Lecture 2

(1) Permanent Income Hypothesis

(2) Precautionary Savings

Erick Sager

February 6, 2018

Econ 606: Adv. Topics in Macroeconomics
Johns Hopkins University, Spring 2018

Housekeeping (2/6/2018)

- Updated syllabus
  - Office hours set
- Problem set due 2/27
- Referee report selection due 2/14
- Presentation paper selection due 2/14
  - Email both me and Joseph

Last Time (1/30/18)

- **Considered data**
  - Wealth, Consumption, Income
  - Cross-Sectional, Times-Series, Life-Cycle
  - Inequality

Last Time (1/30/18)

- **Considered data**
  - Wealth, Consumption, Income
  - Cross-Sectional, Times-Series, Life-Cycle
  - Inequality
- **Talked about aggregation**
  - Started with Gorman Form

$$c^i(p, w_i) = a^i(p) + b(p)w_i \qquad \Longrightarrow \qquad C(p, w) = C(p, W)$$

Last Time (1/30/18)

- **Negishi Planner's Problem:**

$$V(\{\mu_i\}_{i=1}^N, k_0) = \max_{\{\{c_t^i\}_{t=1}^N, k_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \sum_{i=1}^N \mu_i u(c_t^i)$$

$$\text{s.t.} \quad \left(\textstyle\sum_{i=1}^N \mu_i c_t^i\right) + k_{t+1} = f(k_t) + (1-\delta)k_t$$

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- **Constantinides Decomposition:**

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- **Constantinides Decomposition:**

1. Individual Allocation

$$U(C_t) = \max_{\{c_t^i\}_{i=1}^N} \left\{ \sum_{i=1}^{N} \mu_i u(c_t^i) \quad \text{s.t.} \quad \sum_{i=1}^{N} \pi_i c_t^i \leq C_t \right\}$$

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2. Aggregate Allocation

$$\max_{\{C_t, k_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{s.t.} \quad C_t + k_{t+1} = f(k_t) + (1-\delta)k_t$$

$$k_0 \text{ given}$$

Last Time (1/30/18)

Maliar and Maliar (2003)

- Complete markets
- Idiosyncratic labor productivity shocks

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Planner's Problem

$$U(C_t, 1 - L_t) = \max_{\{c_t^i, h_t^i\}_{i \in \mathcal{I}}} \int_{\mathcal{I}} \alpha^i u(c_t^i, h_t^i) d\mu^i$$

$$\begin{aligned} \text{s.t.} \qquad \textstyle\int_{\mathcal{I}} c_t^i d\mu^i &\leq C_t \\ \textstyle\int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i &\leq L_t \end{aligned}$$

Last Time (1/30/18)

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$$\int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i \leq L_t$$

First Order Conditions:

$$c_t^i = \frac{\left(\alpha^i\right)^{\frac{1}{\sigma}}}{\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i} C_t \quad , \quad 1 - h_t^i = \frac{\left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{-\frac{1}{\gamma}}}{\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i} (1 - L_t)$$

Last Time (1/30/18)

Maliar and Maliar (2003)

$$\int_{\mathcal{I}} \alpha^i \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} + \psi \frac{(1-h_t^i)^{1-\gamma}}{1-\gamma} \right] d\mu^i = \frac{(C_t)^{1-\sigma}}{1-\sigma} + \Psi_t \frac{(1-L_t)^{1-\gamma}}{1-\gamma}$$

Last Time (1/30/18)

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- "Labor Wedge" endogenously arises:

$$\Psi_t \equiv \frac{\psi \left(\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i \right)^{\gamma}}{\left(\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i \right)^{\sigma}}$$

Last Time (1/30/18)

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- Labor Wedge:

$$\psi(1-h_t)^{-\gamma} = (1-\tau_t) w_t c_t^{-\sigma} \quad \Longrightarrow \quad \tau_t = 1 - \frac{1}{(1-\alpha)Y_t/L_t} \cdot \frac{\psi(1-h_t)^{-\gamma}}{c_t^{-\sigma}}$$

Today (2/6/18)

Permanent Income Hypothesis (PIH)

- Restrict asset space
- Derive results for consumption, savings, wealth
- Empirical evaluation of theory
    - Excess Sensitivity Puzzle
    - Excess Smoothness Puzzle
    - Reconciliation of Puzzles (Campbell and Deaton (1989))

Next: Precautionary Savings

Permanent Income Hypothesis

Next Time

Asset Markets

- Are complete markets a good representation of the data?
- Consider two extremes:
  - Complete Markets
  - Autarky
- Which does the data better support?
- Consider some intermediate case?

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Asset Markets

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- Consider two extremes:
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- Which does the data better support?
- Consider some intermediate case? ($\leftarrow$ this one)

Asset Markets

Preliminaries

- s_t : state of the economy at t
- S_t : set of possible states s.t. $s_t \in S_t$
- $s^t = \{x_0, \ldots, s_t\} \in S^t$: history of states up to t
- $\pi(s^t)$: probability of a particular history
- $y_t^i(s^t)$: agent i's income following history s^t at time t

Asset Markets

Autarky

- No possibility for intertemporal substitution of resources
  - No access to asset markets
  - No access to storage technology
- Then: $c_t^i(s^t) = y_t^i(s^t)$
  - No insurance against income shocks
  - No *risk sharing*

Asset Markets

Complete Markets

- Access to Arrow securities $a^i_{t+1}(s_{t+1}, s^t)$ with price $q_{t+1}(s_{t+1}, s^t)$
- Sequential budget constraint (as before):

$$c^i_t(s^t) + \sum_{s_{t+1} \in S_{t+1}} q_t(s_{t+1}, s^t) a^i_{t+1}(s_{t+1}, s^t) \leq y^i_t(s^t) + a^i_t(s^t)$$

- Impose a no Ponzi condition:

$$\lim_{t \to \infty} \sum_{s_{t+1} \in S_{t+1}} q_t(s_{t+1}, s^t) a^i_{t+1}(s_{t+1}, s^t) \geq 0$$

- Constant relative MUCs & only aggregate risk:

$$\frac{u'(c^i_t(s^t))}{u'(c^j_t(s^t))} = \frac{\alpha^j}{\alpha^i} \quad \Longrightarrow \quad c^i_t(s^t) = \frac{(\alpha^i)^{\frac{1}{\sigma}}}{\sum_{j \in \mathcal{I}} (\alpha^j)^{\frac{1}{\sigma}}} \, C_t(s^t)$$

Asset Markets

Empirical Evaluation

- Individual consumption growth vs aggregate consumption growth:

$$\log\left(c_t^i/c_{t-1}^i\right) = \beta \log\left(C_t/C_{t-1}\right)$$

- Include income growth (c.f. Mace (1991))

$$\log\left(c_t^i/c_{t-1}^i\right) = \beta_1 \log\left(C_t/C_{t-1}\right) + \beta_2 \log\left(y_t^i/y_{t-1}^i\right) + \varepsilon_t^i$$

Asset Markets

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- $\beta_1 = 1$ and $\beta_2 = 0$
- $\beta_1 = 0$ and $\beta_2 = 1$

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- $\beta_1 = 1$ and $\beta_2 = 0$
- $\beta_1 = 0$ and $\beta_2 = 1$
- Data shows something in between

Asset Markets

Restrictions

- Want alternative that supports *partial risk sharing*
- *Incomplete Markets:*
    - cannot write contracts contingent on all histories s^t

Asset Markets

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- *Incomplete Markets:*
  - cannot write contracts contingent on all histories s^t
- *Exogenously Incomplete Markets:*
  - cannot write contracts on **any** future contingencies

Asset Markets

Restrictions

- Want alternative that supports *partial risk sharing*
- *Incomplete Markets:*
  - cannot write contracts contingent on all histories s^t
- *Exogenously Incomplete Markets:*
  - cannot write contracts on **any** future contingencies
- Then:

$$c_t^i(s^t) + q_t(s^t)a_{t+1}^i(s^t) \leq y_t^i(s^t) + a_t^i(s^t)$$

- Going forward: suppress (s^t) notation

$$c_t^i + q_t a_{t+1}^i \leq y_t^i + a_t^i$$

Asset Markets

Next Steps

- Write down model of consumer behavior
- Suppose access to exogenously incomplete markets
- Make additional assumptions: Permanent Income Hypothesis
- What are the implications for consumption, savings, income, wealth?

Incomplete Markets

Canonical Consumption Savings Problem:

$$v^i(a_0^i, y_0^i) = \max_{\{c_t^i, a_{t+1}^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i)$$

$$\begin{aligned} \text{s.t.} \quad c_t^i + \frac{1}{1+r_t} a_{t+1}^i &\leq y_t^i + a_t^i \\ a_{t+1}^i &\geq \underline{a}_{t+1}^i \\ c_t^i &\geq 0 \end{aligned}$$

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Recursive Form:

$$v(a_t, y_t) = \max_{c_t, a_{t+1}} u(c_t) + \beta \mathbb{E}_t \left[v(a_{t+1}, y_{t+1}) \right]$$

$$\begin{aligned} \text{s.t.} \quad c_t + \frac{1}{1+r_t} a_{t+1} &\leq y_t + a_t \\ a_{t+1} &\geq \underline{a}_{t+1} \\ c_t &\geq 0 \end{aligned}$$

Permanent Income Hypothesis

Restrictions on Canonical Problem - Review -

- Quadratic utility specification:

$$u(c) = -\frac{\alpha}{2}(c_t - \bar{c})^2$$

  - $\bar{c}$ is a "bliss point" of maximium utility
  - α is a utility parameter

- One-period returns are certain and pinned down by the discount rate:

$$\beta(1+r) = 1$$

- Borrowing constraints are replaced by the No Ponzi Condition for all $t \geq 0$:

$$\mathbb{E}_t \left[\lim_{j\to\infty} \left(\frac{1}{1+r} \right)^j a_{t+j} \right] \geq 0$$

PIH Characterization

- Quadratic utility:

$$u'(c) = \frac{d}{dc}\left[-\frac{\alpha}{2}(c_t - \bar{c})^2\right] = -\alpha(c_t - \bar{c})$$

- Euler equation:

$$u'(c) \geq \beta(1+r)\mathbb{E}_t\left[u'(c_{t+1})\right]$$

- Quadratic utility, $\beta(1+r) = 1$ and the No Ponzi condition, imply:

$$\alpha\bar{c} - \alpha c_t = \mathbb{E}_t[\alpha\bar{c} - \alpha c_{t+1}] \qquad \Rightarrow \qquad c_t = \mathbb{E}_t[c_{t+1}]$$

- Consumption is a *random walk* or *martingale*

- By the Law of Iterated Expectations:

$$c_t = \mathbb{E}_t[c_{t+j}] \quad \forall\ j \geq 0$$

PIH Characterization

- Define human wealth at time t as: $h_t \equiv \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \mathbb{E}_t[y_{t+j}]$
- Define financial wealth as a_t and total wealth as $a_t + h_t$
- *Permanent income* is the annuity value of total wealth: $\frac{r}{1+r}(a_t + h_t)$.
- Iterate forward on the budget constraint and divide by $(1+r)^j$ for each iteration j:

$$c_t = y_t + a_t - \left(\frac{1}{1+r}\right) a_{t+1}$$

$$\left(\frac{1}{1+r}\right) c_{t+1} = \left(\frac{1}{1+r}\right) y_{t+1} + \left(\frac{1}{1+r}\right) a_{t+1} - \left(\frac{1}{1+r}\right)^2 a_{t+2}$$

$$\left(\frac{1}{1+r}\right)^j c_t = \left(\frac{1}{1+r}\right)^j y_{t+j} + \left(\frac{1}{1+r}\right)^j a_{t+j} - \left(\frac{1}{1+r}\right)^{j+1} a_{t+1+j}$$

- Summing gives:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j c_{t+j} = a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j y_{t+j} - \lim_{j\to\infty}\left(\frac{1}{1+r}\right)^j a_{t+j}$$

- Taking expectations and applying the No Ponzi Condition yields:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t[c_{t+j}] = a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t[y_{t+j}]$$

- But because consumption is a random walk, $\mathbb{E}_t[c_{t+j}] = c_t$ for all $j \geq 0$:

$$c_t = \frac{r}{1+r}\left(a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t[y_{t+j}]\right)$$

$$\equiv \frac{r}{1+r}\Big(a_t + h_t\Big)$$

- Therefore, consumption equals permanent income

PIH Characterization

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- Certainty Equivalence

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- Certainty Equivalence
  - Consumption does not depend on income variance
  - Any stochastic process for income with same expected value (e.g., $= h_t$) yields same consumption
  - The result of quadratic preferences

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- Next: Consumption and Wealth Dynamics

PIH Characterization

$$
\begin{aligned}
\Delta c_t &= c_t - c_{t-1} \\
&= c_t - \mathbb{E}_{t-1} c_t \\
&= \frac{r}{1+r}\Big((a_t + h_t) - \mathbb{E}_{t-1}\left[(a_t + h_t)\right]\Big) \\
&= \frac{r}{1+r}\left(a_t - \mathbb{E}_{t-1}[a_t] + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \Big(\mathbb{E}_t[y_{t+j}] - \mathbb{E}_{t-1}[\mathbb{E}_t[y_{t+j}]]\Big)\right)
\end{aligned}
$$

$$
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$$

The change in consumption between times $t-1$ and t is proportional to new information agents receive about their discounted expected income.

PIH Characterization

- Similarly for wealth dynamics

$$\Delta a_{t+1} = a_{t+1} - a_t = ra_t + (1+r)(y_t - c_t)$$

- Substitute expression for consumption and iterate

$$\frac{1}{1+r}\Delta a_{t+1} = -\sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t[\Delta y_{t+j}]$$

- Change in savings proportional to discounted expected income change
- Agents use savings to offset expected income fluctuations

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- Suppose $\Delta y_{t+j} = (1+g)^{j-1}$ for $g < r$

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$$\Delta a_{t+1} = -\sum_{j=1}^{\infty}\left(\frac{1+g}{1+r}\right)^{j-1} = -\frac{1+r}{r-g}$$

- Income growth increases the rate at which consumer decreases savings

PIH Characterization - Review -

- Optimal Consumption: Consumption is random walk, $\mathbb{E}_t[c_{t+1}] = c_t$
- Permanent Income: Consumption equals permanent income

$$c_t = \frac{r}{1+r}\left(a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t[y_{t+j}]\right) \equiv \frac{r}{1+r}\Big(a_t + h_t\Big)$$

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- Certainty Equivalence
  - Consumption does not depend on income variance
- Consumption and Wealth Dynamics
  - Response to news

$$\Delta c_t = \frac{r}{1+r}\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \Big(\mathbb{E}_t[y_{t+j}] - \mathbb{E}_{t-1}[y_{t+j}]\Big)$$

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$$\Delta c_t = \frac{r}{1+r}\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \big(\mathbb{E}_t[y_{t+j}] - \mathbb{E}_{t-1}[y_{t+j}]\big)$$

  - Offsets expected income fluctuations

$$\frac{1}{1+r}\Delta a_{t+1} = -\sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t[\Delta y_{t+j}]$$

Permanent Income Hypothesis

Next Steps

- What are the implications for consumption, savings, income, wealth?
- Empirically evaluate
- Excess sensitivity and Excess smoothness
- Reconciliation of puzzles

Empirical Evaluation

Excess Sensitivity

- Is consumption a random walk?

$$c_t = \gamma_0 + \gamma_1 c_{t-1} + \gamma_2 z_{t-1}$$

- z_{t-1} is any variable known at $t-1$
  (suppose income or stock market returns)

- PIH implies $\gamma_1 = 1$, $\gamma_2 = 0$

- Hall (1978): $\gamma_1 \approx 1$, $\gamma_2 > 0$ and significant

Empirical Evaluation

Excess Sensitivity

- Consumption growth a random walk?

$$\Delta c_t = +\mu_0 + \mu_1 z_{t-1}$$

- Suppose z_{t-1} is income growth Δy_t
- PIH implies $\mu_0 = 0$ and $\mu_2 = 0$
- Flavin (1981):

$$\begin{array}{cccccc} \Delta c_t & = & 11.39 & + & 0.121 & \Delta y_{t-1} \\ & & (9.7) & & (3.20) & \end{array}$$

- *"Excess sensitivity"* of current consumption to lagged income

Empirical Evaluation

Time Aggregation

- Suppose consumption data were collected annually
  - t represents a year
  - τ represents six months: $\tau + (\tau + 1)$ is a year
- Annual consumption growth:

$$\begin{aligned}
\Delta c_t^A &= (c_\tau + c_{\tau+1}) - (c_{\tau-1} + c_{\tau-2}) \\
&= \Delta c_\tau + c_{\tau+1} - c_{\tau-2} \\
&= \Delta c_\tau + c_{\tau+1} - c_{\tau-2} + (\underbrace{\Delta c_{\tau+1} + \Delta c_\tau + \Delta c_{\tau-1} + c_{\tau-2} - c_{\tau+1}}_{=0}) \\
&= \Delta c_{\tau+1} + 2\Delta c_\tau + \Delta c_{\tau-1}
\end{aligned}$$

- Annual income growth:

$$\Delta y_t^A = \Delta y_{\tau+1} + 2\Delta y_\tau + \Delta y_{\tau-1}$$

Empirical Evaluation

Time Aggregation

$$\Delta c_t^A = \Delta c_{\tau+1} + 2\Delta c_\tau + \Delta c_{\tau-1}$$
$$\Delta y_t^A = \Delta y_{\tau+1} + 2\Delta y_\tau + \Delta y_{\tau-1}$$

- $\tau - 1$ is the second half of $t - 1$
- Δc_t^A and Δy_t^A depend on $\tau - 1$
- Therefore measure a portion of lagged income/consumption

Empirical Evaluation

Time Aggregation

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- $\tau - 1$ is the second half of $t - 1$
- Δc_t^A and Δy_t^A depend on $\tau - 1$
- Therefore measure a portion of lagged income/consumption
- Instrument Δy_{t-1} with Δy_{t-2}:

$$\Delta c_t = \underset{(6.83)}{10.63} + \underset{(2.18)}{0.174} \Delta y_{t-1}$$

- Excess sensitivity still present!

Empirical Evaluation

Predicatable Income Change

- What if a fraction λ of consumers are "hand-to-mouth"?
  - Hand-to-Mouth: consume all income each period

$$\Delta c_t = \lambda \Delta y_t + (1 - \lambda)\varepsilon_t$$

- If past income is a good predictor of future income:
  - Excess sensitivity might be due to large fraction λ
- Campbell and Mankiw (1989):

$$\Delta c_t \quad = \quad \mu \quad + \quad 0.506 \quad \Delta y_t$$

  - Resolution *if* $\lambda \approx 1/2$ of aggregate income consumed by "Hand-to-Mouth"

Permanent Income Hypothesis

Next Steps:

- Excess smoothness puzzle
  - If income is *persistent*, then PIH predicts:
    consumption variability > income variability
  - Data:
    consumption variability < income variability
- Two resolutions of the Excess Sensitivity and Excess Smoothness puzzles
  - Inefficient inference / Bias (Campbell and Deaton (1989))
  - Precautionary savings motives

Excess Smoothness

Process 1: $y_t = \epsilon_t + \gamma \epsilon_{t-1}$

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- Assume $r = 0.04$, $\gamma = (1+r)/2$

$$\sigma_{\Delta c} \;=\; \frac{r}{1+r}\left(1 + \frac{\gamma}{1+r}\right)\sigma_\epsilon \;=\; \frac{0.04}{1.04}(1+0.5)\sigma_\epsilon \;\approx\; 0.06\sigma_\epsilon$$

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- Then too smooth: $\sigma_{\Delta c}/\sigma_\epsilon$ nearly zero

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Excess Smoothness

Process 3: $\Delta y_t = \gamma \Delta y_{t-1} + \epsilon_t$

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- Assume $r = 0.04$, $\gamma = 0.26$

$$\sigma_{\Delta c} = \frac{1+r}{1+r-\gamma}\sigma_\epsilon = \frac{1.04}{1.04-0.26}\sigma_\epsilon = 1.33\sigma_\epsilon$$

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- Then too volatile: $\sigma_{\Delta c} > \sigma_\epsilon$
- If income changes persistent ($\gamma < 1$), consumption variability *larger* than income variability
- Data shows the opposite!

Resolution

Campbell and Deaton (1989)

- Suppose individuals have more information than econometrician

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$$\frac{1}{1+r}\left(\Delta a^i_{t+1} - \Delta a^e_{t+1}\right) = -\sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j \left(\mathbb{E}_t[\Delta y_{t+j} \mid \Omega_t] - \mathbb{E}_t[\Delta y_{t+j} \mid \mathcal{I}_t]\right)$$

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- Assume:

$$\begin{pmatrix} \Delta y_t \\ \frac{1}{1+r}\Delta a_{t+1} \end{pmatrix} = \begin{pmatrix} \zeta_{11} & \zeta_{12} \\ \zeta_{21} & \zeta_{22} \end{pmatrix} \begin{pmatrix} \Delta y_{t-1} \\ \frac{1}{1+r}\Delta a_t \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

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- Rewrite using $e_1 = (1, 0)$ and $e_2 = (0, 1)$:

$$e_1 x_t = \Delta y_t$$

$$e_2 x_t = \frac{1}{1+r}\Delta a_{t+1}$$

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- Implies parameter restrictions:

$$\zeta_{21} = \zeta_{11}$$

$$\zeta_{22} = \zeta_{12} + (1+r)$$

Resolution

- Restrictions imply consumption is a random walk:

$$\Delta c_t \;=\; \Delta y_t + \Delta a_t - \frac{1}{1+r}\Delta a_{t+1} \;=\; u_{1t} - u_{2t}$$

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- Implies excess sensitivity:

$$\Delta c_t = \chi \Delta y_{t-1} + (u_{1t} - u_{2t})$$

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  - Cases: $\chi \to 0$, $\chi \to 1 + r$
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- If past savings predicts income, then savings affects permanent income

Next Time

- Other mechanisms that generate excess sensitivity and smoothness?

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