Lecture 1
Empirical Facts and Background

Erick Sager

January 30, 2018

Econ 606: Adv. Topics in Macroeconomics
Johns Hopkins University, Spring 2018

This Course

Study theory, empirics and computation

This Course

Study theory, empirics and computation

- Heterogeneous agents
  - Idiosyncratic shocks and demographics

This Course

Study theory, empirics and computation

- Heterogeneous agents
  - Idiosyncratic shocks and demographics
- Asset Market Incompleteness
  - Wealth distribution

This Course

Study theory, empirics and computation

- Heterogeneous agents
  - Idiosyncratic shocks and demographics
- Asset Market Incompleteness
  - Wealth distribution
  - Consumption and wealth dynamics

This Course

Study theory, empirics and computation

- Heterogeneous agents
  - Idiosyncratic shocks and demographics
- Asset Market Incompleteness
  - Wealth distribution
  - Consumption and wealth dynamics
- Aggregate Uncertainty
  - Time-varying aggregate variables
  - Microfoundations

Agenda

- Aggregation

Agenda

- Aggregation
- Heterogeneity with Complete Markets

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
    - Aiyagari-Bewley-Huggett models & Computation

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
    - Aiyagari-Bewley-Huggett models & Computation
    - Add Aggregate Uncertainty (business cycles)
    - Add Life Cycle (individual demographics)

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework
  - Public Finance, Life Cycle Taxation

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework
  - Public Finance, Life Cycle Taxation
  - Consumer Debt and Default

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework
  - Public Finance, Life Cycle Taxation
  - Consumer Debt and Default
  - Housing Wealth and the Great Recession

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework
  - Public Finance, Life Cycle Taxation
  - Consumer Debt and Default
  - Housing Wealth and the Great Recession
- Student Presentations:

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework
  - Public Finance, Life Cycle Taxation
  - Consumer Debt and Default
  - Housing Wealth and the Great Recession
- Student Presentations:
  - The Distributional Effects of Policy

Agenda

- Aggregation
- Heterogeneity with Complete Markets
- Permanent Income Hypothesis / Buffer Stock Savings
- Neoclassical Growth Model with Incomplete Markets
  - Aiyagari-Bewley-Huggett models & Computation
  - Add Aggregate Uncertainty (business cycles)
  - Add Life Cycle (individual demographics)
- (Joseph Briggs) Models that build from this framework
  - Public Finance, Life Cycle Taxation
  - Consumer Debt and Default
  - Housing Wealth and the Great Recession
- Student Presentations:
  - The Distributional Effects of Policy
  - Research Proposal

Some Housekeeping

The Syllabus

Some Housekeeping

The Syllabus → See: www.ErickSager.com

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager?

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?
- Course Requirements and Grading Policy

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

- Course Requirements and Grading Policy

1. Participate
2. Problem Sets
3. Referee Report (R)
4. Paper Presentation (P)
5. Written Research Proposal and Presentation

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

- Course Requirements and Grading Policy

1. Participate
2. Problem Sets → Will put one up before next lecture
3. Referee Report (R)
4. Paper Presentation (P)
5. Written Research Proposal and Presentation

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

- Course Requirements and Grading Policy

1. Participate
2. Problem Sets → Will put one up before next lecture
3. Referee Report (R) → Paper selection: 2/14, Due: 5/4
4. Paper Presentation (P)
5. Written Research Proposal and Presentation

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

- Course Requirements and Grading Policy

1. Participate
2. Problem Sets → Will put one up before next lecture
3. Referee Report (R) → Paper selection: 2/14, Due: 5/4
4. Paper Presentation (P) → Paper selection: 2/14
5. Written Research Proposal and Presentation

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

- Course Requirements and Grading Policy

1. Participate
2. Problem Sets → Will put one up before next lecture
3. Referee Report (R) → Paper selection: 2/14, Due: 5/4
4. Paper Presentation (P) → Paper selection: 2/14
5. Written Research Proposal and Presentation → Write-up due 5/4

Some Housekeeping

The Syllabus → See: www.ErickSager.com

- Who is Erick Sager? Aren't you glad you asked?

- Course Requirements and Grading Policy

  1. Participate
  2. Problem Sets → Will put one up before next lecture
  3. Referee Report (R) → Paper selection: 2/14, Due: 5/4
  4. Paper Presentation (P) → Paper selection: 2/14
  5. Written Research Proposal and Presentation → Write-up due 5/4

- Tools

  - Zotero
  - HACK

Motivation

Why this theory and set of computational methods?

Motivation

Why this theory and set of computational methods?

- Concerned with Inequality

Motivation

Why this theory and set of computational methods?

- Concerned with Inequality
  - Wealth
  - Consumption
  - Income

Motivation

Why this theory and set of computational methods?

- Concerned with Inequality
  - Wealth
  - Consumption
  - Income
- Welfare, policy design and distributions

Motivation

Why this theory and set of computational methods?

- Concerned with Inequality
  - Wealth
  - Consumption
  - Income
- Welfare, policy design and distributions
- Need heterogeneous agents to make progress

Today

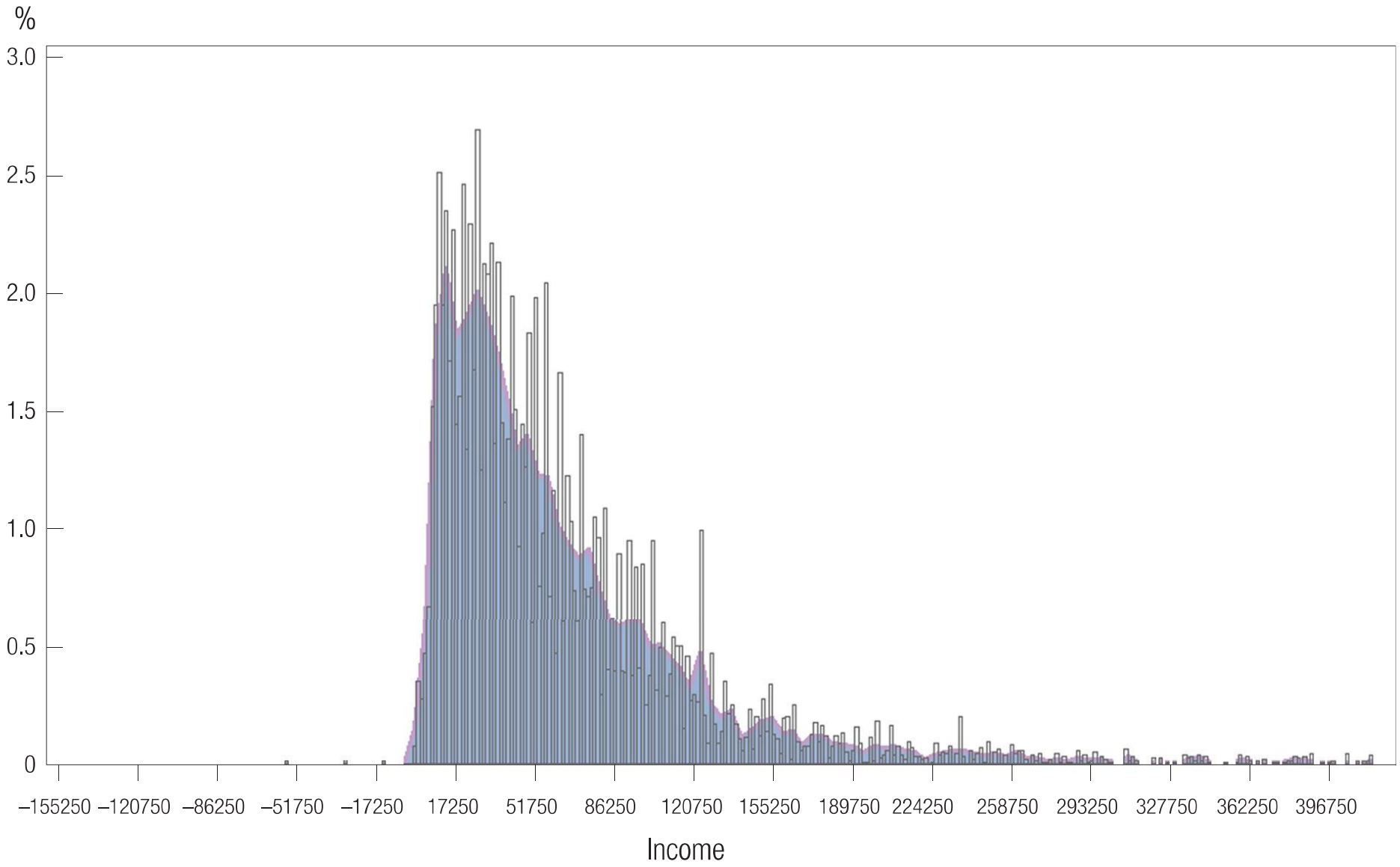
Overview of some relevant data

- Cross-sectional / Life Cycle
- Time-series / Aggregates

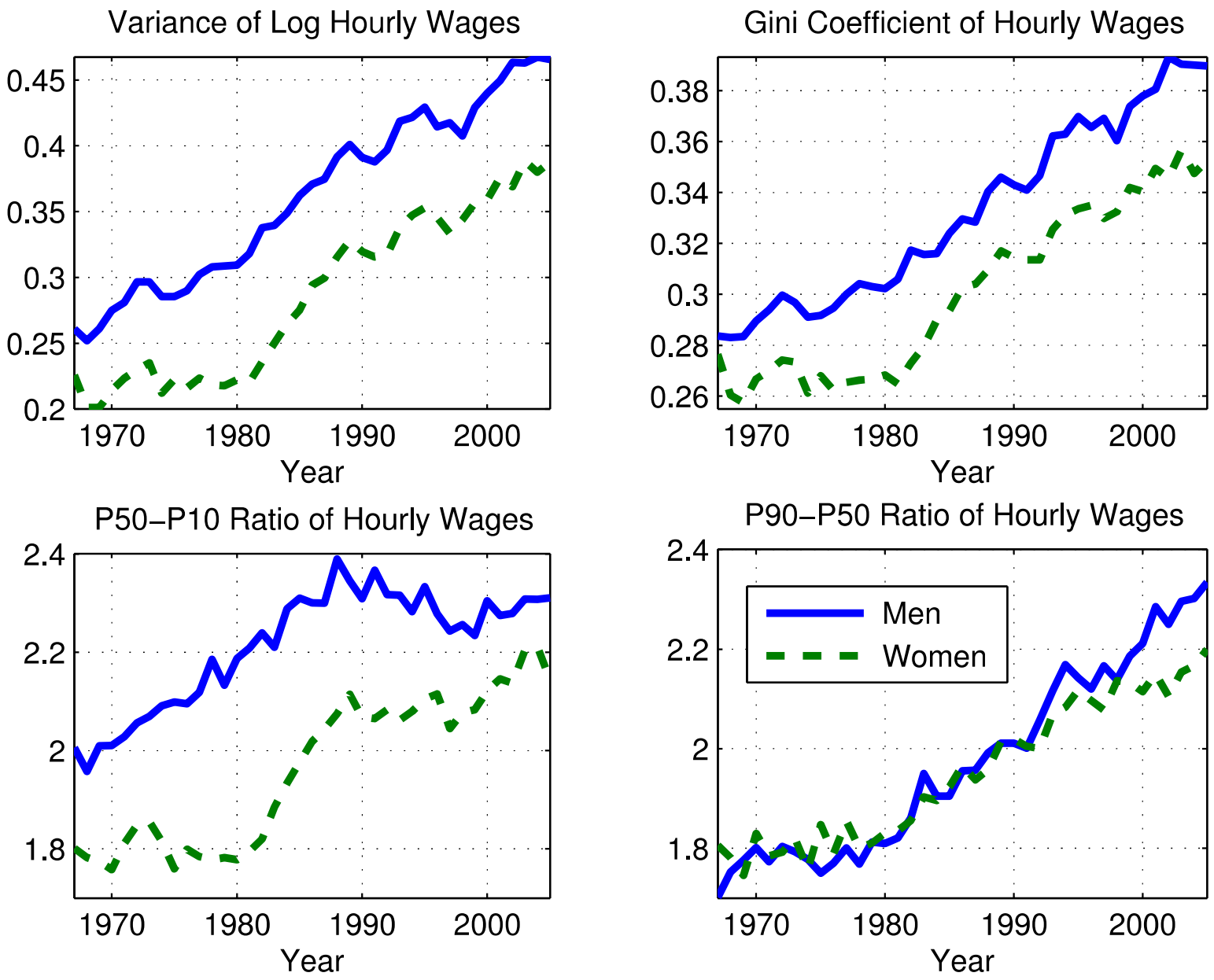
Three main papers:

- Heathcote, Perri and Violante (RED, 2010)
- Kaplan (QE, 2012)
- Díaz-Giménez, Ríos-Rull and Glover (Mpls Fed Qrtly, 2011)

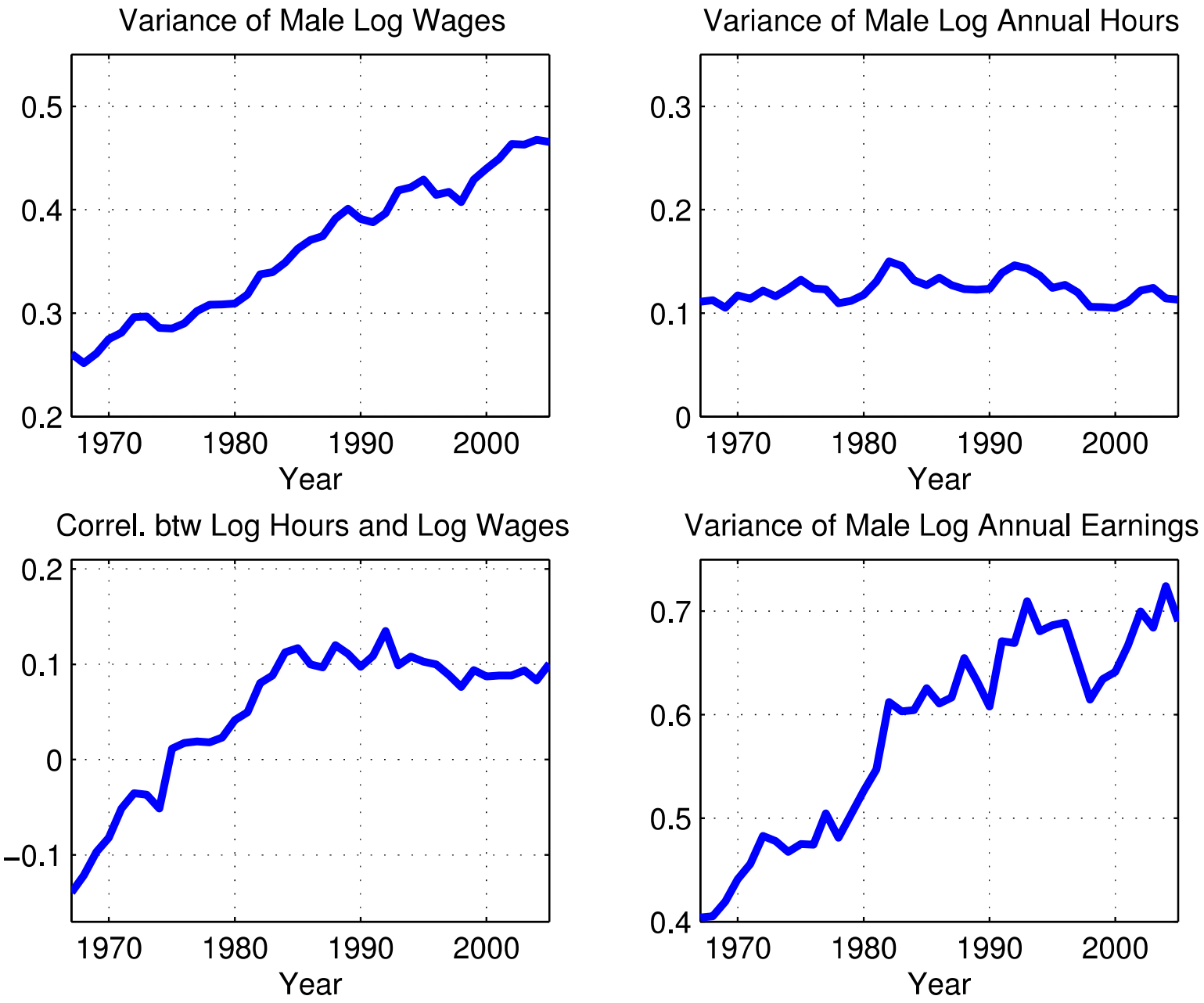
Income Distribution (2007)



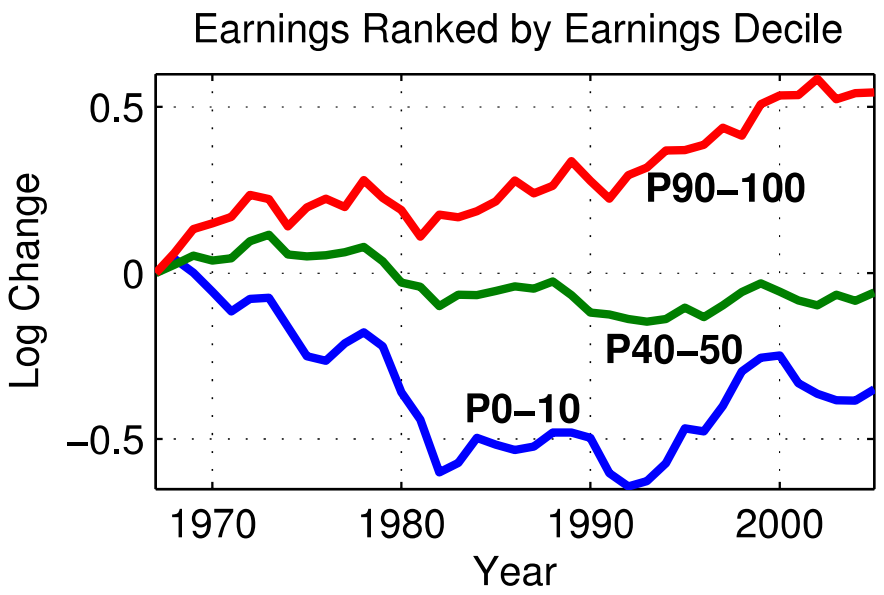
Wage Inequality

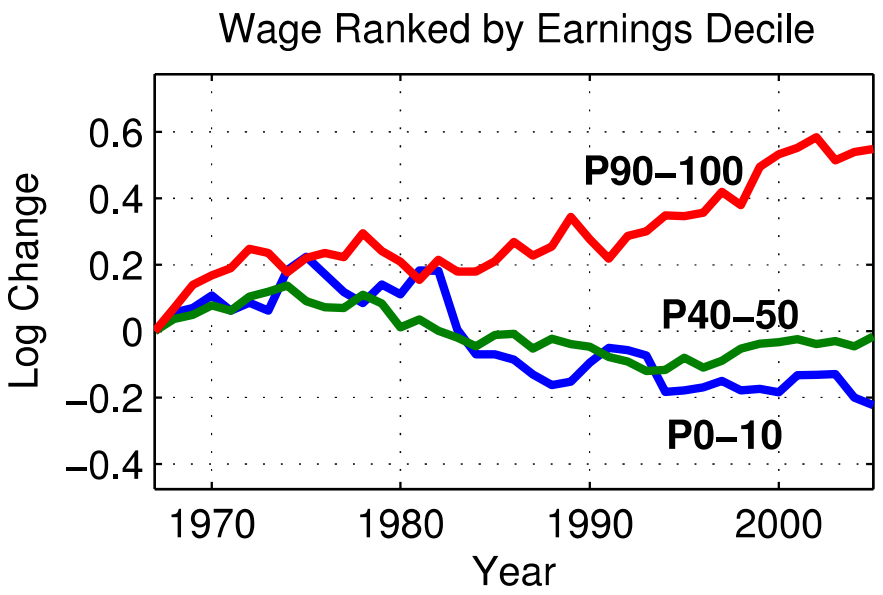


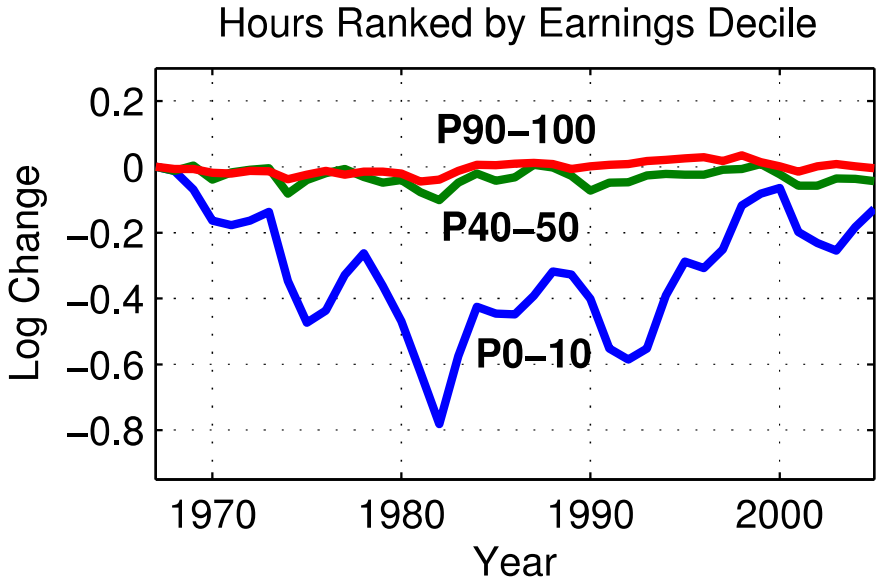
Labor Supply

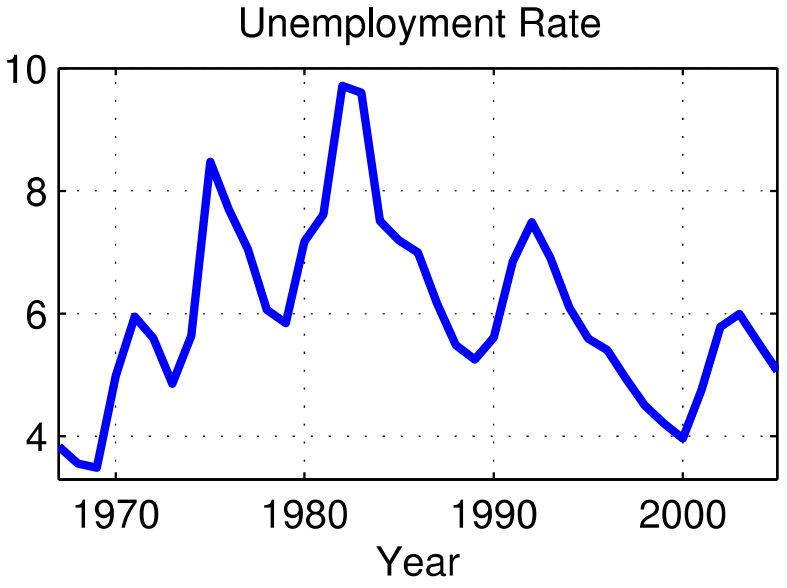


Earnings Inequality

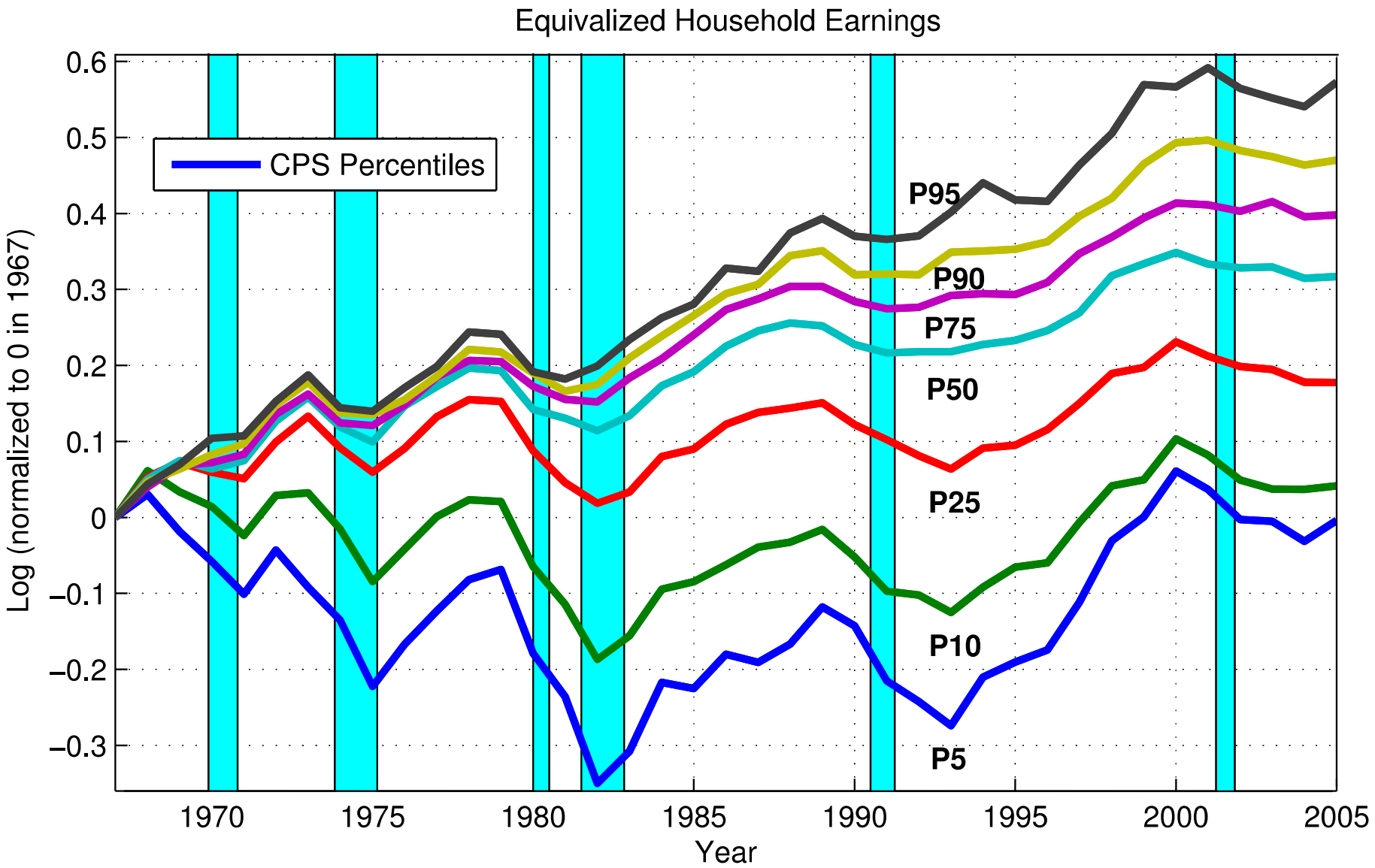




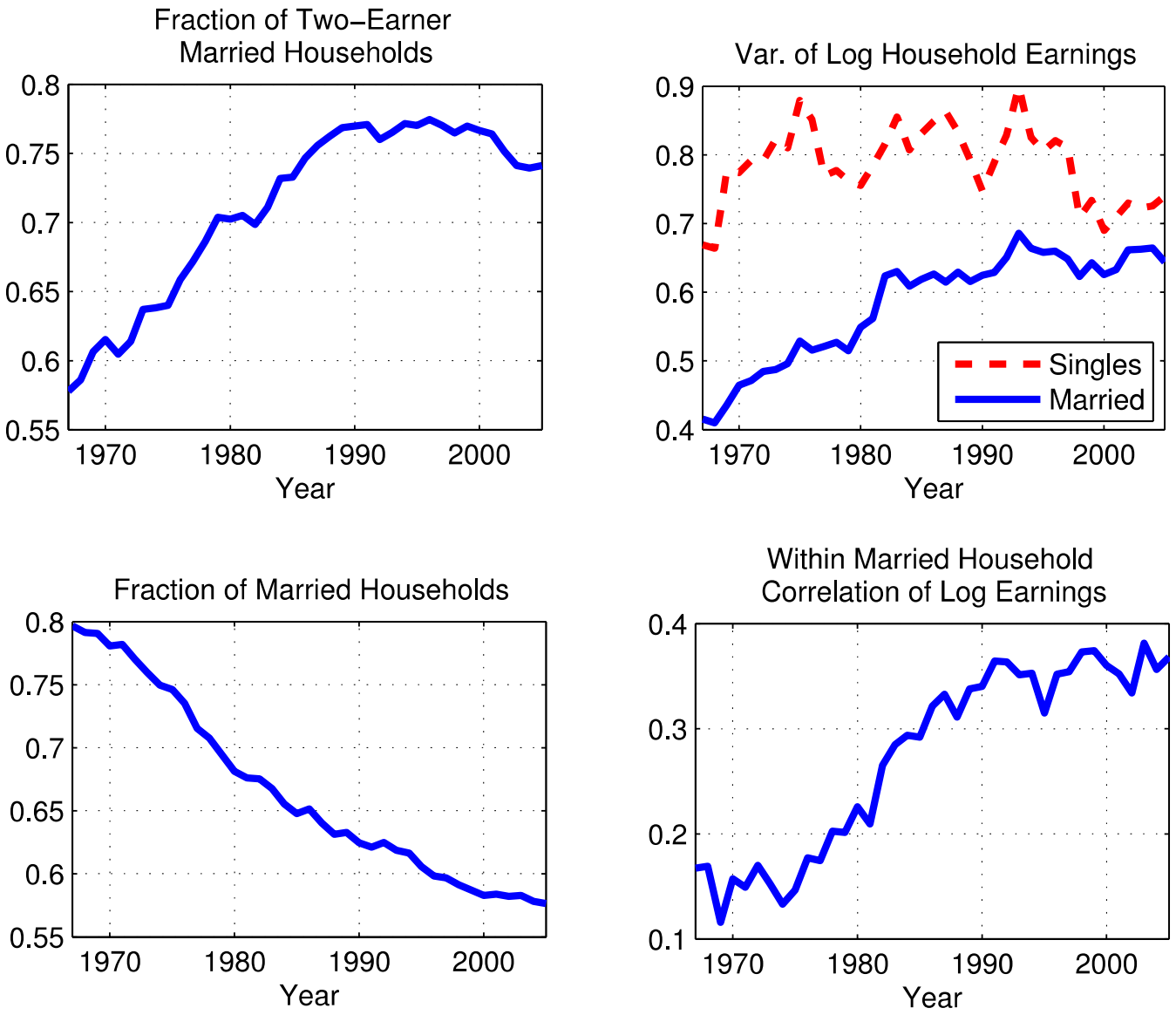




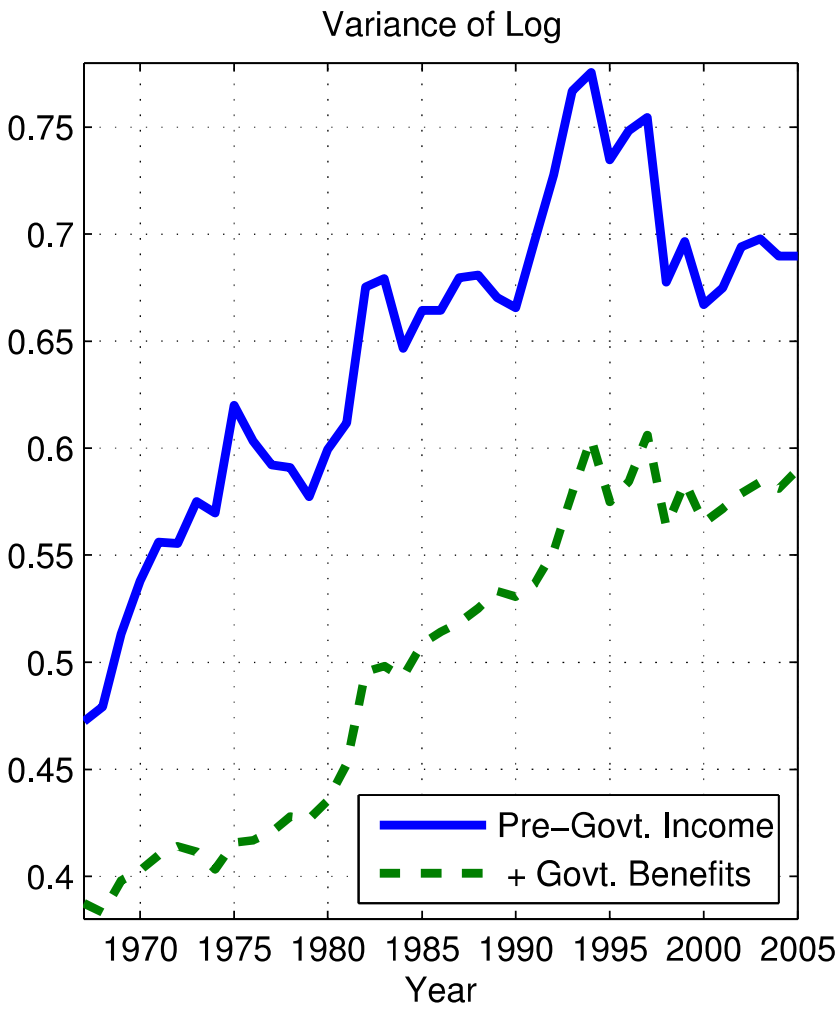
Earnings Distriubtion Dynamics

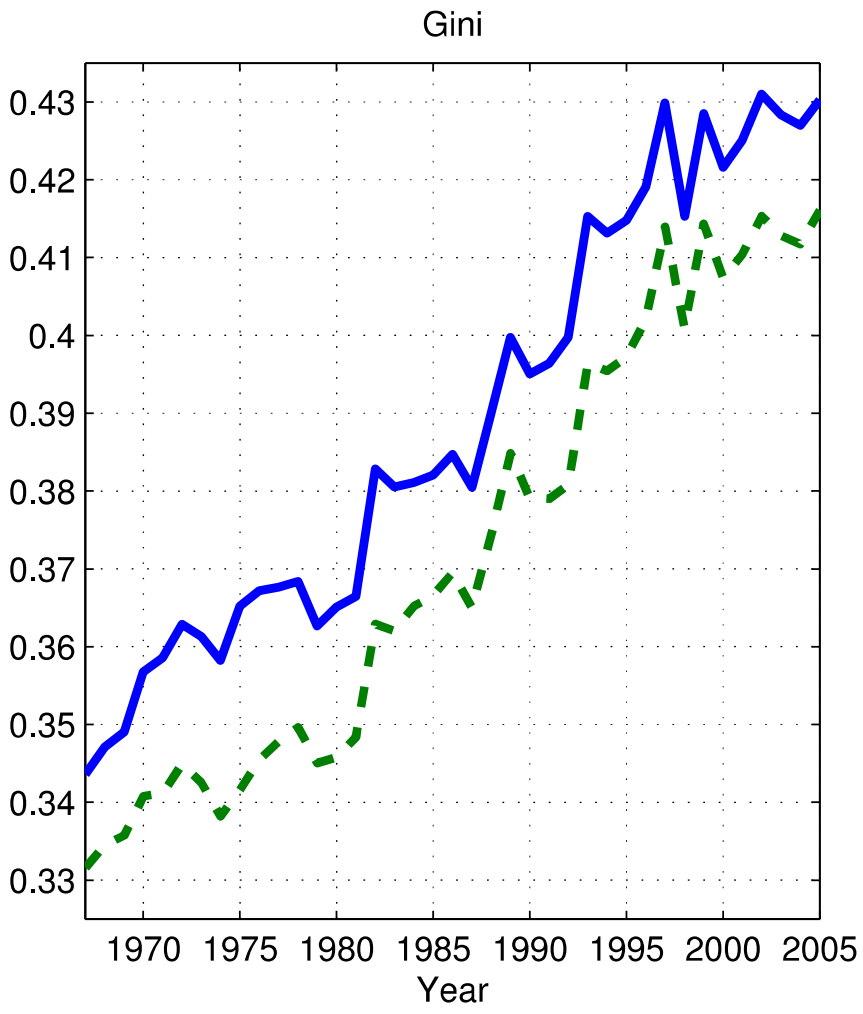


Within-Household Insurance

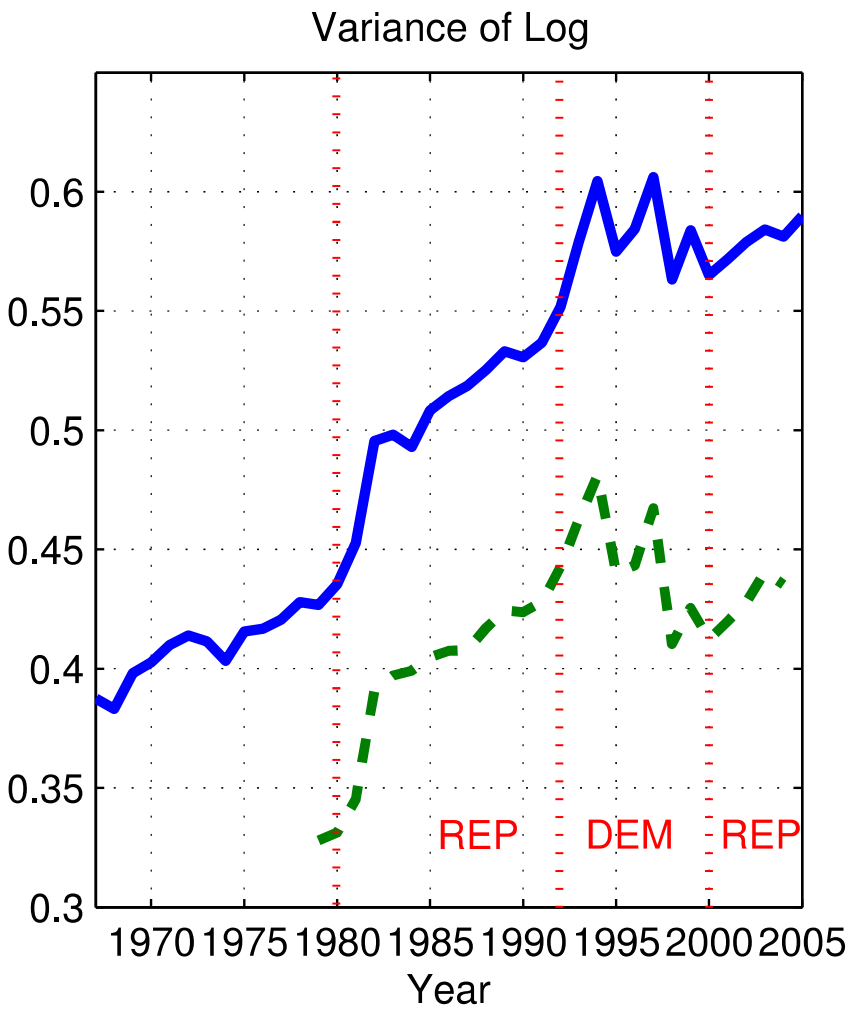


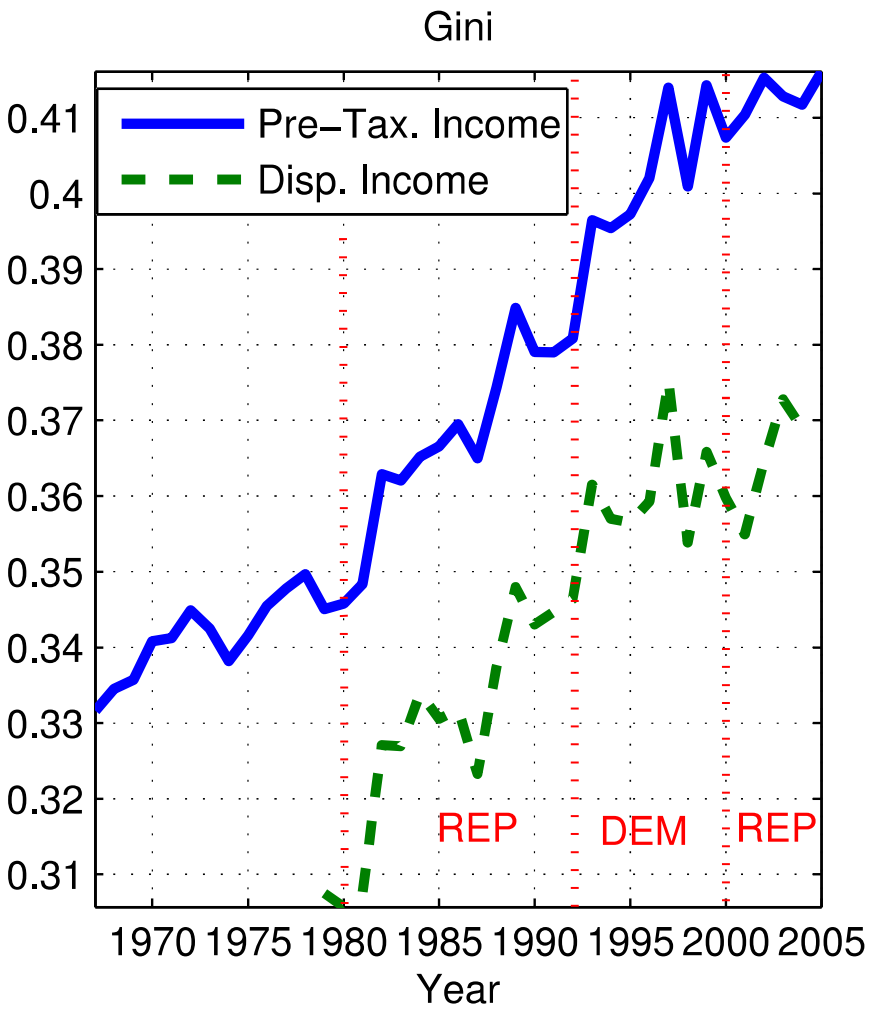
Gov't Insurance



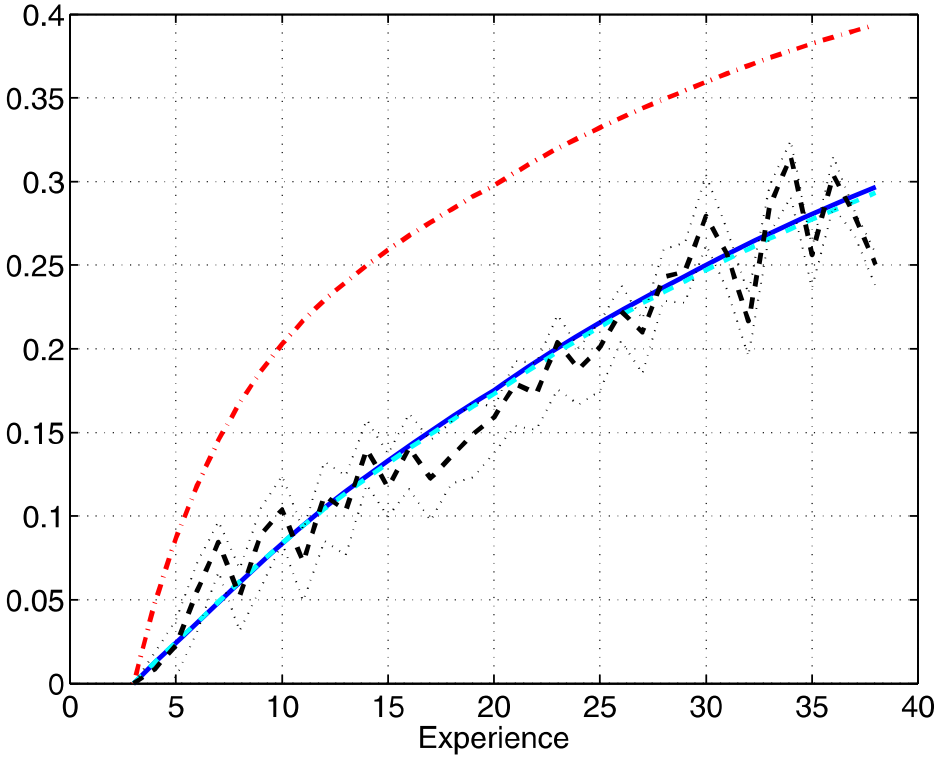


Taxation

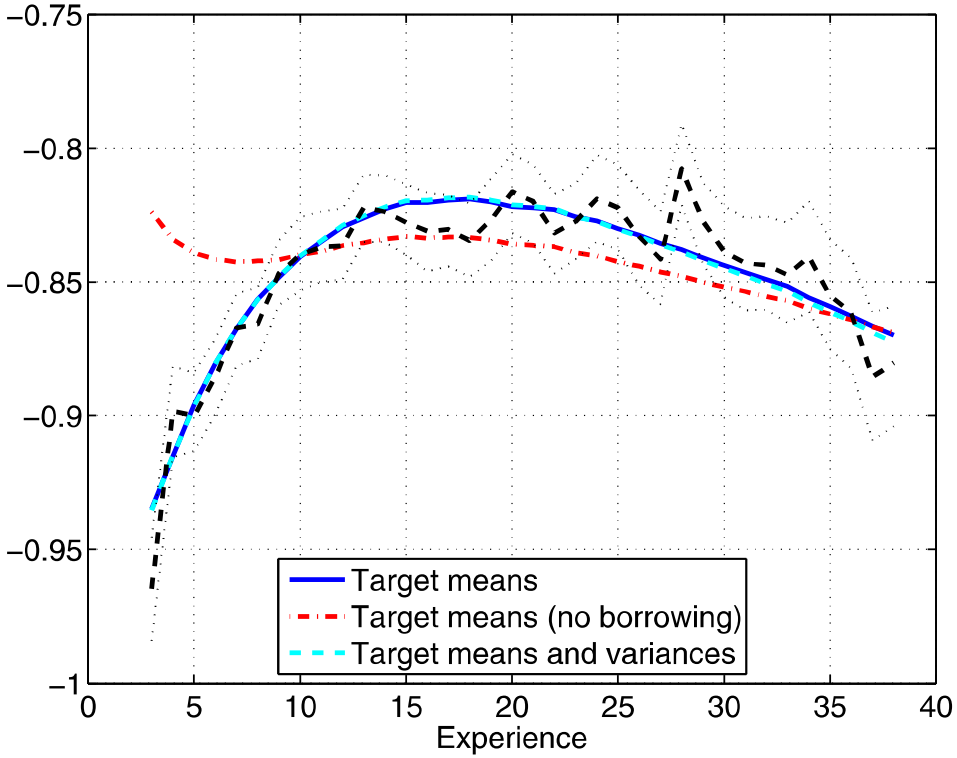

Variance of Log
0.6
0.55
0.5
0.45
0.4
0.35
0.3
REP
DEM
REP
1970 1975 1980 1985 1990 1995 2000 2005
Year


Gini
Pre−Tax. Income
Disp. Income
0.41
0.4
0.39
0.38
0.37
0.36
0.35
0.34
0.33
0.32
0.31
REP
DEM
REP
1970 1975 1980 1985 1990 1995 2000 2005
Year

Life Cycle Means

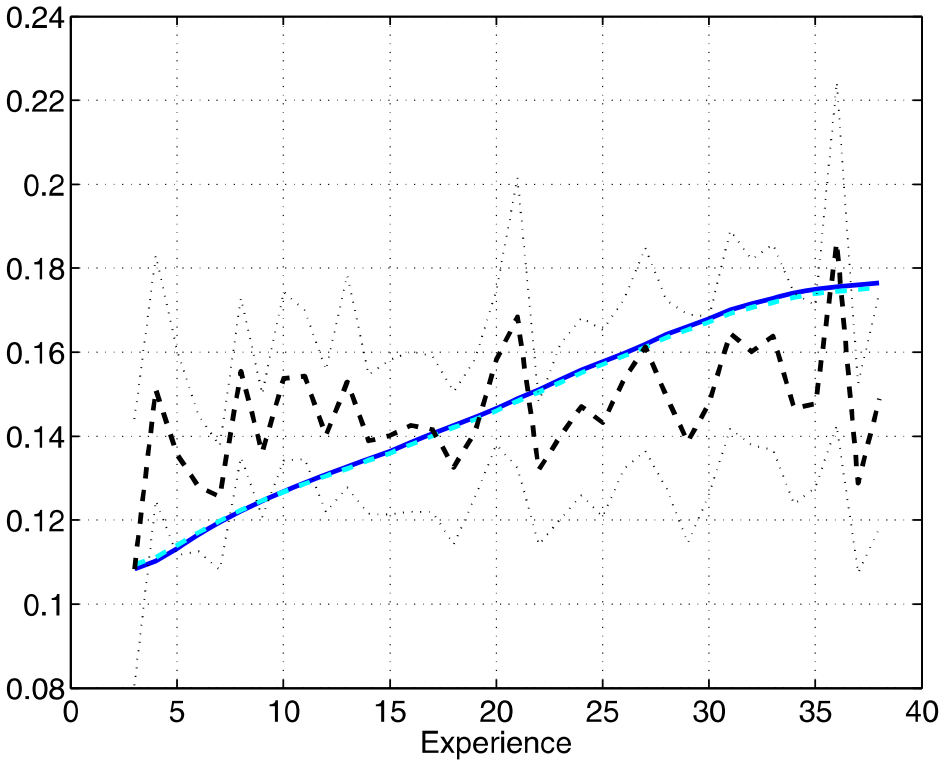


(a) Mean log consumption

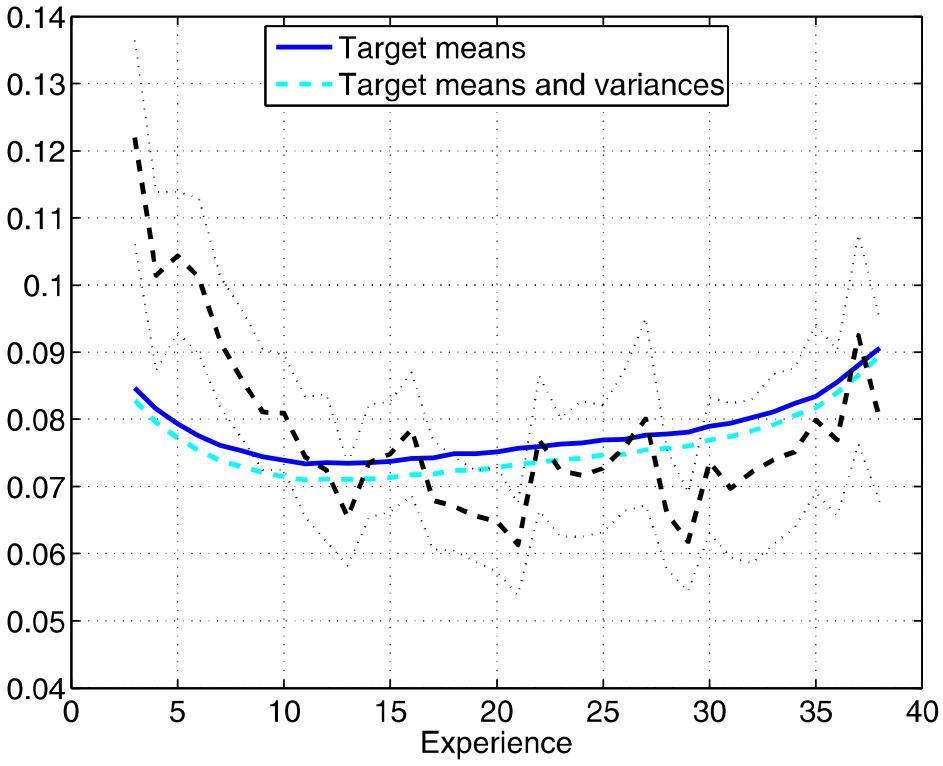


(b) Mean log hours

Life Cycle Inequality

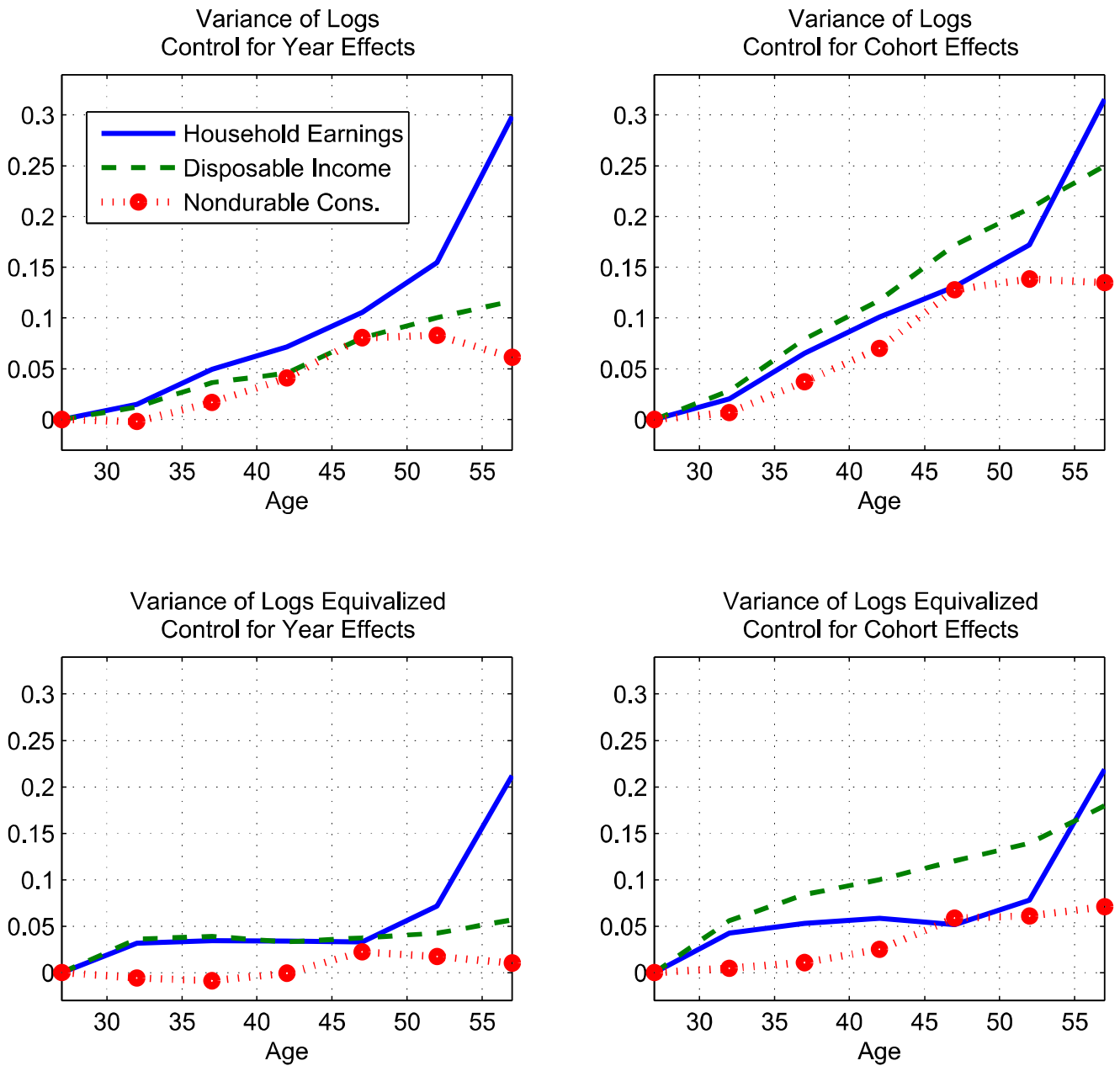


(a) Variance log consumption



(b) Variance log hours

Life Cycle Inequality



Motivation

Inequality is

- Large, time-varying, pervasive

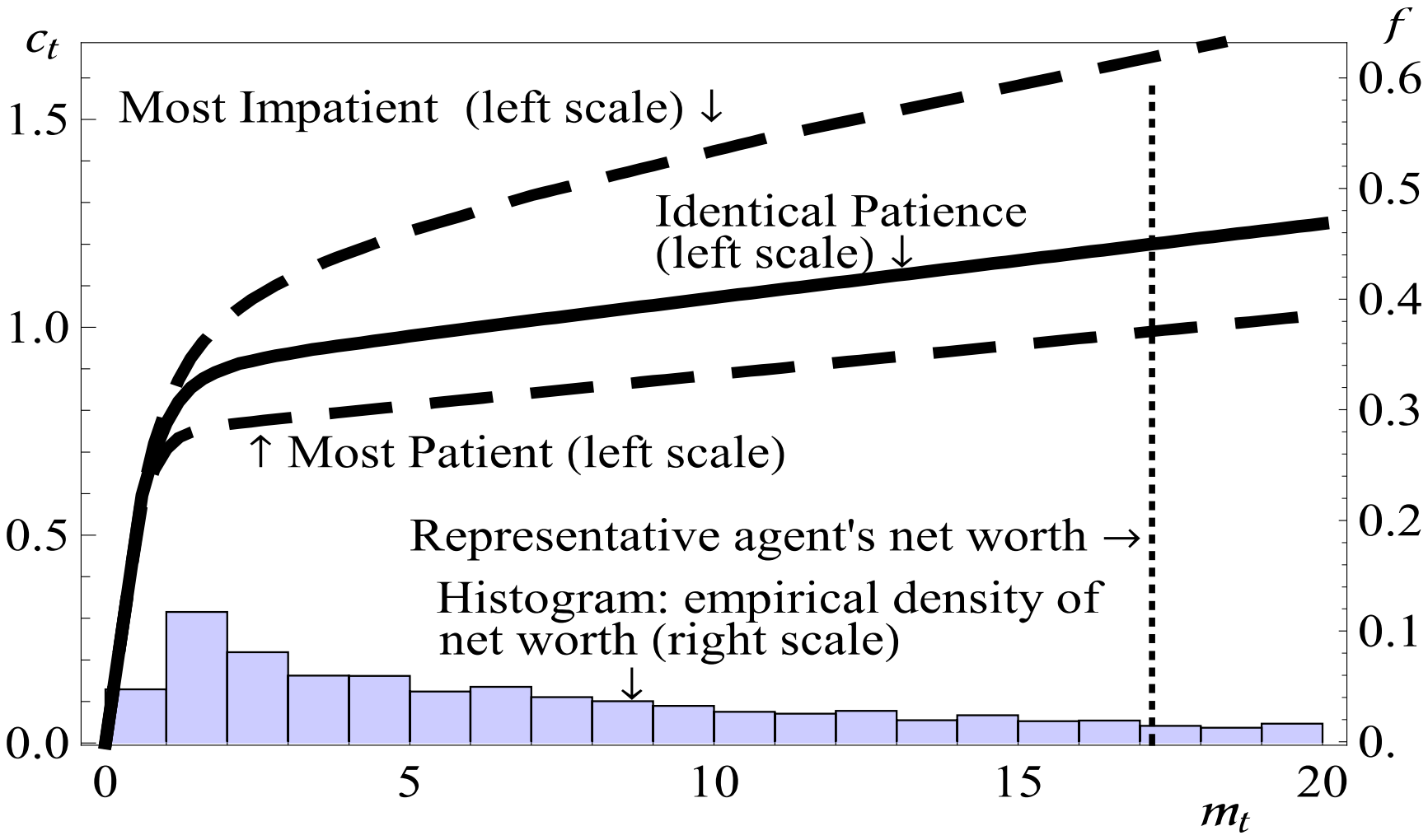
To understand it

- Need a model of individual choice
- Model frictions / institutions that influence choice
- Operational measure of welfare

Inequality over the business cycle

- Increases in recessions
- But is it important for aggregates?

Representative Agent?



Aggregation

Today

- **We just considered data**
  - Wealth, Consumption, Income
  - Cross-Sectional, Times-Series, Life-Cycle
  - Inequality

Today

- **We just considered data**
  - Wealth, Consumption, Income
  - Cross-Sectional, Times-Series, Life-Cycle
  - Inequality

- **Next, aggregation with complete markets**
  - Gorman Form

$$c^i(p, w_i) = a^i(p) + b(p)w_i \qquad \Longrightarrow \qquad C(p, w) = C(p, W)$$

Today

- **We just considered data**
  - Wealth, Consumption, Income
  - Cross-Sectional, Times-Series, Life-Cycle
  - Inequality

- **Next, aggregation with complete markets**
  - Gorman Form

$$c^i(p, w_i) = a^i(p) + b(p)w_i \qquad \Longrightarrow \qquad C(p, w) = C(p, W)$$

- **<u>Next time</u>: Permanent Income Hypothesis (PIH)**
  - Restrict to incomplete markets

Aggregation Under Complete Markets

Overview

- Pre-trade heterogeneity undone by complete markets
- Usually admits a representative agent
- Highly tractable, useful baseline
- Little/no role for distributional effects

Gorman Form Aggregation

- Suppose N consumers, indexed by $i = 1, \ldots, N$
- M consumption goods with given prices $p \equiv \{p_1, \ldots, p_M\}$
- $u : \mathbb{R}_+^M \to \mathbb{R}$ s.t. $u' > 0$, $u'' < 0$, $u \in \mathcal{C}^2$, Inada
- Endowed with wealth $w \equiv \{w_1, \ldots, w_N\}$

Gorman Form Aggregation

- Suppose N consumers, indexed by $i = 1, \ldots, N$
- M consumption goods with given prices $p \equiv \{p_1, \ldots, p_M\}$
- $u : \mathbb{R}_+^M \to \mathbb{R}$ s.t. $u' > 0$, $u'' < 0$, $u \in \mathcal{C}^2$, Inada
- Endowed with wealth $w \equiv \{w_1, \ldots, w_N\}$
- Suppose agents with wealth w_i and taking prices p as given, choose consumption according to decision rule $c^i : \mathbb{R}_+^M \times \mathbb{R} \to \mathbb{R}^M$
- Aggregate demand over each agent's consumption, $c^i(p, w_i)$ is:

$$C(p, w) = \sum_{i=1}^{N} c^i(p, w_i)$$

Gorman Form Aggregation

- Suppose N consumers, indexed by $i = 1, \ldots, N$
- M consumption goods with given prices $p \equiv \{p_1, \ldots, p_M\}$
- $u : \mathbb{R}^M_+ \to \mathbb{R}$ s.t. $u' > 0$, $u'' < 0$, $u \in \mathcal{C}^2$, Inada
- Endowed with wealth $w \equiv \{w_1, \ldots, w_N\}$
- Suppose agents with wealth w_i and taking prices p as given, choose consumption according to decision rule $c^i : \mathbb{R}^M_+ \times \mathbb{R} \to \mathbb{R}^M$
- Aggregate demand over each agent's consumption, $c^i(p, w_i)$ is:

$$C(p, w) = \sum_{i=1}^{N} c^i(p, w_i)$$

- Aggregate demand depends on the *distribution* of wealth across agents
- When can we write aggregate demand as a function of aggregate wealth?

$$W \equiv \sum_{i=1}^{N} w_i$$

Gorman Form Aggregation

- When can we write aggregate demand as a function of aggregate wealth?
- Must be: W doesn't change in response to redistribution across agents
- Take two wealth distributions w and w' for which $W = W'$
- Want to show: $C(p, w) = C(p, w')$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w')$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w')$
- Take some arbitrary redistribution $dw = (dw_1, \ldots, dw_2)$ s.t. $\sum_{i=1}^{N} dw_i = 0$
- Assume decision rules are differentiable

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w')$
- Take some arbitrary redistribution $dw = (dw_1, \ldots, dw_2)$ s.t. $\sum_{i=1}^{N} dw_i = 0$
- Assume decision rules are differentiable
- <u>Want</u>: no change in aggregate demand in response to redistribution:

$$\sum_{i=1}^{N} \frac{\partial c_j^i(p, w_i)}{\partial w_i} dw_i = 0 \quad \forall\ j = 1, \ldots, M$$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w')$
- Take some arbitrary redistribution $dw = (dw_1, \ldots, dw_2)$ s.t. $\sum_{i=1}^{N} dw_i = 0$
- Assume decision rules are differentiable
- <u>Want</u>: no change in aggregate demand in response to redistribution:

$$\sum_{i=1}^{N} \frac{\partial c_j^i(p, w_i)}{\partial w_i} dw_i = 0 \quad \forall\ j = 1, \ldots, M$$

- <u>One way</u>: True if all agents change consumption in the *same way*:

$$\frac{\partial c_j^i(p, w_i)}{\partial w_i} = \frac{\partial c_j^k(p, w_k)}{\partial w_k} \quad \forall\ i, k = 1, \ldots, N \quad \forall j = 1, \ldots, M$$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w')$
- Take some arbitrary redistribution $dw = (dw_1, \ldots, dw_2)$ s.t. $\sum_{i=1}^{N} dw_i = 0$
- Assume decision rules are differentiable
- <u>Want</u>: no change in aggregate demand in response to redistribution:

$$\sum_{i=1}^{N} \frac{\partial c_j^i(p, w_i)}{\partial w_i} dw_i = 0 \quad \forall\ j = 1, \ldots, M$$

- <u>One way</u>: True if all agents change consumption in the *same way*:

$$\frac{\partial c_j^i(p, w_i)}{\partial w_i} = \frac{\partial c_j^k(p, w_k)}{\partial w_k} \quad \forall\ i, k = 1, \ldots, N \quad \forall j = 1, \ldots, M$$

- A sufficient condition for this, all agents have same MPC out of wealth:

$$c^i(p, w) = a^i(p) + b(p) w_i$$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w') = C(p, W)$
- If all agents have same MPC out of wealth: $c^i(p, w) = a^i(p) + b(p)w_i$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w') = C(p, W)$
- If all agents have same MPC out of wealth: $c^i(p, w) = a^i(p) + b(p)w_i$

$$C(p, w) = \sum_{i=1}^{N} c^i(p, w_i)$$

$$C(p, w) = \sum_{i=1}^{N} \Big[a^i(p) + b(p)w_i\Big]$$

$$C(p, W) = \Big(\sum_{i=1}^{N} a^i(p)\Big) + b(p)W$$

Gorman Form Aggregation

- Want to show: $C(p, w) = C(p, w') = C(p, W)$
- If all agents have same MPC out of wealth: $c^i(p, w) = a^i(p) + b(p)w_i$

$$C(p, w) = \sum_{i=1}^{N} c^i(p, w_i)$$

$$C(p, w) = \sum_{i=1}^{N} \left[a^i(p) + b(p)w_i\right]$$

$$C(p, W) = \left(\sum_{i=1}^{N} a^i(p)\right) + b(p)W$$

- (**Gorman Form**) $C(p, w) = C(p, W)$ iff preferences admit indirect utility functions with form

$$v_i(p, w_i) = a^i(p) + b(p)w_i$$

Aggregation Under Complete Markets

Issue

- Consider heterogeneous agents with complete markets
- Suppose Gorman aggregation fails
- $c^i(p, w_i) \neq a^i(p) + b(p)w_i$

Aggregation Under Complete Markets

Issue

- Consider heterogeneous agents with complete markets
- Suppose Gorman aggregation fails
- $c^i(p, w_i) \neq a^i(p) + b(p)w_i$
- How do we write the preferences of a representative agent?
- Can we compute prices and allocation with heterogeneity?

Aggregation Under Complete Markets

Issue

- Consider heterogeneous agents with complete markets
- Suppose Gorman aggregation fails
- $c^i(p, w_i) \neq a^i(p) + b(p)w_i$
- How do we write the preferences of a representative agent?
- Can we compute prices and allocation with heterogeneity?

Negishi Approach

- FWT: Competitive Equilibrium allocation = Pareto Efficient allocation
- Allocation can be found as solution to Social Planner Problem
- Choose Planner weights that recover Competitive Equilibrium allocation

Aggregation Under Complete Markets

Issue

- Consider heterogeneous agents with complete markets
- Suppose Gorman aggregation fails
- $c^i(p, w_i) \neq a^i(p) + b(p)w_i$
- How do we write the preferences of a representative agent?
- Can we compute prices and allocation with heterogeneity?

Negishi Approach $\rightarrow$ Study lecture notes

- FWT: Competitive Equilibrium allocation = Pareto Efficient allocation
- Allocation can be found as solution to Social Planner Problem
- Choose Planner weights that recover Competitive Equilibrium allocation

Constantinides Approach

Next Slides:

- Generalization of Negishi Approach
- Decompose into
  - allocation across agents
  - representative consumer
- Example: Maliar and Maliar (2003)

Constantinides Approach

- $i = 1, 2, \ldots, N$ consumers
- π_i population weight s.t. $\sum_{i=1}^{N} \pi_i = 1$
- Social Planner's Problem with given welfare weights $(\mu_1, \ldots, \mu_N)$

$$\max_{\{c_t^1, \ldots, c_t^N, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{N} \mu_i u(c_t^i)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \pi_i c_t^i + K_{t+1} = f(K_t) + (1-\delta)K_t$$

Constantinides Approach

- $i = 1, 2, \ldots, N$ consumers
- π_i population weight s.t. $\sum_{i=1}^{N} \pi_i = 1$
- Social Planner's Problem with given welfare weights $(\mu_1, \ldots, \mu_N)$

$$\max_{\{c_t^1, \ldots, c_t^N, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{N} \mu_i u(c_t^i)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \pi_i c_t^i + K_{t+1} = f(K_t) + (1-\delta)K_t$$

- Constantinides Decomposition: Allocation across agents

$$U(C_t) \equiv \max_{\{c_t^i\}_{i=1}^{N}} \left\{ \sum_{i=1}^{N} \mu_i u(c_t^i) \quad \text{s.t.} \quad \sum_{i=1}^{N} \pi_i c_t^i \leq C_t \right\}$$

Constantinides Approach

- $i = 1, 2, \ldots, N$ consumers
- π_i population weight s.t. $\sum_{i=1}^{N} \pi_i = 1$
- Social Planner's Problem with given welfare weights $(\mu_1, \ldots, \mu_N)$

$$\max_{\{c_t^1, \ldots, c_t^N, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{N} \mu_i u(c_t^i)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \pi_i c_t^i + K_{t+1} = f(K_t) + (1-\delta)K_t$$

- Constantinides Decomposition: Allocation across agents

$$U(C_t) \equiv \max_{\{c_t^i\}_{i=1}^{N}} \left\{ \sum_{i=1}^{N} \mu_i u(c_t^i) \quad \text{s.t.} \quad \sum_{i=1}^{N} \pi_i c_t^i \leq C_t \right\}$$

- Constantinides Decomposition: Representative consumer

$$\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t) \quad \text{s.t.} \quad C_t + K_{t+1} = f(K_t) + (1-\delta)K_t, \ K_0 \text{ given} \right\}$$

Taking Stock

- Method for computing Representative Agent with heterogeneity
- Only assumption on preferences is strict concavity
  - Gorman Form (homotheticity or quasilinearity) not required
- Requires complete markets
  - Gorman Form makes no assumption on asset markets
- Next: example with closed-form solution

Maliar and Maliar (2003)

- Unit continuum of infinitely lived **households**
- indexed by $i \in \mathcal{I} \equiv [0, 1]$
- Let μ^i be measure of type i agents s.t. $\int_{\mathcal{I}} d\mu_i = 1$

Maliar and Maliar (2003)

- Unit continuum of infinitely lived **households**
- indexed by $i \in \mathcal{I} \equiv [0, 1]$
- Let μ^i be measure of type i agents s.t. $\int_{\mathcal{I}} d\mu_i = 1$
- Endowed with one unit of time, labor (h_t^i) vs leisure $(1 - h_t^i)$
- Idiosyncratic labor productivity shocks: $\varepsilon_t^i \in \mathcal{E}$ s.t. $\mathbb{E}[\varepsilon] = 1$
- Trade Arrow securities $a_{t+1}(\varepsilon)$ at price $p_t(\varepsilon)$
- Endowed with k_0 and a_0

Maliar and Maliar (2003)

- Unit continuum of infinitely lived **households**
- indexed by $i \in \mathcal{I} \equiv [0,1]$
- Let μ^i be measure of type i agents s.t. $\int_{\mathcal{I}} d\mu_i = 1$
- Endowed with one unit of time, labor (h_t^i) vs leisure $(1 - h_t^i)$
- Idiosyncratic labor productivity shocks: $\varepsilon_t^i \in \mathcal{E}$ s.t. $\mathbb{E}[\varepsilon] = 1$
- Trade Arrow securities $a_{t+1}(\varepsilon)$ at price $p_t(\varepsilon)$
- Endowed with k_0 and a_0
- Household i's problem

$$\max_{\{c_t, h_t, k_{t+1}, a_{t+1}(\varepsilon)\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} + \psi \frac{(1-h_t^i)^{1-\gamma}}{1-\gamma} \right]$$

$$\text{s.t.} \quad c_t^i + k_{t+1}^i + \int_{\mathcal{E}} p_t(\varepsilon) a_{t+1}^i(\varepsilon) d\varepsilon \leq w_t \varepsilon_t^i h_t^i + (1+r_t) k_t^i + a_t^i(\varepsilon_t^i)$$

Maliar and Maliar (2003)

Representative Firm

- CRS production technology, $Y_t = z_t F(K_t, L_t)$
- z_t is an aggregate shock
- Spot markets for capital and labor (rental rate and wage)
- Static profits:

$$\Pi_t = z_t F(K_t, L_t) - w_t L_t - (r_t + \delta) K_t$$

Maliar and Maliar (2003)

Equilibrium

- household allocation $\{c_t^i, h_t, k_{t+1}^i, a_{t+1}^i(\varepsilon)\}_{t=0}^{\infty}$ for each i,
- firm allocation $\{K_t, h_t, L_t\}_{t=0}^{\infty}$
- and prices $\{w_t, h_t, r_t, p_t(\varepsilon)\}_{t=0}^{\infty}$ s.t.

(I) satisfy household optimality,
(II) satisfy firm optimality
(III) satisfy market clearing conditions for capital, labor and resources:

$$K_t = \int_{\mathcal{I}} k_t^i d\mu^i$$

$$L_t = \int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i$$

$$\int_{\mathcal{I}} c_t^i d\mu^i + K_{t+1} = z_t F(K_t, L_t) + (1-\delta)K_t$$

(IV) Arrow securities are in net zero supply

Maliar and Maliar (2003)

Planner's Problem

$$U(C_t, 1 - L_t) = \max_{\{c_t^i, h_t^i\}_{i \in \mathcal{I}}} \int_{\mathcal{I}} \alpha^i u(c_t^i, h_t^i) d\mu^i$$

$$\begin{aligned} \text{s.t.} \qquad \int_{\mathcal{I}} c_t^i d\mu^i &\leq C_t && [\lambda_t^c] \\ \int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i &\leq L_t && [\lambda_t^l] \end{aligned}$$

Maliar and Maliar (2003)

Planner's Problem

$$U(C_t, 1 - L_t) = \max_{\{c_t^i, h_t^i\}_{i \in \mathcal{I}}} \int_{\mathcal{I}} \alpha^i u(c_t^i, h_t^i) d\mu^i$$

$$\begin{array}{llcll} \text{s.t.} & \int_{\mathcal{I}} c_t^i d\mu^i & \leq & C_t & [\lambda_t^c] \\ & \int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i & \leq & L_t & [\lambda_t^l] \end{array}$$

- First Order Conditions:

$$c_t^i = \left(\frac{\alpha^i}{\lambda_t^c}\right)^{\frac{1}{\sigma}} \qquad \text{and} \qquad h_t^i = 1 - \left(\frac{\psi \alpha^i}{\lambda_t^l \varepsilon_t^i}\right)^{\frac{1}{\gamma}}$$

Maliar and Maliar (2003)

Planner's Problem

$$U(C_t, 1 - L_t) = \max_{\{c_t^i, h_t^i\}_{i \in \mathcal{I}}} \int_{\mathcal{I}} \alpha^i u(c_t^i, h_t^i) d\mu^i$$

$$\begin{array}{lrcll} \text{s.t.} & \int_{\mathcal{I}} c_t^i d\mu^i & \leq & C_t & [\lambda_t^c] \\ & \int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i & \leq & L_t & [\lambda_t^l] \end{array}$$

- First Order Conditions:

$$c_t^i = \left(\frac{\alpha^i}{\lambda_t^c}\right)^{\frac{1}{\sigma}} \qquad \text{and} \qquad h_t^i = 1 - \left(\frac{\psi \alpha^i}{\lambda_t^l \varepsilon_t^i}\right)^{\frac{1}{\gamma}}$$

- Aggregate:

$$C_t = \int_{\mathcal{I}} c_t^i d\mu^i = (\lambda_t^c)^{-\frac{1}{\sigma}} \int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i$$

$$L_t = \int_{\mathcal{I}} \varepsilon_t^i h_t^i d\mu^i = 1 - \left(\frac{\psi}{\lambda_t^l}\right)^{\frac{1}{\gamma}} \int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i$$

Maliar and Maliar (2003)

- First Order Conditions:

$$c_t^i = \left(\frac{\alpha^i}{\lambda_t^c}\right)^{\frac{1}{\sigma}} \qquad \text{and} \qquad h_t^i = 1 - \left(\frac{\psi\alpha^i}{\lambda_t^l \varepsilon_t^i}\right)^{\frac{1}{\gamma}}$$

- Aggregate:

$$C_t = (\lambda_t^c)^{-\frac{1}{\sigma}} \int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i \qquad \text{and} \qquad L_t = 1 - \left(\frac{\psi}{\lambda_t^l}\right)^{\frac{1}{\gamma}} \int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i$$

Maliar and Maliar (2003)

- First Order Conditions:

$$c_t^i = \left(\frac{\alpha^i}{\lambda_t^c}\right)^{\frac{1}{\sigma}} \qquad \text{and} \qquad h_t^i = 1 - \left(\frac{\psi\alpha^i}{\lambda_t^l \varepsilon_t^i}\right)^{\frac{1}{\gamma}}$$

- Aggregate:

$$C_t = (\lambda_t^c)^{-\frac{1}{\sigma}} \int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i \qquad \text{and} \qquad L_t = 1 - \left(\frac{\psi}{\lambda_t^l}\right)^{\frac{1}{\gamma}} \int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i$$

- Rewrite FOCs:

$$c_t^i = \frac{\left(\alpha^i\right)^{\frac{1}{\sigma}}}{\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i} C_t$$

$$1 - h_t^i = \frac{\left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{-\frac{1}{\gamma}}}{\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i} (1 - L_t)$$

Maliar and Maliar (2003)

Substitute:

$$\int_{\mathcal{I}} \alpha^i \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} + \psi \frac{(1-h_t^i)^{1-\gamma}}{1-\gamma} \right] d\mu^i$$

$$= \int_{\mathcal{I}} \alpha^i \left[\left(\frac{\left(\alpha^i\right)^{\frac{1}{\sigma}}}{\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i} \right)^{1-\sigma} \frac{(C_t)^{1-\sigma}}{1-\sigma} + \psi \left(\frac{\left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{-\frac{1}{\gamma}}}{\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i} \right)^{1-\gamma} \frac{(1-L_t)^{1-\gamma}}{1-\gamma} \right] d\mu^i$$

Rewrite:

$$U(C_t, L_t; \Psi_t) \equiv \frac{(C_t)^{1-\sigma}}{1-\sigma} + \Psi_t \frac{(1-L_t)^{1-\gamma}}{1-\gamma}$$

where by normalization we obtain:

$$\Psi_t \equiv \frac{\psi \left(\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\gamma}} \left(\varepsilon_t^i\right)^{1-\frac{1}{\gamma}} d\mu^i \right)^{\gamma}}{\left(\int_{\mathcal{I}} \left(\alpha^i\right)^{\frac{1}{\sigma}} d\mu^i \right)^{\sigma}}$$

Taking Stock

- Analytical expression for the representative consumer's preferences
- Nearly identical to the individual agent's preferences
  - Labor Wedge (Chari, Kehoe and McGrattan (2006))
- Ψ_t depends on the distribution of
  - idiosyncratic productivity shocks
  - Welfare weights
- Given $U(C, 1-L; \Psi)$, representative agent's allocation:

$$\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t)^{1-\sigma}}{1-\sigma} + \Psi_t \frac{(1-L_t)^{1-\gamma}}{1-\gamma} \right]$$

$$\text{s.t.} \quad C_t + K_{t+1} = z_t F(K_t, L_t) + (1-\delta) K_t$$

Permanent Income Hypothesis

Next Time

Asset Markets

- Are complete markets a good representation of the data?
- Consider two extremes:
  - Complete Markets
  - Autarky
- Which does the data better support?
- Consider some intermediate case?

Next Time

Asset Markets

- Are complete markets a good representation of the data?
- Consider two extremes:
  - Complete Markets
  - Autarky
- Which does the data better support?
- Consider some intermediate case? ($\leftarrow$ this one)