:

$$\Gamma_t^{1-\sigma} V_t(X_t/\Gamma_t, 1) = \max_{C_t} \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[\Gamma_{t+1}^{1-\sigma} V_{t+1}(X_{t+1}/\Gamma_{t+1}, 1) \right]$$

$$\begin{aligned} \text{s.t.} \quad X_{t+1} &= (1+r)(X_t - C_t) + Y_{t+1} \\ Y_t &= \Gamma_t z_t \\ \Gamma_{t+1} &= \Gamma_t g_{t+1} \end{aligned}$$

Consumption-Savings Problem

- Define: $v(x) = V_t(X_t/\Gamma_t, 1)$ and $x = X_t/\Gamma_t$:

$$\Gamma_t^{1-\sigma} v(x) = \max_c \Gamma_t^{1-\sigma} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[\Gamma_{t+1}^{1-\sigma} v(x') \right]$$

$$\text{s.t.} \quad \Gamma_{t+1} x' = (1+r)\Gamma_t (x-c) + \Gamma_{t+1} z'$$

Consumption-Savings Problem

- <u>Substitute</u>: $\Gamma_{t+1}/\Gamma_t = (1+\gamma)e^{\eta_{t+1}}$ and $z = e^{\varepsilon}$:

$$v(x) = \max_c \frac{c^{1-\sigma}}{1-\sigma} + \beta(1+\gamma)^{1-\sigma}\mathbb{E}_t\left[e^{(1-\sigma)\eta'}v(x')\right]$$

$$\text{s.t.} \quad x' = e^{-\eta'}\frac{1+r}{1+\gamma}(x-c) + e^{\varepsilon'}$$

Consumption-Savings Problem

$$v(x) = \max_{c} \frac{c^{1-\sigma}}{1-\sigma} + \beta(1+\gamma)^{1-\sigma}\mathbb{E}_t\left[e^{(1-\sigma)\eta'}v(x')\right]$$

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- Euler equation:

$$c^{-\sigma} = \frac{\beta(1+r)}{(1+\gamma)^{\sigma}}\mathbb{E}\left[\left(e^{\eta'}c'\right)^{-\sigma}\right]$$

- Impatience condition (stationarity of normalized economy):

$$\beta(1+r)\mathbb{E}\left[\frac{1}{[(1+\gamma)e^{\eta'}]^{\sigma}}\right] < 1$$

Next Steps

Deterministic Income Characterization

- consumption policy function: $c(x)$
- MPC out of transitory income: $c'(x)$
- MPC out of permanent income: $\frac{d}{d\Gamma}c(x)$

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Stochastic Income Characterization

- MPC out of transitory income: $c'(x)$
- MPC out of permanent income: $\frac{d}{d\Gamma}c(x)$
- MPC's relationship with savings

Next Steps

- Start with **deterministic income** case

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- Analytically tractable MPC

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Next Steps

- Start with **deterministic income** case
- Analytically tractable MPC
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- Stochastic income case relies on computation

Deterministic Income

- Assume $z_t = 1$ and $\eta_t = 0$ for all t

$$Y_t = \Gamma_t = (1+\gamma)^t$$

Deterministic Income

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$$C_t^{-\sigma} = \beta(1+r)C_{t+1}^{-\sigma} \quad \Longrightarrow \quad C_{t+j} = \left((\beta(1+r))^{\frac{1}{\sigma}}\right)^j C_t$$

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$$\sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j C_{t+j} = (1+r)A_t + \sum_{j=0}^{\infty} \left(\frac{1+\gamma}{1+r}\right)^j \Gamma_t$$

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- Substitute Euler equation:

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- Rewrite Intertemporal Budget Constraint:

$$C_t = \left(1 - \frac{\left(\beta(1+r)\right)^{\frac{1}{\sigma}}}{1+r}\right)\underbrace{\left((1+r)A_t + \frac{1+r}{r-\gamma}\Gamma_t\right)}_{\equiv W_t + H_t}$$

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$$c_t=\left(1-\frac{\left(\beta(1+r)\right)^{\frac{1}{\sigma}}}{1+r}\right)\underbrace{\left(w_t+\frac{1+r}{r-\gamma}\right)}_{\equiv w_t+\bar{h}}$$

Deterministic Income

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- Let $\beta = 0.96$, $r = 0$, $\sigma = 2.0$,

Deterministic Income

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- Small consumption response

Deterministic Income

- Define: **MPC out of permanent income**

$$\frac{\partial C_t}{\partial \Gamma_t} = \left(1 - \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right) \frac{\partial (W_t + H_t)}{\partial \Gamma_t}$$

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- If impatience condition violated, $\beta(1+r) = (1+\gamma)^\sigma$

$$\frac{\partial C_t}{\partial \Gamma_t} = \left(1 - \frac{1+\gamma}{1+r}\right)\frac{1+r}{r-\gamma} = 1$$

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$$\frac{\partial C_t}{\partial \Gamma_t} = \left(1 - \frac{1+\gamma}{1+r}\right)\frac{1+r}{r-\gamma} = 1$$

- Otherwise:

$$\frac{\partial C_t}{\partial \Gamma_t} = \left(1 - \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)\frac{1+r}{r-\gamma} > \left(1 - \frac{1+\gamma}{1+r}\right)\frac{1+r}{r-\gamma} = 1$$

Deterministic Income

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- $\partial C_t / \partial \Gamma_t$ is increasing as $\gamma \to r$

Deterministic Income

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- $\partial C_t / \partial \Gamma_t$ is increasing as $\gamma \to r$
- $\gamma \to r$ implies income grows at a faster rate

Deterministic Income

$$\frac{\partial C_t}{\partial \Gamma_t} = \left(1 - \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)\frac{1+r}{r-\gamma} > \left(1 - \frac{1+\gamma}{1+r}\right)\frac{1+r}{r-\gamma} = 1$$

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- Concave utility implies a desire to smooth

Deterministic Income

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- $\partial C_t / \partial \Gamma_t$ is increasing as $\gamma \to r$
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Deterministic Income

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- $\partial C_t / \partial \Gamma_t$ is increasing as $\gamma \to r$
- $\gamma \to r$ implies income grows at a faster rate
- Concave utility implies a desire to smooth
- Impatience condition implies intertemporal substitution from future
- Therefore: Consumer increases C_t more than one-for-one today

Next Steps

- **Add Stochastic Income**
  - Transitory shocks:

$$z_t = e^{\varepsilon_t}$$

$$\varepsilon \overset{iid}{\sim} \mathcal{N}\left(-\frac{\sigma_\varepsilon^2}{2}, \sigma_\varepsilon^2\right)$$

  - Permanent shocks:

$$g_t = (1+\gamma)e^{\eta_t}$$

$$\eta \overset{iid}{\sim} \mathcal{N}\left(-\frac{\sigma_\eta^2}{2}, \sigma_\eta^2\right)$$

Stochastic Income

Marginal Propensity to Consume out of Transitory Income

$$\frac{\partial c_t(x)}{\partial z_t} = c_t'(x)$$

- If $\sigma_\varepsilon > 0$, must solve numerically
- Let $r = 0$ so that β indexes patience

Table: Baseline Parameters

γ	β	r	σ	σ_η^2	σ_ε^2
0.02	0.96	0	2.0	0.1	0.1

Stochastic Income

Comparative Statics

(1) MPC out of Transitory Income, (2) Average Wealth, (3) Target Wealth:

TABLE: Baseline and Parameter Deviations

	(1)	(2)	(3)
Baseline	0.33	0.35	0.32
$\beta = 1$			

Stochastic Income

Comparative Statics

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TABLE: Baseline and Parameter Deviations

	(1)	(2)	(3)
Baseline	0.33	0.35	0.32
$\beta = 1$	0.15	0.66	0.61
$\beta = 0.90$			

Stochastic Income

Comparative Statics

(1) MPC out of Transitory Income, (2) Average Wealth, (3) Target Wealth:

TABLE: Baseline and Parameter Deviations

	(1)	(2)	(3)
Baseline	0.33	0.35	0.32
$\beta = 1$	0.15	0.66	0.61
$\beta = 0.90$	0.46	0.25	0.23
$r = 0.02$			

Stochastic Income

Comparative Statics

(1) MPC out of Transitory Income, (2) Average Wealth, (3) Target Wealth:

TABLE: Baseline and Parameter Deviations

	(1)	(2)	(3)
Baseline	0.33	0.35	0.32
$\beta = 1$	0.15	0.66	0.61
$\beta = 0.90$	0.46	0.25	0.23
$r = 0.02$	0.26	0.45	0.42
$r = 0.04$			

Stochastic Income

Comparative Statics

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r	$=$	0.02	0.26	0.45	0.42
r	$=$	0.04	0.17	0.65	0.61
σ	$=$	1			

Stochastic Income

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Stochastic Income

Marginal Propensity to Consume out of Permanent Income

- Change in C_{t+1} w.r.t. the change in Γ_{t+1} caused by η_{t+1}

$$(1+\gamma)\Gamma_t \mu(a_{t+1}) \equiv \mathbb{E}_t \left[\frac{\partial}{\partial e^{\eta_{t+1}}} C_{t+1} \right]$$

$$(1+\gamma)\Gamma_t \mu(a_{t+1}) \equiv \mathbb{E}_t \left[\frac{\partial}{\partial e^{\eta_{t+1}}} \Gamma_{t+1} c(x_{t+1}) \right]$$

$$= (1+\gamma)\Gamma_t \mathbb{E}_t \left[c(x_{t+1}) + e^{\eta_{t+1}} c'(x_{t+1}) \frac{\partial x_{t+1}}{\partial e^{\eta_{t+1}}} \right]$$

$$\mu(a_{t+1}) = \mathbb{E}_t \left[c(x_{t+1}) - c'(x_{t+1}) e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} a_{t+1} \right]$$

Stochastic Income

Take $\sigma_\varepsilon \to 0$ and $\sigma_\eta \to 0$:

$$\mu(a_{t+1}) = c(x_{t+1}) - c'(x_{t+1})\frac{1+r}{1+\gamma}a_{t+1}$$

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$$\mu(a_{t+1}) = c(x_{t+1}) - c'(x_{t+1})\frac{1+r}{1+\gamma}a_{t+1}$$

$$= \left(1 - \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)\left(w_{t+1} + \bar{h}\right) - \left(1 - \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)\frac{1+r}{1+\gamma}a_{t+1}$$

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$$= \left(1 - \frac{(\beta(1+r))^{\frac{1}{\sigma}}}{1+r}\right)\frac{1+r}{r-\gamma} = \frac{\partial C_t}{\partial \Gamma_t} \quad \text{for Deterministic Case}$$

Stochastic Income

Characterizing $\mu(a')$

- Proposition 1: If $a^* \equiv x^* - c(x^*) > 0$, then $\mu(a^*) < 1$.
    - Proof: See lecture notes, similar to "target cash-in-hand" x^*.
    - *Intuition:* Near wealth target a^*, agent weakly responds (< 1) to shocks

Stochastic Income

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  - Proof: See lecture notes, similar to "target cash-in-hand" x^*.
  - *Intuition:* Near wealth target a^*, agent weakly responds (< 1) to shocks
- Proposition 2: The MPC out of permanent shocks is increasing in savings:

$$\frac{\partial}{\partial a_{t+1}} \mu(a_{t+1}) > 0$$

  - Proof: Simply take the derivative.

Stochastic Income

Characterizing $\mu(a')$

Proof: Take a straightforward derivative:

$$\frac{\partial \mu(a_{t+1})}{\partial a_{t+1}} = \frac{\partial}{\partial a_{t+1}} \mathbb{E}_t \left[c(x_{t+1}) + c'(x_{t+1}) e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} a_{t+1} \right]$$

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$$= \mathbb{E}_t \Big[c'(x_{t+1}) e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} - c''(x_{t+1}) \left(e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} \right)^2 a_{t+1}$$

$$- c'(x_{t+1}) e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} \Big]$$

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$$- c'(x_{t+1}) e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} \Big]$$

$$= \mathbb{E}_t \left[-c''(x_{t+1}) \left(e^{-\eta_{t+1}} \frac{1+r}{1+\gamma} \right)^2 a_{t+1} \right] > 0$$

Stochastic Income

Numerical Characterization of $\mu(a')$

TABLE: Comparative Statics with respect to β

β	Mean a_{t+1}	Mean $c'(x_t)$	Mean $\mu(a_{t+1})$
1.00	1.14	0.08	0.91
0.98			

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- Consume more and save less today

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1.00	1.14	0.08	0.91
0.98	0.77	0.18	

- Consume more and save less today
- Consumes more and is more responsive to transitory shocks

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- Consume more and save less today
- Consumes more and is more responsive to transitory shocks
- But less responsive to permanent shocks

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β	Mean a_{t+1}	Mean $c'(x_t)$	Mean $\mu(a_{t+1})$
1.00	1.14	0.08	0.91
0.98	0.77	0.18	0.87
0.96	0.64	0.24	0.85
0.94	0.59	0.28	0.84

- Consume more and save less today
- Consumes more and is more responsive to transitory shocks
- But less responsive to permanent shocks

Testable Implications

Ludvigson and Michaelides (2001) -overview-

- Evaluate in Buffer Stock Model:
  Excess Sensitivity and *Excess Smoothness*

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- Excess Smoothness:

	Data (quarterly)	Buffer Stock Model	PIH Model
$\sigma_{\Delta_c}/\sigma_{\Delta_y}$	0.68	1.09	1.26

  - Accounts for $(1.26 - 1.09)/(1.26 - 0.68) \approx 1/3$ of the gap
  - Note $\mu(a_{t+1}) < 1$ is the *immediate response* to shocks

Testable Implications

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- Excess Sensitivity:

$$\Delta \log(C_t) = \beta_0 + \beta_1 \Delta \log(Y_{t-1}) + \epsilon_t$$

	Data (quarterly)	Buffer Stock Model	PIH Model
β_1	0.16	0.06	0.00

  - Accounts for approximately 1/2 of the gap

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- Excess Smoothness:
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- Excess Sensitivity:
  - Accounts for approximately 1/2 of the gap
- Any way to account for a larger portion of gap?

Testable Implications

- Incomplete Information:

Testable Implications

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    - Include aggregate shocks

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  - Weakly respond to transitory shocks

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