Demand Uncertainty, Selection, and Trade *

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Abstract

This paper shows that the information available to firms affects participation decision of exporters and therefore the social value of trade. When firms have complete information about profitability in foreign markets, selection into exporting is stringent as firms would not knowingly enter an unprofitable market. With incomplete information, a larger number of firms engage in risky export activity - both firms that will be profitable and unprofitable after learning their demand. Although trade flows are more elastic to changes in variable trade costs under incomplete information, the welfare gains from trade are lower because more (ex post) unprofitable firms enter the market. We obtain these results from a structural trade model that accommodates varying the degree of firm-level uncertainty. We prove the results theoretically, then quantify the magnitude of gains from trade using Brazilian microdata to estimate model parameters, and finally estimate a gravity equation that lends support to the model's testable implications.

Keywords: Demand uncertainty, firm size distribution, extensive margin, selection, trade elasticities, welfare.

JEL: F12, F13.

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1 Introduction

When variable trade costs vary, not only do existing exporters change the size of their shipments abroad, but also the set of exporters varies through entry and exit. Participation decisions for exporters – that is, entry into export markets and the subsequent decision of how intensively to produce and ship goods to foreign destinations – is a conceptually and quantitatively important dimension of trade, and factors affecting these decisions can play a central role in determining the social value of trade for an economy. However, the benchmark framework for measuring the gains from trade considers a special set of circumstances surrounding potential exporters' decisions: firms have complete information about the demand for their products in all foreign markets. While this benchmark has elucidated central mechanisms that drive the gains from trade, particularly the role of selection, less is known about the gains from trade in the empirically more relevant case under which firms face some amount of uncertainty about how profitable their venture into foreign markets will be.

In this paper, we show that uncertainty about demand and profits in foreign markets lowers the social value of trade. This is because uncertainty induces a reallocation of factors of production from ex post profitable firms to ex post unprofitable firms. Since no firm knows whether it will be profitable, both firms that will be profitable and unprofitable after learning their demand in foreign markets choose to export, and consequently generate less net surplus in the aggregate. Accordingly, ignoring the possibility of firm-level uncertainty would lead one to overstate the true magnitude of the welfare gains from trade, and we find this bias is particularly severe for countries and industries in which exporters face high uncertainty about the demand for their product.

We proceed by establishing a structural framework that incorporates uncertainty and can be used to quantitatively evaluate the gains from trade. Specifically, we compare a benchmark trade model in which monopolistically competitive firms possess full information about their profitability prior to making export decisions (e.g., Melitz (2003)), to a variant that incorporates uncertainty similarly to Jovanovic (1982), by requiring firms make export decisions prior to the revelation of their product's demand in foreign markets. In the benchmark model, gains from trade operate through increased production in response to lower costs and new firms' self-selection into export markets. Incorporating uncertainty, however, alters the mechanism governing self-selection into export markets. Firms choose whether and how much to export based on ex ante information and beliefs, and accept risk that the ex post realization of demand for their product could deviate from expectations. Using this structural framework we study the effect of information on the welfare gains from trade in three ways: theoretically, quantitatively and empirically.

Theoretically, we prove that uncertainty lowers welfare gains from trade because the response of new or potential exporters to lower trade costs, which characterizes the effects of uncertainty on trade flows and welfare, is larger when firms' ex post profitability is uncertain. The result follows from recognizing that when firms only observe the ex ante component of their profits, the distribution over firms' information sets is less dispersed than the distribution over information sets for firms that have full information about their profits. When decisions are informed by a less dispersed information set, there is less perceived risk for any individual firm and, as a result, uncertainty creates a larger set of firms who would be now willing to enter export markets in response to lower trade costs. In this case, there is a greater reallocation of factor inputs from ex post profitable firms to ex post unprofitable firms, leading to a larger loss in net surplus due to less stringent self-selection into exporting.

While we show that uncertainty increases the elasticity of trade due to selection, the magnitude of the response is a quantitative issue. Accordingly, we estimate the model using Brazilian microdata on export quantities and export sales, and show that the structurally estimated trade elasticities are systematically larger in the economy with uncertainty than in the complete information economy, on average 7% larger. Therefore, uncertainty amplifies an increase in export sales from new entrants in response to a decline in variable trade costs relative to the model with complete information. We further find that this amplification effect is larger when an export destination exhibits larger demand uncertainty, as measured by the variance of ex post demand shocks. Finally, we use a stylized symmetric two-country example to show that complete information economies overstate the welfare gains from trade and the magnitude of the bias is larger when the variance of ex post demand shocks is larger.

Methodologically, we show that identifying the partial trade elasticities under uncertainty requires data on quantities traded while the complete information case can be identified from sales data alone.[1] We consider a simple identification strategy that relies on the revealed preference of firms. Firms export a greater quantity when they have lower costs or believe destination markets highly demand their exports, and thus we recover the distribution of ex ante heterogeneity from these export decisions.[2] Thus, the sales distribution is simply the ex post realization of these destination-specific demand shocks. In the complete information benchmark, on the other hand, the sales distribution contains all information that reveals firms' information at the time of making export participation decisions. The distri-

[1]While the benchmark model remains close to standard assumptions in the trade literature, we show that our identification strategy also holds in several extensions including quality upgrading (Kugler and Verhoogen (2012)) and variable markups (Melitz and Ottaviano (2008)).

[2]The key identifying assumption is that firms choose quantities and allow prices to clear markets. In Appendix E, we show that our results are robust to allowing firms to choose a price instead, and then committing to export a quantity demanded under that price. In either case, our structural estimation does not make requirements on knowing how much information firms have at the time of their export decision, it simply allows that amount of information is imperfect and there is some uncertainty about outcomes.

butions of quantity exported and sales therefore embody all the relevant information from the perspective of the underlying models.

Finally, we find reduced form empirical evidence for the effect of information uncertainty on the partial trade elasticity. In particular, we perform a series of cross-sectional gravity regressions that control for information availability and demand dispersion.[3] We use ethno-linguistic diversity as a proxy for limited information on demand, which captures the notion that idiosyncratic demand shocks are less observable by exporters in a demographically diverse export destination, since exporters will have less precision in predicting demand from demographics in destinations with greater demographic heterogeneity, and that greater homogeneity in the underlying population would make demand more predictable. We further construct a theory-consistent measure of demand dispersion, where the variation in the dispersion of sales across export destinations occurs solely due to differences in the dispersion of the destination specific demand shocks. We find support for the main theoretical and quantitative results of the paper: (i) trade is more elastic when demand is harder to predict and is therefore more uncertain, and (ii) this elasticity is larger in markets with more dispersed demand.[4]

This paper shows that the information structure faced by firms is crucially important for measuring the extensive margin response to a decline in trade costs. For countries or industries in which exporters face demand uncertainty, assuming away information asymmetries understates the magnitude of extensive margin adjustments to changes in variable trade costs and, therefore, overstates the welfare gains from trade.

This paper is related to several strands of the literature on international trade. First, the benchmark model is based on Melitz (2003) and is further developed in many influential papers, such as Chaney (2008), Bernard, Redding, and Schott (2010), Arkolakis, Costinot, and Rodriguez-Clare (2012), Melitz and Redding (2015). A growing branch of the literature has demonstrated that models incorporating uncertainty along the lines of Jovanovic (1982) are well suited to match salient patterns of empirically observed firm behavior such as firm growth as a function of age and size (Arkolakis, Papageorgiou, and Timoshenko (2018)), firm product switching behavior (Timoshenko (2015b)), and firm input and output pricing be-

[3] We use the distance data from the *CEPII GeoDist* database (Mayer and Zignago, 2011), bilateral aggregate trade data and export sales dispersion data from the World Bank Exporter Dynamics Database (Fernandes et al., 2016), and proxy information availability with ethic, linguistic, and religious diversity scores from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003).

[4] We find that the interaction term on distance and each of the diversity scores is negative and statistically significant, lending support for our quantitative results. The triple interaction term between distance, information uncertainty and dispersion in a standard gravity regression will therefore measure the additional effect of demand dispersion on the amplification effect of uncertainty. We find that the triple interaction term is also negative and in some specifications statistically significant, lending further support for our quantitative results.

havior (Bastos, Dias, and Timoshenko (2018)). Although models that follow the benchmark have focused on decomposing and measuring trade elasticities, the normative implications of models that incorporate uncertainty, particularly for measurements of trade elasticities and the welfare gains from trade, are not yet well understood.[5]

In terms of decomposing trade elasticities, this paper shows that selection into exporting (and hence the extensive margin of trade elasticity), depends on the information structure faced by firms. Previous work has shown that the partial elasticity of trade with respect to variable trade costs can be decomposed into an intensive and an extensive margin of adjustment components (Chaney (2008)), and that the extensive margin adjustment crucially depends on the distributional assumptions with respect to the sources of firm-level heterogeneity (Melitz and Redding (2015)). Sager and Timoshenko (2019) characterize a flexible distribution that well describes firm-level heterogeneity and find the extensive margin trade elasticity to be small. With respect to trade elasticity measurement, this paper uses a structural model with alternative assumptions about information and specifies firm-level data requirements necessary for identification. Existing work focuses on full information benchmarks that estimate trade elasticities using aggregate trade flows and prices data (see Eaton and Kortum (2002) and Simonovska and Waugh (2014)) or trade flows and tariff data (Caliendo and Parro (2015)).[6]

Finally, this paper relates to several related papers on information asymmetries in trade. Recent work uses data and theory to infer information available to firms when making export participation decisions. This work finds that past continuous export history predicts current export choice (Timoshenko (2015a)), and that firm-level sales and industry averages predict exporting for large firms but not for small firms (Dickstein and Morales (2016)). Our findings capture these previous results in a reduced form manner, insofar as firms with positive ex ante information about productivity levels are more likely to be large and export. Furthermore, this paper's results are complementary, as it focuses on understanding the aggregate implications of imperfect information on the gains from trade through the mechanism of export participation. Other recent work considers trade policy uncertainty and the timing of trade investment. Handley and Limao (2015) similarly find that uncertainty lowers welfare, but show that there is less entry into foreign markets when trade policy is uncertain because firms find it costly to engage in irreversible investments. Those authors examine a wait-and-see mechanism with intertemporal uncertainty, while this paper studies a static

[5]A notable exception is Arkolakis, Papageorgiou, and Timoshenko (2018), who characterize constrained efficiency of a model in which firms learn about demand but do not engage in international trade.

[6]This literature further finds elasticities estimated from aggregate trade flows are smaller than those estimated from disaggregated industry-level data (Imbs and Mejean (2015)), and that there is substantial heterogeneity in bilateral trade elasticities due to heterogeneity in countries' industrial production (Imbs and Mejean (2017)).

entry decision in which cross-sectional, idiosyncratic uncertainty is only resolved after a export participation decision is made. Finally, Baley, Veldkamp, and Waugh (2019) develop a model in which firms export more when there is greater uncertainty about the terms of trade in bilateral trade relationships, which can also be thought of as trade policy uncertainty, yet the welfare effects are ambiguous and depend on preferences.[7]

The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 theoretically characterizes the difference between trade elasticities in economies with and without uncertainty, and describes the implications of uncertainty for identifying key model parameters. Section 4 details our estimation procedure for measuring trade elasticities. Section 5 presents estimation results and computes a counterfactual trade liberalization to quantify welfare in the two information environments. Section 6 estimates a gravity equation that controls for information availability and demand dispersion. Section 7 concludes. All proofs, derivations and theoretical elaborations are relegated to the Appendix.[8]

2 Theoretical Framework

This section outlines our main theoretical framework. We consider an economic environment in which heterogeneous firms export products to monopolistically competitive markets. This environment is similar to that in Melitz (2003), and we assume exogenous entry as in Chaney (2008). All derivations are relegated to Appendix A.

There are N countries and K sectors in each country. Each country is indexed by j and each sector is indexed by k.

2.1 Demand

Each country is populated by a mass of L_j identical consumers. Each consumer within country j owns an equal share of domestic firms and is endowed with a unit of labor that is inelastically supplied to the labor market. The preferences of a representative consumer in

[7] There are other papers that consider the effects of information on trade. Bergin and Lin (2012) show that the entry of new varieties increases at the time of the announcement of the future implementation of the European Monetary Union, suggesting that changes in the information available to firms have immediate consequences for firms' decisions; Lewis (2014) studies the effect of exchange rate uncertainly on trade; Allen (2014) shows that information frictions help to explain price variation across locations; Fillat and Garetto (2015) show that aggregate demand fluctuations can explain variation in stock market returns between multinational and non-multinational firms.

[8] Appendix A provides a detailed description of the demand uncertainty model and complete information model. Appendix B provides proofs to propositions. Appendix C incorporates endogenous quality into the model framework, while Appendix D incorporates variable markups, in order to show how quantitative results are robust to alternative assumptions. Appendix E demonstrates that our results under uncertainty are robust to firms choosing price instead of quantities. Appendix F provides model parameter estimates.

country j are represented by a nested constant elasticity of substitution utility function

$$U_j = \prod_{k=1}^{K}\left[\left(\sum_{i=1}^{N}\int_{\omega\in\Omega_{ijk}}\left(e^{z^p_{ijk}(\omega)}\right)^{\frac{1}{\epsilon_k}} c_{ijk}(\omega)^{\frac{\epsilon_k-1}{\epsilon_k}}d\omega\right)^{\frac{\epsilon_k}{\epsilon_k-1}}\right]^{\mu_k}, \tag{1}$$

where Ω_{ijk} is the set of varieties in sector k consumed in country j originating from country i, $c_{ijk}(\omega)$ is the consumption of variety $\omega \in \Omega_{ijk}$, ϵ_k is the elasticity of substitution across varieties within sector k, $z^p_{ijk}(\omega)$ is the demand shock for variety $\omega \in \Omega_{ijk}$, and μ_k is the Cobb-Douglas utility parameter for goods in sector k such that $\sum_{k=1}^{K}\mu_k = 1$.[9]

Cost minimization yields a standard expression for the optimal demand for variety $\omega \in \Omega_{ijk}$, given by

$$c_{ijk}(\omega) = e^{z^p_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}, \tag{2}$$

where $p_{ijk}(\omega)$ is the price of variety $\omega \in \Omega_{ijk}$, Y_{jk} is total expenditures in country j on varieties from sector k, and P_{jk} is the aggregate price index in country j in sector k.[10]

2.2 Supply

Each variety $\omega \in \Omega_{ijk}$ is supplied by a monopolistically competitive firm that has access to a linear production technology that transforms labor into output, $q = \exp(z^a)\ell$. Upon entry, a firm selling from country i to country j in sector k is endowed with an idiosyncratic labor productivity level z^a_{ijk} and a set of idiosyncratic product-destination-sector specific demand shocks, $\{z^p_{ijk}\}_{j=1,\dots,N}$.[11] Productivity and demand shocks are drawn from independent distributions. Firms draw productivity, z^a_{ijk}, from the cumulative distribution function denoted by $G^a_{izk}(z^a_{ijk})$ with associated density function $g^a_{ijk}(.)$, and draw demand, z^p_{ijk}, from the cumulative distribution function denoted by $G^p_{izk}(z^p_{ijk})$ with associated density function $g^p_{ijk}(.)$.

[9] The superscript p in $z^p_{ijk}(\omega)$ stands for an ex post shock. This will become clear when we discuss information structure in Section 2.3.

[10] Due to the assumed Cobb-Douglas utility specification over consumption bundles across sectors, $Y_{jk} = \mu_k Y_j$, where Y_j is aggregate income in country j.

[11] The idiosyncratic demand shocks are realized by consumers, but are a payoff relevant state for the firms. Thus, when firms enter they draw their realization of the idiosyncratic demand of consumers that determines their sales. Following Foster, Haltiwanger, and Syverson (2008), who document that idiosyncratic firm-level demand shocks, rather than productivity, account for a greater variation of sales across firms, we focus on the demand shocks that are firm specific. Each firm from country i draws a separate demand shock for each destination j and sector k. The ijk subscript on z^p_{ijk} therefore indicates that idiosyncratic firm-level demand shocks are origin-destination-sector specific. The subscript does not refer to an aggregate origin-destination-sector level shock that is common across firms. We abstract from such potential aggregate shocks since they do not affect properties of the distribution of quantities or sales across firms, which is the focus of our analysis.

Without loss of generality, we normalize the mean of $g^p_{ijk}(.)$ to zero.[12]

Firms from country i selling output in sector k to country j face fixed costs, f_{ijk}, and variable 'iceberg' trade costs, τ_{ij}. Fixed and variable costs are denominated in units of labor, and w_j denotes the wage rate in country j.

Each firm can potentially supply one variety of a product from each sector. Firms decide in which markets to export (the extensive margin decision) and how much to export to each of the chosen markets (the intensive margin decision). Without loss of generality, we assume that firms choose a quantity to export. Prices are the result of market clearing given the exported quantity of the variety, and then export sales are realized along with prices.[13]

2.3 Information Structure

We consider two information structures: an environment with complete information and an environment with uncertainty.

Complete Information: In the environment with complete information, firms observe all idiosyncratic shocks before making decisions. Namely, firms observe their productivity, z^a_{ijk}, and demand shocks, z^p_{ijk}, before deciding where to export and how much to export. As we demonstrate below, the firm's decision relevant variable in the complete information environment is the profitability shock, denoted by z_{ijk}, which is a linear combination of the productivity and the demand shock:

$$z_{ijk} = \alpha^a z^a_{ijk} + \alpha^p z^p_{ijk}. \tag{3}$$

Uncertainty: In the environment with uncertainty, firms do not observe all idiosyncratic shocks before making decisions. We will refer to the observed component of the profitability shock as an *ex ante* shock, and denote it by z^a_{ijk} so that the superscript a specifies it as an *ex ante* shock. Without loss of generality, we therefore assume that firms observe only their productivity shock, z^a_{ijk}, before deciding where to export and how much to export.

After all export decisions have been made, demand shocks, z^p_{ijk}, are realized and prices clear markets. We will refer to the initially unobserved component of the profitability shock as an *ex post* shock, hence superscript p. Therefore, in this set-up and in contrast to the

[12]Because the distributions are drawn independently, we can normalize the mean of $G^p_{ijk}(.)$ to zero so that the distribution $G^a_{ijk}(.)$ determines the mean of the overall idiosyncratic shock, described below in equation (3).

[13]Alternatively, we can assume that firms choose and commit to a price at which they will export and engage in production. After prices are set, demand shocks are realized and foreign markets order market-clearing quantities. While we conform to the standard learning model of Timoshenko (2015b) by assuming that firms choose quantities, we confirm in Appendix E that both firm-decision setups yields similar results.

complete information environment, the firm's decision relevant variable is the productivity shock, z^a_{ijk}, which is only one component of the profitability shock, z_{ijk}.

In sum, in the environment with complete information, the selection into exporting and the intensive margin decisions will be based on the realization of a profitability shock, z_{ijk}, while in the environment with uncertainty, the same decisions will be based only on the realization of an ex ante component of a profitability shock, z^a_{ijk}. This difference leads to different implications regarding the magnitude of the partial trade elasticities with respect to variable trade costs across information environments, and provides novel insights into which data are suited to structurally identify the partial trade elasticities under different information environments.

2.4 Environment with Complete Information

In the complete information environment, a firm's problem selling from country i to country j in sector k consists of maximizing profit

$$\pi_{ijk}(z_{ijk}) = \max_{q_{ijk}} p_{ijk}(z^p_{ijk})q_{ijk} - \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}} q_{ijk} - w_i f_{ijk}, \tag{4}$$

subject to the demand equation (2). The firm's optimal revenue can be expressed as

$$r_{ijk}(z_{ijk}) = B_{ijk} f^\tau(\tau_{ij}) f^z(z_{ijk}), \tag{5}$$

where B_{ijk} is an origin-destination-sector fixed effect common across firms exclusive of the variable trade costs. Function $f^\tau(.)$ is a strictly monotonically decreasing function, and function $f^z(.)$ is a strictly monotonically increasing function.[14] Variable z_{ijk} is a firm specific idiosyncratic profitability draw, where in the context of equation (3) the weights on the productivity and demand shocks are given by $\alpha^a = (\epsilon_k - 1)$ and $\alpha^p = 1$ respectively.

A firm exports if its profit from exporting is positive: $\pi_{ijk}(z_{ijk}) \geq 0$. Further, CES preferences imply that a firm's variable profit is a constant share of its revenue and is given by $r_{ijk}(z_{ijk})/\epsilon_k$. Hence, selection into exporting is determined by the variable profit being greater than the fixed export cost:

$$r_{ijk}(z_{ijk})/\epsilon_k \geq w_i f_{ijk}. \tag{6}$$

Since revenue function is strictly increasing in profitability, z_{ijk}, equation (6) determines the profitability entry threshold z^*_{ijk} that is implicitly determined by $r_{ijk}(z^*_{ijk})/\epsilon_k = w_i f_{ijk}$. Therefore, a firm exports if $z_{ijk} \geq z^*_{ijk}$, and does not export otherwise.

[14]Specifically, $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}$, $f^\tau(\tau_{ij}) = \tau_{ij}^{1-\epsilon_k}$, and $f^z(z_{ijk}) = e^{z_{ijk}}$.

Trade Elasticity: Given the endogenous selection into exporting that is based on the realization of profitability shocks, the total trade flows from country i to country j in sector k, X_{ijk}, are defined as

$$X_{ijk} = J_i(1 - G^z_{ijk}(z^*_{ijk})) \int_{z^*_{ijk}}^{+\infty} B_{ijk} f^\tau(\tau_{ij}) f^z(z) \frac{g^z_{ijk}(z)}{1 - G^z_{ijk}(z^*_{ijk})} dz, \tag{7}$$

where J_i is the exogenous mass of potential entrants in country i, and $g^z_{ijk}(.)$ and $G^z_{ijk}(.)$ are the probability and the cumulative distribution functions of the profitability shock z_{ijk}.

The partial elasticity of trade with respect to the iceberg trade costs, τ_{ij}, can then be written as

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} = \underbrace{\frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}}}_{\substack{\text{level of the} \\ \text{partial trade elasticity}}} \left(\underbrace{1}_{\substack{\text{intensive margin} \\ \text{contribution}}} + \underbrace{\frac{e^{z^*_{ijk}} g^z_{ijk}(z^*_{ijk})}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz}}_{\substack{\text{extensive margin} \\ \text{contribution}}} \right). \tag{8}$$

Notice that since function $f^\tau(.)$ is a strictly decreasing function, $\partial \log f^\tau(\tau_{ij}) / \partial \log \tau_{ij} < 0$, which implies that the partial trade elasticity is negative, as expected.

2.5 Environment with Uncertainty

In an environment with uncertainty, a firm from country i chooses the quantity it will export to country j in sector k in order to maximize its *expected* profit

$$E_{z^p_{ijk}}[\pi_{ijk}(z^a_{ijk}, z^p_{ijk})] = \max_{q_{ijk}} E_{z^p_{ijk}} \left(p_{ijk}(z^p_{ijk}) q_{ijk} - \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}} q_{ijk} \right) - w_i f_{ijk} \tag{9}$$

subject to the demand equation (2). Notice that in contrast to the firm's problem under complete information presented in equation (4), under uncertainty, a firm's decision relevant variable is the observed ex ante component, z^a_{ijk}, of the profitability shock, z_{ijk}. The expectation is then computed based on the distribution of the unobserved ex post component, z^p_{ijk}, of the profitability shock, z_{ijk}.

The resulting realized revenue of the firm can be written as

$$r_{ijk}(z^a_{ijk}, z^p_{ijk}) = B_{ijk} f^\tau(\tau_{ij}) f^p(z^p_{ijk}) f^a(z^a_{ijk}), \tag{10}$$

where B_{ijk} is an origin-destination-sector fixed effect common across firms exclusive of the variable trade costs. Functions $f^p(.)$ and $f^a(.)$ are strictly monotonically increasing functions.[15]

[15]Specifically, $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{1-\epsilon_k} \left(E_{z^p_{ijk}} \left(e^{\frac{z^p_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k - 1}$, $f^p(z^p_{ijk}) = e^{\frac{z^p_{ijk}}{\epsilon_k}}$, and $f^a(z^a_{ijk}) =$

A firm exports if its expected profit from exporting is positive. Hence, selection into exporting occurs when expected profit is greater than zero:

$$E_{z^p_{ijk}}[\pi_{ijk}(z^a_{ijk}, z^p_{ijk})] \geq 0. \tag{11}$$

When held with equality, equation (11) determines the entry threshold z^{a*}_{ijk} such that a firm exports if $z^a_{ijk} \geq z^{a*}_{ijk}$, and does not export otherwise. Hence, selection into exporting under uncertainty is determined solely by the realization of an ex ante component of the profitability shock.

Trade Elasticity: Given that selection into exporting is based on the realization of the *ex ante* component, z^a_{ijk}, of the profitability shock, z_{ijk}, the total trade flows from country i to country j in sector k, X_{ijk}, are given by

$$X_{ijk} = J_i(1 - G^a_{ijk}(z^{a*}_{ijk})) \int_{z^{a*}_{ijk}}^{+\infty} \left[\int_{-\infty}^{+\infty} B_{ijk} f^p(z^p_{ijk}) f^\tau(\tau_{ij}) f^a(z) g^p_{ijk}(z^p_{ijk}) dz^p_{ijk} \right] \frac{g^a_{ijk}(z)}{1 - G^a_{ijk}(z^{a*}_{ijk})} dz,$$

where $g^a_{ijk}(.)$ and $G^a_{ijk}(.)$ are the probability and the cumulative distribution functions of the ex ante component, z^a_{ijk}, of the profitability shock, z_{ijk}.

The partial elasticity of trade with respect to the iceberg trade costs, τ_{ij}, can then be written as

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} = \underbrace{\frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}}}_{\substack{\text{level of the} \\ \text{partial trade elasticity}}} \left(\underbrace{1}_{\substack{\text{intensive margin} \\ \text{contribution}}} + \underbrace{\frac{e^{\bar{z}^{a*}_{ijk}} g^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk})}{\int_{\bar{z}^{a*}_{ijk}}^{+\infty} e^z g^{\bar{z},a}_{ijk}(z) dz}}_{\substack{\text{extensive margin} \\ \text{contribution}}} \right), \tag{12}$$

where $\bar{z}^a_{ijk}$ is the weighted ex ante component of the profitability shock, and $\bar{z}^{a*}_{ijk}$ and $g^{\bar{z},a}_{ijk}(.)$ are the corresponding entry threshold and the probability density function. Corresponding with equation (3), $\bar{z}^a_{ijk} \equiv \alpha^a z^a_{ijk}$, where $\alpha^a = (\epsilon_k - 1)$.

3 Characterization of Trade Elasticities

In this section we develop two main theoretical results. In Section 3.1, we prove that uncertainty increases the partial trade elasticity because the extensive margin contribution of the partial trade elasticity, which captures the effect of uncertainty on trade flows, is larger when firm's ex post component of the profitability shock is uncertain. In Section 3.2, we show that identification of the extensive margin contribution to the partial trade elasticity requires different data in the two environments. Under complete information export sales

$e^{(\epsilon_k - 1) z^a_{ijk}}$.

data identify the elasticity, while under uncertainty identification requires the use of data on quantities exported. Lastly, in Section 3.3 we show that the identification strategy is robust to alternate model assumptions. We demonstrate that the identification result holds in economies with various frictions including endogenous quality choice (Kugler and Verhoogen, 2012) and variable markups (Melitz and Ottaviano, 2008).

3.1 Size of Trade Elasticities Across Information Environments

The partial elasticities of trade flows with respect to variable trade costs in the environments with complete information and with uncertainty are defined in equations (8) and (12) respectively. Notice that in both information environments the partial trade elasticity depends on an entry threshold and the corresponding distribution of the underlying state variable. Accordingly, the partial trade elasticity admits the same functional form across the two information environments,

$$\frac{\partial \ln X_{ijk}}{\partial \tau_{ij}}(x, g^x(x)) \quad = \quad \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(1 + \gamma(x)\right), \tag{13}$$

where the extensive margin contribution is

$$\gamma(x) \equiv \frac{e^x g^x(x)}{\int_x^{+\infty} e^u g^x(u) du}. \tag{14}$$

The variable x is the entry threshold, which is z^*_{ijk} in the complete information environment and is $\bar{z}^{a*}_{ijk}$ in the environment with uncertainty. Furthermore, $g^x(.)$ is the probability density function of the underlying decision-relevant idiosyncratic shock, and is given by the distribution, $g^z_{ijk}(.)$, of the profitability shock, z_{ijk}, in the complete information economy and is given by the distribution, $g^{\bar{z},a}_{ijk}(.)$, of the weighted ex ante component of the profitability shock, $\bar{z}^a_{ijk} = \alpha^a z^a_{ijk}$, in the environment with uncertainty.

Equations (13) and (14) make it clear that the difference in the partial trade elasticities across the two information environments arises solely from differences in the extensive margin component, $\gamma(x)$. We next show that the extensive margin of the partial trade elasticity is larger under uncertainty than under complete information, which is summarized as Result 1 below.[16]

Result 1: *Under a mild set of assumptions, the extensive margin of the partial trade elasticity is larger under uncertainty compared to the complete information.*

Conceptually, the result follows from a straightforward application (under mild conditions) of seminal characterizations of uncertainty, specifically that if a distribution second

[16] See Appendix B for a proof of Result 1.

order stochastically dominates another then the dominant distribution has a higher mean over any concave function of the random variable. Given that the profitability shock, z_{ijk}, is a mean-preserving spread of the weighted ex ante component of the profitability, $\bar{z}^a_{ijk}$, the hazard rate associated with the profitability distribution is lower than the hazard rate associated with the distribution the weighted ex ante component of the profitability. This insight allows us to conclude that the extensive margin of the partial trade elasticity, and therefore the overall partial trade elasticity, is higher in the environment with uncertainty than with complete information. Thus, trade is more elastic when firms face greater uncertainty which subsequently lowers welfare gains from trade all else equal.

To compare the extensive margin component, $\gamma(x)$, across the two information environments notice that $\gamma(x)$ can further be expressed as a hazard rate

$$\gamma(x) = \frac{h(x)}{1 - H(x)}, \tag{15}$$

where the probability density function $h(x)$ is defined as

$$h(x) \equiv \frac{e^x g^x(x)}{\displaystyle\int_{-\infty}^{+\infty} e^u g^x(u) du}, \tag{16}$$

and the corresponding cumulative distribution function is defined as

$$H(x) \equiv \frac{\displaystyle\int_{-\infty}^{x} e^u g^x(u) du}{\displaystyle\int_{-\infty}^{+\infty} e^u g^x(u) du}. \tag{17}$$

Result 1 relies on two properties of the hazard rate function. The first property is that the hazard rate is a monotonically increasing function. The second property is that for an increasing hazard rate function, above a certain value, the hazard rate associated with a mean preserving spread of a distribution is lower than the hazard rate associated with the base-line distribution. We relegate the rigorous proofs of these properties and Result 1 to Appendix B. Here, we discuss what those properties mean for the distributions of the idiosyncratic firm-level shocks, and provide a basic intuition for Result 1.

Assumption 1 below ensures that the extensive margin elasticity, $\gamma(x)$, is a monotonically increasing hazard rate function.

Assumption 1 (A1) *The probability density function $g^x(x)$ has the following properties:*

(i) $x \in \mathbb{R}$ is the support of the distribution,

(ii) $E(e^x) \equiv \int_{-\infty}^{+\infty} e^u g^x(u) du$ exists and is finite, and

(iii) the function $\log\left(\int_x^{+\infty} e^u g^x(u)du\right)$ *is concave in* x*.*

Assumption (i) is trivial and Assumption (ii) ensures that the probability and the cumulative distribution functions $h(.)$ and $H(.)$ are well defined. Assumption (iii) ensures that function $\gamma(x)$ is a monotonically increasing function of x. Assumption (iii) states that the log of the conditional expectation of an exponential function is a concave function of the threshold value. Intuitively, this assumption requires that the upper tail of the distribution $g^x(x)$ does not have too much mass.[17] Without such a restriction, total sales of marginal firms relative to average sales could become very small as the threshold increases, and the extensive margin elasticity, $\gamma(x)$, might not be monotonically increasing in x. The standard distributional assumptions made in the literature all meet this requirement.[18]

Next, notice from equation (3) that the profitability shock z_{ijk} is a mean preserving spread of the weighted ex ante component of profitability $\bar{z}^a_{ijk} = \alpha^a z^a_{ijk}$. Therefore, distribution $g^z(.)$ is a mean preserving spread of distribution $g^{\bar{z},a}(.)$. As a result, the corresponding cumulative distribution function, $G^z(.)$, crosses $G^{\bar{z},a}(.)$ once from above. This single-crossing property of the cumulative distribution functions is preserved by the transformations in equations (16) and (17), and subsequently by the corresponding hazard rate function defined by equation (15). Hence, the entire hazard rate function $\gamma(x)$ under complete information crosses the hazard rate function under uncertainty once from above. Therefore, provided the corresponding entry thresholds are sufficiently high, the extensive margin of the partial trade elasticity under complete information is lower than under uncertainty.

To summarize, trade is less elastic under complete information because the distribution of the shock that determines the partial trade elasticity is more dispersed under complete information than under uncertainty. Under uncertainty, trade elasticity is determined by the distribution of the ex ante component of the profitability shock, while under complete information trade elasticity is determined by the distribution of the entire profitability shock, a mean preserving spread of the ex ante component. As a result, the associated hazard rate (the extensive margin of the partial trade elasticity), and consequently the partial trade elasticity, is lower under complete information than under uncertainty.

3.2 Identification of Trade Elasticities

Equations (13) and (14) inform what data are needed to structurally identify the partial trade elasticities. First, notice that the overall level of the partial trade elasticity is determined

[17] Heavy-tailed distributions, e.g. distributions that violate assumption (ii), are sometimes said to have the property of log-convexity.

[18] For example, the Normal distribution, Exponential distribution (with an appropriate restriction on the scale parameter) and the Double Exponentially Modified Gaussian distribution all satisfy this requirement.

by the direct effect of changes in variable trade costs on the sales of individual exporters, the intensive margin of trade elasticity $\partial \log f^{\tau}(\tau_{ij})/\partial \log \tau_{ij}$. This component does not depend on the information structure. In the model with CES preferences considered here, $\partial \log f^{\tau}(\tau_{ij})/\partial \log \tau_{ij} = (1 - \epsilon_k)$, and hence is entirely determined by the preferences.

Second, the intensive margin is then augmented by the changes in trade flows generated by the selection mechanism, the extensive margin of trade represented by $\gamma(x)$. The extensive margin of trade indicates changes in trade flows due to entry and exit of new exporters. Observe that the extensive margin of the partial trade elasticity is entirely dependent on the entry threshold, x, and the distribution of the underlying idiosyncratic shock associated with the entry threshold, $g^x(.)$. Hence, to structurally estimate the partial trade elasticity, more specifically the extensive margin component, one needs to estimate the distribution governing export decision-relevant idiosyncratic shocks and the respective selection threshold. How these two objects can be recovered from the data crucially depends on the information environment, as we now explain in detail.

3.2.1 Complete Information

In the the complete information environment, the profitability entry threshold z^*_{ijk} and the distribution of the profitability shock $g^z_{ijk}(.)$ can be directly recovered from export sales data. Consider export sales equation (5). The logarithm of equation (5) yields $\log r_{ijk}(z_{ijk}) = \log B_{ijk} + \log f^{\tau}(\tau_{ij}) + \log f^z(z_{ijk})$. In our set-up, the functional form $f^z(z) = e^z$. Hence, the logarithn of export sales is given by

$$\log r_{ijk}(z_{ijk}) = \log B_{ijk} + \log f^{\tau}(\tau_{ij}) + z_{ijk}, \tag{18}$$

if $z_{ijk} \geq z^*_{ijk}$. From equation (18) it is clear that the cross-sectional distribution of export sales, mimics the distribution of the underlying idiosyncratic profitability shocks augmented by a constant. Hence, by fitting a parametric distribution to the cross-sectional distribution of log-export sales of firms from country i in destination j and sector k, one can directly recover the distribution $g^z_{ijk}(.)$, up to a mean.[19] The entry threshold z^*_{ijk} can subsequently be recovered by matching the model-implied average-to-minimum ratio of export sales to that in the data, following the methodology of Bas et al. (2017):

$$\text{Average-to-Minimum Ratio Sales} \;\; = \;\; e^{-z^*_{ijk}} \int_{z^*_{ijk}}^{+\infty} \frac{e^z g^z_{ijk}(z)}{Prob^z_{ijk}(z > z^*_{ijk})} dz. \tag{19}$$

[19] Given that the hazard rate of a distribution does not depend on the mean of that distribution, the mean can further be normalized.

3.2.2 Uncertainty

In the model with uncertainty, sales alone are no longer sufficient to identify the partial trade elasticity. Based on the partial trade elasticity equation (8), under uncertainty one needs to recover the weighted ex ante profitability entry threshold, $\bar{z}^{a*}_{ijk}$, and the corresponding distribution $g^{\bar{z},a}_{ijk}(.)$. Consider following the same steps as under complete information and take the logarithm of the export sales equation (10):

$$\log r_{ijk}(z^a_{ijk}, z^p_{ijk}) = \log B_{ijk} + \log f^{\tau}(\tau_{ij}) + (1/\epsilon_k)\, z^p_{ijk} + \bar{z}^a_{ijk}, \tag{20}$$

if $\bar{z}^a_{ijk} \geq \bar{z}^{a*}_{ijk}$. There are two distinct idiosyncratic shocks that determine the distribution of log-export sales: the weighted ex ante shock $\bar{z}^a_{ijk}$ and the ex post shock z^p_{ijk}. Only one of those shocks is relevant for the identification of the partial trade elasticity under uncertainty: the weighted ex ante shock $\bar{z}^a_{ijk}$.

Hence, under uncertainty, the one-to-one mapping between the decision relevant shocks (weighted ex ante shock $\bar{z}^a_{ijk}$) and sales is broken, and sales data alone are insufficient to identify the partial trade elasticity. In fact, it is only the ex ante portion of sales that is needed to infer partial trade elasticities. To extract the ex ante component of sales, additional data are needed. The structure of the model can further be used to guide the identification strategy.

Recall from the firm's problem described in equation (9) that firms choose quantities based on the realization of ex ante shocks. Hence, the quantity data can contain information regarding the needed distribution of the ex ante shocks. The optimal quantity of a firm is given by

$$q_{ijk}(\bar{z}^a_{ijk}) = B_{ijk} f^{\tau,q}(\tau_{ij}) f^{\bar{z},a}(\bar{z}^a_{ijk}), \tag{21}$$

where B_{ijk} is an origin-destination-sector fixed effect common across firms exclusive of the variable trade costs, and the functions $f^{\tau,q}(\tau_{ij}) = \tau_{ij}^{-\epsilon_k}$ and $f^{\bar{z},a}(\bar{z}^a_{ijk}) = e^{\frac{\epsilon_k}{\epsilon_k - 1}\bar{z}^a_{ijk}}$.[20] The logarithm of equation (21) therefore yields

$$\log q_{ijk}(\bar{z}^a_{ijk}) = \log B_{ijk} + \log f^{\tau,q}(\tau_{ij}) + \frac{\epsilon_k}{\epsilon_k - 1}\bar{z}^a_{ijk}, \tag{22}$$

if $\bar{z}^a_{ijk} > \bar{z}^{a*}_{ijk}$. Hence, for a given value of the elasticity of substitution ϵ_k, the cross-sectional distribution of log-export quantity data completely recovers the underlying distribution of the decision-relevant shock under uncertainty: the weighted ex ante component of the profitability shock $\bar{z}^a_{ijk}$. The entry threshold $\bar{z}^{a*}_{ijk,a}$ can then be recovered by matching the model-

[20]Specifically, $B_{ijk} = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} w_i^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \left(E_{z^p_{ijk}}\left(e^{\frac{z^p_{ijk}}{\epsilon_k}}\right)\right)^{\epsilon_k}$.

implied average-to-minimum ratio of export *quantity* to that in the data:

$$\text{Average-to-Minimum Ratio Quantity} \;=\; e^{-\frac{\epsilon_k}{\epsilon_k-1}\bar{z}^{a*}_{ijk}} \int_{\bar{z}^{a*}_{ijk}}^{+\infty} \frac{e^{\frac{\epsilon_k}{\epsilon_k-1}z} g^{\bar{z},a}_{ijk}(z)}{Prob^{\bar{z},a}_{ijk}(z > \bar{z}^{a*}_{ijk})} dz. \quad (23)$$

To summarize, quantity data identifies the extensive margin contribution to the partial trade elasticity by enabling inference about the ex ante component of the profitability shock. In contrast, it is only appropriate to identify the partial trade elasticity using export sales data when firms possess complete information about their demand.

3.3 Robustness

The finding that the identification of the partial trade elasticity relies on using the quantity data under uncertainty and on using the sales data under complete information is more general than appears at first. The finding also holds in economies with various frictions. In Appendix C and D we formally demonstrate the robustness of this result with respect to two common frictions: endogenous quality choice and variable markups respectively. It continues to hold that, regardless of the frictions, quantity versus sales data contain separate information about ex ante versus ex post shocks that are needed to identify the partial trade elasticities under each of the information environments. Below, we succinctly demonstrate these insights.

3.3.1 Endogenous Quality Choice

In the context of a model with endogenous quality choice along the lines Kugler and Verhoogen (2012), the partial elasticity of trade with respect to variable trade cost admits the same general functional form as in equations (8) and (12). With endogenous quality, function $f^{\tau}(.)$ is augmented with parameters of the quality production function, yet preserves the monotonically decreasing property.[21] Hence, the overall level of trade elasticity in a model with endogenous quality choice may be different from the baseline model regardless of the information environments. The distinction in the partial trade elasticities between information environments, however, arises from the extensive margin component that depends on the distribution of profitability shocks versus the weighted ex ante profitability shocks that are identified using sales versus quantity data.

Complete Information: In Kugler and Verhoogen (2012), under complete information, the revenue function admits the same general functional for as in the baseline model equation

[21] Specifically, $f^{\tau}(\tau_{ij}) = \tau_{ij}^{\frac{(1-\epsilon_k)\beta_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}}$, where β_k and γ_k are parameters of the quality production function. See Appendix C for details.

(5). The revenue similarly depends on the profitability shock, z_{ijk}. The profitability shock is defined as the weighted sum of the ex ante and the ex post shocks, as in equation (3). The only distinction from the baseline model is that the weights α^a and α^p are augmented with the parameters of the quality production function.[22] Given that these weights do not impact the relation between the profitability shocks and log-export sales and do not factor into the estimation of the extensive margin of the partial trade elasticity, the sales data alone continues to identify the extensive margin of the partial trade elasticity under complete information in a model with endogenous quality choice in exactly the same way as in the baseline model.

Uncertainty: Under uncertainty, the log-export sales data embeds both the ex ante and ex post shocks, while only weighted ex ante shocks determine selection in exporting and the extensive margin of the partial trade elasticity. Hence, similarly to the baseline model, one needs the log-quantity data to recover the underlying distribution of the weighted ex ante shock. With endogenous quality choice, the weight on the ex ante shock is augmented with the quality production function parameters. This weight however does not impact the relation between the weighted ex ante shocks and the log-export quantity.

Hence, from the perspective of structural estimation of the partial trade elasticities, the two frameworks are isomorphic.

3.3.2 Variable Markups

To examine the implications for identification of trade elasticities in the presence of variable markups, consider a model along the lines of Melitz and Ottaviano (2008). To generate variable markups, the model assumes quadratic preferences in the differentiated goods sector. This assumption breaks the log-linearity of the revenue and quantity functions in the profitability shocks. Hence, the estimation strategy that we developed for the CES economy has to be altered. In this section, we demonstrate that the identification of the partial trade elasticity relies on using the sales (and, in the case of quadratic preferences, quantity) data under complete information, and relies on using the quantity data alone under uncertainty, as suggested in the baseline model, and leave the development of the estimation method for our future work. To ease explication, we abstract from multi-sector environment and consider a model with one differentiated goods sector.

Complete Information: In Melitz and Ottaviano (2008), under complete information, the

[22]Specifically, $\alpha^a = (\epsilon_k - 1)\beta_k/[\beta_k - (1-\gamma_k)(\epsilon_k - 1)]$ and $\alpha^p = \beta_k/\beta_k/[\beta_k - (1-\gamma_k)(\epsilon_k - 1)$, where β_k and γ_k are parameters of the quality production function. See Appendix C for details.

partial trade elasticity can be expressed as follows:

$$\frac{\partial \log X_{ij}}{\partial \log \tau_{ij}} = -2\left(\frac{\int_{-\infty}^{+\infty}(u_{ij}^{p})^2 G_{ij}^{a}(u_{ij}^{p}/\tau_{ij})g_{ij}^{p}(u_{ij}^{p})du_{ij}^{p}}{\int_{-\infty}^{+\infty}(\tau_{ij}u_{ij}^{a})^2\left(1-G_{ij}^{p}(\tau_{ij}u_{ij}^{a})\right)g_{ij}^{a}(u_{ij}^{a})du_{ij}^{a}}-1\right)^{-1}, \tag{24}$$

where u_{ij}^{a} and u_{ij}^{p} are monotonic transformations of the respective ex ante and ex post profitability shocks, $g_{ij}^{a}(.)(G_{ij}^{a}(.))$ and $g_{ij}^{p}(.)(G_{ij}^{p}(.))$ are the probability (cumulative) density functions of u_{ij}^{a} and u_{ij}^{p} respectively. Hence, to estimate the trade elasticity, one needs to recover the distributions of both of the profitability shocks. That information is contained in the sales and quantity given by

$$r_{ij}(u_{ij}^{p}, u_{ij}^{a}) = \frac{L_j}{4\gamma}\left((u_{ij}^{p})^2-(\tau_{ij}u_{ij}^{a})^2\right), \tag{25}$$

$$q_{ij}(u_{ij}^{p}, u_{ij}^{a}) = \frac{L_j}{2\gamma}\left(u_{ij}^{p}-\tau_{ij}u_{ij}^{a}\right), \tag{26}$$

where L_j is the country j's population size, and γ is a parameter of the utility function. According to equations (25) and (26), the distributions of the needed ex ante, u_{ij}^{a}, and ex post, u_{ij}^{p}, shocks can be recovered from the joint distribution of the sales and quantity data via the respective transformations by assuming a parametric distribution for the underlying shocks and using the maximum likelihood estimation method.

Uncertainty: Under uncertainty, the partial trade elasticity can be expressed as follows:

$$\frac{\partial \log X_{ij}}{\partial \log \tau_{ij}} = -2\left(\frac{\int_{u_{ij}^{a*}}^{+\infty}\left(\frac{u_{ij}^{a}}{u_{ij}^{a*}}\right)^2 g_{ij}^{a}(u_{ij}^{a})du_{ij}^{a}}{1-G_{ij}^{a}(u_{ij}^{a*})}-1\right)^{-1}. \tag{27}$$

In contrast to the complete information environment, under uncertainty, the partial trade elasticity depends only on the distribution of the ex ante profitability shock u_{ij}^{a} and the respective entry threshold, u_{ij}^{a*}, similar to the results in our benchmark economy with uncertainty. Both of these objects can then be recovered from the export quantity given by

$$q_{ij}(u_{ij}^{a}) = \frac{L_j}{2\gamma}\left(E(u_{ij}^{p})-\tau_{ij}u_{ij}^{a}\right). \tag{28}$$

In sum, the main elasticity identification message developed in the baseline model in Section 3 also holds in the environment with variable markups: different data identify trade elasticities under complete versus incomplete information environments. In the Melitz and Ottaviano (2008) framework considered in this section, one requires quantity and sales data to identify trade elasticities under complete information environment, and one requires quantity data alone to identify trade elasticities under uncertainty.

4 Quantifying Trade Elasticities

In this section we use data across Brazilian exporter on the distribution of export quantities and sales by destination-industry over time to quantify trade elasticities between an environment with uncertainty and complete information. Accordingly, Section 4.1 describes the data and presents sample summary statistics. To proceed, Section 4.2 describes four steps to implement the elasticity identification strategy described in Section 3.2: a choice of a parametric distribution for the decision-relevant shocks, a method to estimate the parametric distribution, a method to correct for endogenous selection, and a normalization of values of the elasticity of substitution across varieties. Section 4.3 provides estimates of trade elasticities.

4.1 Data

The data come from the Brazilian customs declarations collected by SECEX (*Secretaria de Comercio Exterior*).[23] The data record export value and weight (in kilograms) of the shipments at the firm-product-destination-year level. A product is defined at the 6-digit Harmonized Tariff System (HS) level. We use the data for the period between 1997 and 2000, when both the sales and the weight data are available.

We proxy the theoretical notion of export quantity with an empirical measure of export weight.[24] The properties of export weight differ substantially across industries. Hence, we further conduct our analysis at the destination-year-industry level where we define an industry as a 6-digit HS code.

We define an observation to be a distribution of export quantity or sales across firms for a given destination-year-industry triplet, and focus on observations where at least 100 firms export.[25]

The final sample consists of 190 destination-year-industry observations, and covers 14 destinations and 35 industries.[26] Table 1 provides summary statistics of log-export quantities and log-export sales distributions in our sample.

[23]For a detailed description of the dataset see Molinaz and Muendler (2013). The data have further been used in Flach (2016) and Flach and Janeba (2017).

[24]Export weight is used as a measure of export quantity in a number of studies including Bastos et al. (2018).

[25]The thresholds of 100 firms ensures that an empirical distribution can be accurately described by percentiles. This threshold is also consistent with the literature. See Fernandes et al. (2015), Sager and Timoshenko (2019).

[26]We note that there are 232,553 destination-year-industry observations in the entire data-set. We focus on the samll sub-sample of 190 observations where at least 100 firms export. Among the remaining 232,363 observations where less than 100 firms export, the median and the average number of exporters is unity and 3.3 respectively. Hence, these markets are unlikely to be characterized by a monopolistic competition environment, and the forces of endogenous market selection that we seek to identify in our paper would not apply.

4.2 Elasticity Estimation Procedure

4.2.1 Parameterizing Distributions

To proceed with estimating the partial trade elasticities we, first, need to parametrize the distribution, $g^{z}_{ijk}(.)$, of the profitability shock, z_{ijk}, and hence the logarithm of export sales, in a model with complete information, and the distribution, $g^{\bar{z},a}_{ijk}(.)$, of the ex ante component of the profitability shock, $\bar{z}^{a}_{ijk}$, and hence the logarithm of export quantity, in a model with uncertainty.

The majority of the trade literature has relied on either a Pareto distribution (Axtell, 2001; Chaney, 2008) or a log-Normal distribution (Bas et al., 2017; Fernandes et al., 2015) in modeling firm level heterogeneity.[27] Note that the logarithm of a Pareto distribution follows an exponential distribution and the logarithm of a log-Normal follows a Normal distribution. In their recent work, Sager and Timoshenko (2019) have shown that a Double Exponentially Modified Gaussian (EMG) distribution (which is derived from the sum of two independent random variables that distributed according to a Double Exponential and a Normal distribution) provides a superior fit to the empirical distribution of the logarithm of export sales compared to an Exponential or a Normal alone.

Hence we proceed by parameterizing distributions $g^{z}_{ijk}(.)$ and $g^{\bar{z},a}_{ijk}(.)$ with a Double EMG distribution, $DEMG(\mu, \sigma^2, \lambda_L, \lambda_R)$, described by the following cumulative distribution function:

$$G(z) = \Phi\left(\frac{z-\mu}{\sigma}\right) - \frac{\lambda_L}{\lambda_L+\lambda_R} e^{-\lambda_R(z-\mu)+\frac{\sigma^2}{2}\lambda_R^2}\Phi\left(\frac{z-\mu}{\sigma}-\lambda_R\sigma\right) + \frac{\lambda_R}{\lambda_L+\lambda_R} e^{\lambda_L(z-\mu)+\frac{\sigma^2}{2}\lambda_L^2}\Phi\left(-\frac{z-\mu}{\sigma}-\lambda_L\sigma\right), \tag{29}$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.[28]

The Double EMG distribution provides a very flexible generalization of common distributional assumptions used in the literature. From equation (29), for example, as $\sigma \to 0$ and $\lambda_L \to 0$, the Double EMG distribution converges to an Exponential (Pareto) distribution, as assumed in Chaney (2008). As $\lambda_L \to +\infty$ and $\lambda_R \to +\infty$, the Double EMG distribution converges to a Normal distribution, as assumed in Bas et al. (2017) and Fernandes et al. (2015). As $\sigma \to 0$, the Double EMG converges to a Double Exponential (Pareto) distribution. By assuming the Double EMG distribution we, therefore, allow the data to recover the best fit of distribution between the Exponential, Normal, Double Exponential or the corresponding convolutions. We estimate parameters of the Double EMG distribution separately for each of the log-export sales and the log-export quantity observations in our sample.

[27] A notable exception includes Nigai (2017) who assumes a mixture of log-Normal and Pareto distributions.

[28] For notational compactness we drop the ijk subscripts in this section.

4.2.2 Distribution Estimation Method

We follow Sager and Timoshenko (2019) in estimating the parameters of the Double EMG distribution using a Generalized Method of Moments (GMM) procedure that minimizes the sum of squared residuals,

$$\min_{(\mu,\sigma^2,\lambda_L,\lambda_R)} \sum_{i=1}^{N_P} \left(x_i^{data} - x_i(\mu,\sigma^2,\lambda_L,\lambda_R)\right)^2,$$

where x_i^{data} is the i-th percentile of the empirical log-export sales or quantity distribution for a given destination-year-industry, $x_i(\mu,\sigma^2,\lambda_L,\lambda_R)$ is the model implied i-th percentile for given parameters $(\mu,\sigma^2,\lambda_L,\lambda_R)$, and N_P is the number of percentiles used in estimation. We use the 1st through 99th percentiles of the empirical log-export sales or quantity distribution to estimate parameters. In practice, this choice eases computational burden compared to using each data point, without significantly changing the parameter estimates we recover. Furthermore, note that choosing parameters to minimize the sum of squared residuals is equivalent to Head et al.'s (2014) method of recovering parameters from quantile regressions.

Hence, for each destination-year-industry observation, we choose distribution parameters $(\mu,\sigma^2,\lambda_L,\lambda_R)$ so that the percentiles of the theoretical log-export sales or quantity distribution match the percentiles of the empirical log-export sales or quantity distribution.

4.2.3 Correcting for Endogenous Selection

In fitting a distribution to the log-export sales or quantity data according to equations (18) or (22), it is important to note that the model implies truncation in the data. Namely, in a model with complete information, equation (18) holds only when $z_{ijk} \geq z^*_{ijk}$ or when $\log r_{ijk} \geq \log r^*_{ijk}$; in a model with uncertainty, equation (22) holds only when $\bar{z}^a_{ijk} > \bar{z}^{a*}_{ijk}$ or when $\log q_{ijk} \geq \log q^*_{ijk}$.

To account for the endogenous selection into exporting, we follow the approach by Sager and Timoshenko (2019). Namely, we proceed by fitting a truncated probability distribution function $g^z_{ijk}(.)$ $(g^{\bar{z},a}_{ijk}(.))$ to the log-export sales (quantity) data and take the truncation point $\log r^*_{ijk}$ $(\log q^*_{ijk})$ to be given by the zeroth percentile of the corresponding log-export sales (quantity) distribution.

4.2.4 Values for ϵ_k

Finally, our quantification method requires assuming values for the elasticities of substitution across varieties, ϵ_k. They are needed to pin down the level of the partial trade elasticity, $\partial \log f^\tau(\tau_{ij})/\partial \log \tau_{ij}$, which, under the assumption of the CES preferences, is given by $(1-\epsilon_k)$;

to recover parameters of the distribution of the weighted ex ante profitability shock according to equation (22); and to solve for the entry thresholds according to equation (23) .

We proceed by using the values of the elasticities of substitution across varieties, ϵ_k, from Soderbery (2015), which refines estimates in Feenstra (1994) and Broda and Weinstein (2006).[29] It must be acknowledged that Soderbery (2015) estimates the elasticities of substitution across varieties under the assumption of complete information. It is plausible that the empirical estimates of elasticities of substitution across varieties are also endogenous with respect to the assumption about the information environment. Hence, to quantify the partial elasticities of trade under uncertainty, one would potentially need to re-estimate the elasticities of substitution across varieties in a model with uncertainty. This, however, is not necessary for our purposes.

In accordance with our modeling assumptions, the elasticities of substitution across varieties are parameters of the utility function that remain unchanged across information environments. We seek to quantify differences in the trade elasticities across information environments that arise solely from the differences in the selection mechanism. Hence it is crucial for our estimation methodology to keep all other potential sources of variation in trade elasticities unchanged. Do to so, we must assume some values for the elasticities of substitution across varieties and keep those values the same when quantifying elasticities across the two information environments. Soderbery (2015) estimates provide a useful benchmark to accomplish such purpose.

4.3 Estimates of Trade Elasticities

Given estimated distribution parameters and entry thresholds according the estimation procedure described in Section 4.2 and presented in Appendix F for completeness, we compute the partial trade elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$, and the extensive margin contribution to the trade elasticity, γ_{ijk}, according to equations (8) and (12) for a model with complete information and uncertainty respectively. Table 2 presents estimation results.

Observe from Panel A in Table 2 that an average contribution of the extensive margin to trade elasticity under uncertainty as measured by the log-export quantity data is 0.07 versus 0.003 under complete information as measured by the log-export sales data. This quantitative finding reiterates our theoretical Result 1 that trade is more elastic under uncertainty due to the differences in the extensive margin response to changes in variable trade costs. In the next section we provide a detailed discussion of the significance of these magnitudes for trade and welfare.

[29] Soderbery (2015) estimates the elasticity of substitution values at the HS-10 digit level using the U.S. import data. To use Soderbery (2015) estimates aggregate the elasticities to the HS-6 digit level equally weighing corresponding HS-10 sub-categories for each HS-6 category.

5 Analysis and Implications of Elasticity Estimates

In this section we compare the estimates of trade elasticities between the two information environments. Given that trade elasticities are larger under uncertainty (see Section 3.1), we quantify the magnitude by which uncertainty amplifies the measurement of trade elasticities, and examine how this "amplification effect" depends on the extent of uncertainty as measured by the dispersion of ex post shocks. Using counterfactual simulations, we subsequently show that the model with complete information overstates the welfare gains from trade. We demonstrate that this occurs because the model with complete information features too strong of a selection mechanism.

5.1 Quantitative Effect of Uncertainty on Trade Elasticities

Result 2: *The sample average partial trade elasticity with respect to variable trade costs is 7% larger when estimated under uncertainty compared to the complete information environment.*

Panel B in Table 2 directly compares the elasticity estimates across the same observations between the two information environments. In particular, it reports summary statistics of the ratio of the quantity implied trade elasticity relative to the sales implied trade elasticity. We refer to this ratio as the *amplification effect*. Notice that the the model with uncertainty produces partial trade elasticities that are on average 7% larger than under complete information.

As argued in Section 3, trade is more elastic under uncertainty solely due to the higher magnitude of the extensive margin contribution to trade elasticity under uncertainty compared to complete information. Our estimates in Panel B in Table 2 produce a higher contribution of the extensive margin to trade by an order of magnitude of 10^5. To motivate this magnitude, consider the following example. Suppose trade increases by a million dollars due to a decline in trade costs. Then, a trade elasticity estimate from a complete information model would attribute approximately \$3,000 out of a million dollars of new trade to trade generated by entrants. In a model with incomplete information, \$65,000 out of a million dollars can be attributed to trade by entrants. Hence, complete information dampens the (already small) contribution of new exporters to trade. Conversely a model with uncertainty amplifies the contribution of the extensive margin to trade.

There are two distinct reasons for why our estimates produce seemingly small magnitudes of the extensive margin contribution of the partial trade elasticity. The first reason is an abundance of small exporters in export sales and quantity distributions in our dataset. The dataset includes the universe of customs declarations and therefore contains smaller firms than most standard datasets where small firms are not always recorded. Sager and Timoshenko (2019) discuss the potential biases that may contaminate estimation on truncated data and yield larger estimates of the extensive margin trade elasticities. The second reason is the static nature of the model combined with the standard assumption of fixed export costs. As we showed in Section 2, the model implies a unique profitability or productivity market-entry threshold. Hence, all entrants and exiters in the model will necessarily be the smallest firms in the economy. As a result, their contribution to export sales will also be small. In Section 5.3, we show that these seemingly small elasticities estimates nonetheless imply large welfare responses to a trade liberalization in General Equilibrium.

5.2 Role of Dispersion of Ex Post Shocks

The magnitude of the uncertainty amplification effect is tightly linked to the extent of variation arising from the ex post shocks. In a model with uncertainty, the log-export sales and quantity are related according to

$$\log r_{ijk} \quad = \quad \frac{\epsilon_k - 1}{\epsilon_k} \log q_{ijk} + FE_{jk} + \frac{z^p_{ijk}}{\epsilon_k}, \tag{30}$$

where $FE_{jk} = \log\left(Y_{jk}^{\frac{1}{\epsilon_k}} P_{jk}^{\frac{\epsilon_k - 1}{\epsilon_k}}\right)$. Notice that the distribution of the ex post shocks, z^p_{ijk}, generates a wedge between the distributions of log-export sales and log-export quantity. This wedge is larger when the variance of ex post shocks is higher. If the variance of the ex post shocks is zero, then the distributions of log-export sales and log-export quantity would coincide, yielding no amplification effect. As the variance of the ex post shock rises, the distributions of log-export sales and log-export quantities are more dissimilar. Hence we would expect a larger amplification effect of uncertainty on the extensive margin contribution to trade elasticity.

Given equation (30), we measure the extent of ex post variation in a given destination-year-industry as the difference between the variance of log-export sales and the variance of log-export quantities. Following the identification strategy of Berman et al. (2019) for estimating the demand shocks, we assume that ex ante (productivity) and ex post (demand) shocks are independent from each other. This assumption implies that the ex post shocks are uncorrelated with log-export quantities, which, under uncertainty, depend only on ex ante shocks. Applying the variance operator to both sides of equation (30) then leads to

$$V\left(\frac{z^p_{jk}}{\epsilon_k}\right) \quad = \quad V(\log r_{jk}) - \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^2 V(\log q_{jk}). \tag{31}$$

We first compute the variance of log-export sales, $V(\log r_{jk})$, and the variance of log-export quantity, $V(\log q_{jk})$, across firms within a given destination-year-industry observation.We then use equation (31) to back out the value of the variance of the ex post shocks, $V\left(z^p_{jk}/\epsilon_k\right)$ for each destination-year-industry observation.

Result 3: *The difference in the trade elasticity estimates between environments with uncertainty and complete information is larger when ex post shocks are more dispersed.*

Figure 1 depicts the relationship between the variance of the ex post shocks and the amplification effect. The x-axis measures the variance of the ex post shocks, while the y-axis measures the ratio of the export quantity implied relative to the export sales implies estimate of the extensive margin elasticity. The figure confirms that the difference in elasticity estimates between the complete information and uncertainty economies increases with an increase in the dispersion of ex post shocks. In the data, exporters do not have full information about product about the realization of ex post (demand) shocks in destination markets and introducing uncertainty into the model leads to a larger extensive margin adjustment.

5.3 Welfare Implications

In this section we quantify the effect of information structure on welfare gains from trade. To do so, using the insights from Arkolakis et al. (2012), we first illustrate a theoretical connection between trade elasticities and welfare. We next perform a counterfactual exercise where we calibrate parameters of the model with uncertainty and simulate counterfactual welfare by changing only the information structure of the model.

5.3.1 Theoretical Mechanism

In both information environments, the total trade flows between country i and j in industry k can be written as

$$X_{ijk} \quad = \quad J_i \epsilon_k w_i f_{ijk} \frac{1}{\gamma_{ijk}(x^*)} g^x_{ijk}(x^*), \tag{32}$$

were in the environment with complete information, x^* is the profitability entry threshold z^*_{ijk}; in the environment with uncertainty, x^* is the rescaled ex ante profitability entry threshold $\bar{z}^{a*}_{ijk}$.

Equation (32) provides a crucial link between the extensive margin of the partial trade elasticity and trade flows and shows that, holding all else constant, the two variables are inversely related. As shown in Section 3, the value of the extensive margin elasticity is lower under complete information relative to an environment with uncertainty. Hence, holding all else constant, the corresponding value of trade flows is higher and the implied domestic trade share is lower in an environment with complete information relative to uncertainty. This leads to higher potential welfare gains from trade as suggested by the Arkolakis et al. (2012) result extended to a multi-sector environment:

$$\ln(W_i) \propto \sum_{k=1}^{K} \frac{\mu_k}{|\eta_{ik}|} \ln\left(\frac{\pi_{ii,k}}{L_{ik}}\right)^{-1}, \tag{33}$$

were W_i denotes welfare (real income) of country i, L_{ik} denotes country i's employment in sector k, and $\pi_{ii,k}$ denotes the share of expenditure in sector k devoted to domestic goods. A lower value of the extensive margin of the partial trade elasticity, γ_{ijk}, leads to a lower absolute value of the partial trade elasticity, $|\eta_{ik}|$, and a lower value of the domestic trade share, $\pi_{ii,k}$. As per equation (33), the welfare is inversely related to both of these magnitudes, and therefore the welfare gains from a decline in variables trade costs are higher for lower values of the extensive margin of the partial trade elasticity. Hence, a model with complete information will predict higher welfare gains from trade relative to a model with uncertainty.

5.3.2 Empirical Significance

In this section we quantify errors in the measurement of the welfare gains from trade arising from changes in the information environment faced by firms. To do so we proceed by calibrating a model with uncertainty to match the estimated trade elasticities in Section 4.3, first and second order moments of the export sales and quantity data. We quantitatively illustrate that a model with complete information leads to significant errors in the measurement of welfare gains from trade.

For expositional simplicity we conduct our exercise using a symmetric two country environment with one industry. We assume that there is no uncertainty in the home market. In the foreign market, firms face ex post demand shocks as described in Section 2. We further assume that ex post shocks are drawn from a Normal distribution with mean m_θ and variance υ_θ^2.

We calibrate the general equilibrium of a model with uncertainty to match moments of the data on export quantities and export sales as follows. For a given distribution of log-export quantity, we recover parameters of the Double EMG productivity distribution (the standard deviation, υ^2, the left and right tail parameters, ξ_L and ξ_R, respectively) using a truncated Double EMG distribution fit to the log-export quantity data as described in Section 4. We next calibrate the fixed export cost, f_x, the variable trade cost, τ, and the mean of the productivity distribution, m, to match the average-to-minimum ratio of export quantity, the minimum export quantity, and the average value of export sales. Hence, the calibrated general equilibrium of the model will reproduce the estimates of the trade elasticities from

Section 4.3.

We next calibrate the variance of ex post demand shocks, υ_θ^2, to match the difference between the variance of log-export quantity and log-export sales as described in equation (31). We further normalize the mean of the demand shocks as follows: $m_\theta = -\upsilon_\theta^2/(2\epsilon)$. Such normalization of the mean ensures that changes in the variance of the ex post demand shocks have no scale effects on the profitability of firms. Hence, potential difference in the predictions between a model with uncertainty and complete information will arise solely from changes in the information environment and not from changes in the level of demand expectations.[30]

As discussed in Section 4.2.4, we take the elasticity of substitution estimates from Soderbery (2015). Without loss of generality we normalize the remaining parameters of the model.[31]

Having calibrated the general equilibrium of a model with uncertainty, our goal is to quantify errors in the measurement of the welfare gains from trade arising from changes in the information environment. Hence, holding all structural parameters at their respective calibrated values we simulate the general equilibrium and trade liberalization starting from that equilibrium using a model with uncertainty and a model with complete information. The results of this simulation are presented in Figure 2. Each dot in Figure 2 represents a simulation for a given observation as defined in Section 4.1.[32]

Result 4: *Complete information environment overstates welfare gains from trade by 1% to 12% from a 10% decline in variable trade costs. The welfare wedge increases when ex post shocks are more dispersed.*

Figure 2 depicts the welfare wedge from a 10% decline in the variable trade costs.[33] The welfare wedge is constructed by subtracting the percentage points increase in welfare in the model with uncertainty from the percentage points increase in welfare in a model with complete information. For example, the value of 2 on the y-axis indicates that if a model

[30] As shown in Appendix A, conditional on the origin-destination-sector fixed effect B_{ijk}, the export thresholds between a model with uncertainty and complete information differ by a factor of $\left(E_{z^p_{ijk}}\left(e^{z^p_{ijk}/\epsilon_k}\right)\right)^{\epsilon_k}$. Given the normalization, this expectation equals to unity.

[31] $L = 10^8$, $f_d = 1$, respective values of J are chosen to ensure labor market clearing.

[32] As described in Section 4.1, our sample consists of 190 observations of the distributions of export sales and export quantities across firms for a given destination, year, and industry triplet. We perform the described calibration procedure separately for each observation where such calibration is numerically feasible. A calibration is numerically feasible when moments of the Double EMG distribution exist and are finite, i.e. when $\xi_R/\epsilon > 1$ and when $\upsilon_\theta^2 > 0$. There are 76 such feasible observations.

[33] We have replicated results for changes in trade cost ranging from a 50% decline to a 50% increase in the variable trade costs. While the magnitude of the welfare wedge increases with the extent of trade liberalization, the positive relation between the welfare wedge and the standard deviation of the demand shock is preserved for any level of a decline in trade costs. These results are available upon request.

with uncertainty predicts an x% increase in welfare due to a 10% decline in the variable trade costs, a model with complete information will predict an increase in welfare amounting to x%+2%.

Figure 2 shows errors in computing the welfare gains from trade arising from the effect of information on exporting: A model with complete information overstates the gains from trade, and the magnitude of the bias is larger in more uncertain environments, as measured by the standard deviation of the ex post demand shock and varies from 1% to 12%.

5.3.3 Discussion

Uncertainty lowers the gains from trade as compared to the complete information environment because uncertainty dampens the forces that lead to selection of firms into export markets, as demonstrated in Figure 3. Figure 3 depicts the ratio of the number of exporters (Panel A) and the average exporter size (Panel B) in the initial simulated equilibrium between a model with complete information and a model with uncertainty.

Observe from the Panel A of Figure 3 that the equilibrium number of exporters in the model with complete information is always lower than in a model with uncertainty. Hence, there is less entry under complete information. Panel B of Figure 3 indicates that those few firms, which export under complete information, generate average trade flows that are orders of magnitude larger than under uncertainty.

The patterns depicted in Figure 3 are driven by selection based on productivity versus profitability introduced in Section 2. In particular, under complete information, firms condition export decisions on the realization of ex ante and ex post shocks. Hence, only those firms that will have high realized profitability shocks, therefore realized export profit and export revenue, choose to export. This results in low levels of entry, but high total trade flows. In contrast, under uncertainty, firms selection into exporting occurs based on the realization of ex ante shocks alone and based on expected profits. This results in more firms trying to export due to incomplete information about their full profitability in foreign markets, but lower total trade flows due to some of the firms being unsuccessful in exporting after ex post shocks are realized.

These two forces lead to higher absolute value of the partial elasticity of trade flows with respect to variable trade costs, higher domestic trade share, and hence lower welfare gains from trade under uncertainty compared to complete information.

6 Empirical Evidence for Main Results

In this section, we provide a simple piece of reduced form evidence for the model mechanism, by showing that information uncertainty increases the partial trade elasticity. In particular, we use bilateral aggregate trade data and distance data from the World Bank Exporter Dynamics Database (Fernandes et al., 2016) and *CEPII GeoDist* database (Mayer and Zignago, 2011) respectively to perform a series of cross-sectional gravity regressions that lend support for testable implications of Result 2 and Result 3 in Section 5.

We consider a number of the standard gravity regressions that are augmented with measures of information uncertainty and the dispersion of the ex post demand shocks as follows:

$$\begin{aligned}\log X_{ij} = \gamma_i + \delta_j \quad &+ \quad \beta_1 \log Distance_{ij} + \\ &+ \quad \beta_2 \log Distance_{ij} \cdot \log Information_j + \\ &+ \quad \beta_3 \log Distance_{ij} \cdot \log Information_j \cdot Dispersion_{ij} + \epsilon_{ij}, \end{aligned} \tag{34}$$

where X_{ij} is the aggregate trade flow from country i to country j; γ_i and δ_j are origin and destination country fixed effects respectively; and $Distance_{ij}$ is a measure of distance between country i and country j.

Result 2 implies that trade is more elastic with respect to the variable trade cost between a bilateral country pair ij compared to a country pair ij' when country i is more uncertain about ex post demand shocks in country j compared to j'. Hence, for a given measure of ex post demand shocks uncertainty or information about the ex post shocks in country j, $Information_j$, our model predicts a negative and statistically significant coefficient β_2 in regression (34). Hence, coefficient β_2 measures the amplification effect of uncertainty on trade elasticity.

Result 3 implies that conditional on the information structure, the amplification effect of uncertainty is larger when the ex post shocks are more dispersed. Hence, for a given measure of the dispersion of ex post demand shocks for country i's goods in country j, $Dispersion_{ij}$, our model predicts a negative and statistically significant coefficient β_3 in regression (34).

Sections 6.1 and 6.2 below describe our measures of $Information_j$ and $Dispersion_{ij}$ respectively, and section 6.3 presents and discusses the regression results.

6.1 A Measure of Demand Uncertainty

The ideal measure for ex post demand shocks uncertainty must capture the 'observability' of demand by exporters, i.e. the extent of the information that exporters have about demand in a foreign market, prior to engaging in export activity in that market. We proxy for the availability of information to exporters with diversity measures from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003), which capture the diversity of ethnicity, religion and language within a country. These proxy measures are not part of the usual suite of gravity covariates in the literature.

An implicit link between a population's underlying heterogeneity and a firm's information about demand has been established in Parrotta, Pozzoli, and Sala (2016). Using Danish firm-level data, Parrotta et al. (2016) demonstrate a robust relationship between a firm's work force diversity and the firm's ability to succeed in export markets. The authors find that the diversity of a work force positively correlates with a firm's probability of exporting, export value, the number of export destinations and products. They argue that an ethnically diverse work force improves the ability of a firm to market its products to foreign markets.[34]

Following this literature, we argue that idiosyncratic demand shocks in a demographically diverse export destination are less observable by exporters, since exporters will have less precision in predicting demand from demographics in destinations with greater demographic heterogeneity. Likewise, greater homogeneity in the underlying population would make demand more predictable.[35]

Alesina et al.'s (2003) diversity measures are constructed as one minus the Herfindahl index of observed shares of different groups within a country,

$$Diversity_j \equiv 1 - \sum_{n=1}^{N} s_{nj}^2$$

where s_{nj} is the share of group n in country j. This diversity construct is meant to capture the probability that two randomly selected individuals from a country's population do not belong to the same group. Alesina et al. (2003) take the data on shares of linguistic and religious groups within a country from *Encyclopedia Britannica* for the year 2001. The source of ethnic data vary by country and the coverage ranges from 1979 to 2001.[36]

Table 3 provides summary statistics of the three diversity measures that are available for up to 195 countries. Notably, according to these measures, South Korea is the least ethnically and linguistically diverse country in the sample. Over 99% of the population is comprised of ethnic South Koreans that also speak the Korean language. In contrast, Uganda is the most ethnically and linguistically diverse country in the sample, with ethnic representation from the Ganda (17.8%), Teso (8.9%), Nkole (8.2%), Soga (8.2%), Gisu (7.2%), Chiga (6.8%), Lango (6.0%) and Rwanda (5.8%) ethnic groups that speak thirty different languages (Alesina et al., 2003).[36]

We argue that exporters have more complete information about demand for their prod-

[34] Furthermore, the marketing and industrial management literatures have identified consumer heterogeneity as increasing "demand uncertainty" and consider it an important impediment to forecasting new product sales. For example, Bartezzaghi and Verganti (1995) and Bartezzaghi et al. (1999) show demand tends to be more lumpy and less predictable when a market exhibits great consumer heterogeneity.

[35] In order to relax concerns that more demographically diverse countries may engage in more (less) trade and therefore have a systematically higher (lower) expected amount of sales, we include destination specific fixed effects.

[36] For a more in depth discussion of the data and the three diversity measures see Alesina et al. (2003).

ucts when exporting to South Korea. In South Korea, exporters would only need to know the demand of a single ethnic group, which they will reach with close to 100% probability. In contrast, when exporting to Uganda, exporters would not only need to know the demand of eight ethnic groups, but the exporters are also uncertain about which specific group(s) will see their exported product. Hence exporting to demographically homogeneous countries can be characterized by a more complete information about demand compared to demographically heterogeneous countries.

6.2 A Measure of Demand Dispersion

A measure of ex post demand shocks dispersion must capture the variance of demand shocks in an export destination regardless of whether those demand shocks are observable (a model with complete information) or unobservable (a model with uncertainty). We therefore construct a theory consistent measure of demand dispersion. Equations (18) and (20) imply that under both information environments the variance of log-export sales across firms in country i selling to country j can be written as a weighted sum of the variance of productivity and variance of demand shocks:

$$V(\log r_{ij}) = \alpha^a V(z_{ij}^a) + \alpha^p V(z_{ij}^p), \tag{35}$$

where, to be consistent with the empirical analysis, we have dropped the industry subscript k.[37] Equation (35) implies that for a given origin country i, the variance of export sales to destination j versus j' differs only due to the difference in the variance of the ex post demand shocks, z_{ij}^p. Hence, for an origin country i, export destinations with larger variance of export sales are associated with a larger variance in demand compared to destinations with a smaller variance of export sales. Hence, our constructed measure of the dispersion of the ex post demand shocks is given by the standard deviation of exports sales from country i to country j normalized by the average standard deviation of export sales from country i across destinations j where country i exports, i.e.

$$Dispersion_{ij} = \frac{st.dev.(r_{ij})}{average_j(st.dev.r_{ij})}.$$

The data on the standard deviation of export sales are derived from the firm-level customs data for each origin country and come from the World Bank Exporter Dynamics Database (Fernandes et al., 2016). For the sample year 2001, the data cover 20 origin countries. Each country in the sample exports to 149 destinations on average. Overall, the $Dispersion_{ij}$ measure is available for 2,978 bilateral country-pairs.

[37] Under uncertainty $\alpha^a = (\epsilon_k - 1)$ and $\alpha^p = 1/\epsilon_k$; under complete information $\alpha^a = (\epsilon_k - 1)$ and $\alpha^p = 1$.

Figure 4 plots the distribution of the $Dispersion_{ij}$ across export destinations j for an average origin country i. Notice that relative to the standard deviation of export sales of an average export destination (normalized to unity), the dispersion measure varies from as low as less than one percent of the average to more than twice the average. Hence the export data exhibit significant variation in the standard deviation of export sales across export destinations. For a given origin country, we attribute these differences to differences in the dispersion of the ex post demand shocks.

6.3 Gravity Results

We estimate equation (34) using cross-country bilateral trade flows data from the World Bank Exporter Dynamics Database (Fernandes et al., 2016). The bilateral distance is measured as the population weighted distance and come from the *CEPII GeoDist* database (Mayer and Zignago, 2011). Our baseline sample year is 2001 and is chosen based on the availability of the diversity measures. Each regression additionally includes an extended set of gravity controls.[38]

Table 4 presents the OLS estimates of regression (34). Column (1) in Table 4 presents results from a simple gravity regression and serves as a benchmark for the rest of the results. Columns (2), (4), and (6) estimate the effect of information structure on the bilateral trade elasticity and provide a test of Result 2. In all three cases the effect of demand uncertainty, the interaction term between $\log Dist_{ij}$ and $\log Information_j$, is negative. In the case of ethnic and linguistic diversity measures of uncertainty presented in columns (2) and (4) respectively the effect is statistically significant at the 5% level, and in the case of the religious diversity measure presented in column (6) the effect is statistically significant at the 10% level. We next compute the amplification effect as the interaction term as a percent of the trade elasticity coefficient on $\log Dist_{ij}$. Depending on the specification, uncertainty increases the trade elasticity by 9 to 17 percent for each one percent increase in a diversity measure. These magnitudes are consistent with the model implied amplification effect presented in Table 2. Hence, we find strong reduced form support for Result 2 based on the aggregate cross-sectional trade data.

[38]The gravity controls are dummy variables for a bilateral trade pair sharing a common border, sharing a common official or primary language, language is spoken by at least 9% of the population, sharing a common colonizer post 1945, pair ever in colonial relationship, pair ever in sibling relationship, sharing a common currency, common legal origins before transition, common legal origins after transition, common legal origin changed since transition, origin is GATT/WTO member, destination is GATT/WTO member, origin is donator, destination is donator, origin is a EU member, destination is a EU member, origin country belongs to African, Caribbean and Pacific Group of States and destination country belongs to EU, origin country belongs to EU and destination country belongs to African, Caribbean and Pacific Group of States, dummy for free trade agreement, dummy for regional trade agreement. These data are taken from *CEPII GeoDist* database (Mayer and Zignago, 2011). See the source for greater details.

Columns (3), (5) and (7) provide a further analysis of the effect of demand dispersion on the partial trade elasticity. In all cases the triple interaction term is negative, and in the case of the religious diversity measure, the triple interaction term is statistically significant at the 10% level. These findings provide further reduced form support for our Result 3 stating that the amplification effect of uncertainty on trade elasticity is stronger when demand is more dispersed.

7 Conclusion

This paper shows that the information structure faced by firms is important for measuring the extensive margin response to a decline in trade costs. By assuming away information asymmetries, trade elasticity estimates will likely understate the true magnitude of extensive margin adjustments, particularly in countries or industries in which exporters face high uncertainty about the demand for their product.

We find that under demand uncertainty trade flows are more elastic to changes in variable trade costs and the welfare gains from trade are lower compared to an environment with complete information. We show that these results arise because selection into export activity is more stringent when potential exporters have complete information, as potential exporters would not knowingly enter an unprofitable market. When potential exporters face uncertainty about demand in foreign markets, a larger number of firms engage in risky export activity but only few firms become large exporters relative to economies with complete information about demand.

We show this in three ways. First, we draw upon recent research that incorporates uncertainty into standard trade models by embedding a learning mechanism along the lines of Jovanovic (1982) into trade models with heterogeneous firms as in Melitz (2003).[39] With demand uncertainty, firms must choose how much of their product to export prior to observing their destination specific demand shock and, therefore, make export decisions based only on their ex ante information (productivity). We show that the basic structure of the model is robust to alternative modeling assumptions, such as including variable markups or allowing for quality upgrading. We use the structure of this model to prove that (under mild conditions) the extensive margin of the partial trade elasticity, which characterizes the effects of uncertainty on trade flows and welfare, is larger when demand is uncertain. This result fills a gap in the literature by characterizing normative implications of such an alternative information structure for measurements of the trade elasticities and the welfare gains from

[39] These models have been used to analyze firm behavior such as growth (Arkolakis et al., 2018), export participation (Timoshenko, 2015a), product switching (Timoshenko, 2015b), and pricing decisions (Bastos et al., 2018).

trade.

Second, we quantify the model using Brazilian microdata on export quantities and export sales, and show that the structurally estimated elasticities are systematically larger in the economy with uncertainty than in the complete information economy. Furthermore, we use a stylized symmetric two-country example to show that complete information economies overstate the welfare gains from trade and the magnitude of the bias is larger when the variance of demand is larger. Methodologically, we show that identification of the extensive margin partial trade elasticity under uncertainty requires data on quantities traded while the complete information case can be identified from sales data.

Third, we provide reduced form empirical evidence for our mechanism by estimating an otherwise standard gravity equation with additional controls for information availability and demand dispersion. The reduced form evidence supports the main theoretical and quantitative results of the paper: (i) trade is more elastic when demand is harder to predict and is therefore more uncertain, and (ii) this elasticity is larger in markets with more dispersed demand.

The environment studied in this paper was made intentionally stark in order to maximize clarity and be most comparable to existing static trade models. Dynamic extensions of this model, that have been shown to capture a large set of empirical patterns of firm growth, could lead to a more nuanced description of the relationship between firms' information and export activity. We leave such extensions of our results to future research.

References

Alesina, A., A. Devleeschauwer, W. Easterly, S. Kurlat, and R. Wacziarg (2003): "Fractionalization," *Journal of Economic Growth*, 8, 155–194.

Allen, T. (2014): "Information Frictions in Trade," *Econometrica*, 82, 2041–2083.

Arkolakis, C., A. Costinot, and A. Rodriguez-Clare (2012): "New Trade Models, Same Old Gains?" *American Economic Review*, 102, 94–130.

Arkolakis, C., T. Papageorgiou, and O. Timoshenko (2018): "Firm Learning and Growth," *Review of Economic Dynamics*, 27, 146–168.

Axtell, R. L. (2001): "Zipf Distribution of U.S. Firm Sizes," *Science*, 293, 1818–1820.

Baley, I., L. Veldkamp, and M. E. Waugh (2019): "Might Global Uncertainty Promote International Trade?" NBER Working Paper No. 25606.

Bartezzaghi, E. and R. Verganti (1995): "Managing demand uncertainty through order overplanning," *International Journal of Production Economics*, 40, 107–120.

Bartezzaghi, E., R. Verganti, and G. Zotteri (1999): "A simulation framework for forecasting uncertain lumpy demand," *International Journal of Production Economics*, 59, 499–510.

Bas, M., T. Mayer, and M. Thoenig (2017): "From Micro to Macro: Demand, Supply, and Heterogeneity in the Trade Elasticity," *Journal of International Economics*, 108, 1–19.

Bastos, P., D. Dias, and O. Timoshenko (2018): "Learning, Prices, and Firm Dynamics," *Canadian Journal of Economics*, 51, 1257–1311.

Bergin, P. R. and C.-Y. Lin (2012): "The dynamic effects of a currency union on trade," *Journal of International Economics*, 87, 191–204.

Berman, N., V. Rebeyrol, and V. Vicard (2019): "Demand learning and firm dynamics: evidence from exporters," *Review of Economics and Statistics*, 101, 91–106.

Bernard, A., S. Redding, and P. Schott (2010): "Multi-Product Firms and Product Switching," *American Economic Review*, 100, 70–97.

Broda, C. and D. Weinstein (2006): "Globalization and the Gains from Variety," *Quarterly Journal of Economics*, 121, 541–585.

Caliendo, L. and F. Parro (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82, 1–44.

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98, 1707–21.

Correia, S. (2017): "Linear Models with High-Dimensional Fixed Effects: An Efficient and Feasible Estimator," Working Paper.

di Giovanni, J., A. A. Levchenko, and R. Rancière (2011): "Power Laws in Firm Size and Openness to Trade: Measurement and Implications," *Journal of International Economics*, 58, 42–52.

Dickstein, M. J. and E. Morales (2016): "What do Exporters Know?" Unpublished.

Eaton, J. and S. Kortum (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.

Feenstra, R. C. (1994): "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84, 157–77.

Fernandes, A., C. Freund, and M. Pierola (2016): "Exporter Behavior, Country Size and Stage of Development: Evidence from the Exporter Dynamics Database," *Journal of Development Economics*, 119, 121–137.

Fernandes, A. M., P. J. Klenow, S. Meleshchuk, M. D. Pierola, and A. Rodríguez-Clare (2015): "The intensive margin in trade: moving beyond pareto," Unpublished.

Fillat, J. L. and S. Garetto (2015): "Risk, Returns, and Multinational Production," *The Quarterly Journal of Economics*, 130, 2027–2073.

Flach, L. (2016): "Quality upgrading and price heterogeneity: Evidence from Brazilian exporters," *Journal of International Economics*, 102, 282–290.

Flach, L. and E. Janeba (2017): "Income inequality and export prices across countries," *Canadian Journal of Economics*, 50, 162–200.

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98, 394–425.

Handley, K. and N. Limao (2015): "Trade and Investment under Policy Uncertainty: Theory and Firm Evidence," *American Economic Journal: Economic Policy*, 7, 189–222.

Head, K., T. Mayer, and M. Thoenig (2014): "Welfare and Trade without Pareto," *American Economic Review*, 104, 310–16.

Imbs, J. and I. Mejean (2015): "Elasticity Optimism," *American Economic Journal: Macroeconomics*, 7, 43–83.

——— (2017): "Trade Elasticties," *Review of International Economics*, 25, 383–402.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50, 649–670.

Kugler, M. and E. Verhoogen (2012): "Prices, Plant Size, and Product Quality," *Review of Economic Studies*, 79(1), 307–339.

Lewis, L. T. (2014): "Exports Versus Multinational Production Under Nominal Uncertainty," *Journal of International Economics*, 94, 371–386.

Mayer, T. and S. Zignago (2011): "Notes on CEPII's distances measures: The GeoDist database," CEPII Working Paper 2011-25.

Melitz, M. and G. Ottaviano (2008): "Market Size, Trade, and Productivity," *Review of Economic Studies*, 75, 295–316.

Melitz, M. and S. Redding (2015): "New Trade Models, New Welfare Implications," *American Economic Review*, 105, 1105–46.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71, 1695–1725.

Molinaz, D. and M.-A. Muendler (2013): "Preparing to Export," NBER Working Paper No. 18962.

Nigai, S. (2017): "A tale of two tails: Productivity distribution and the gains from trade," *Journal of International Economics*, 104, 44–62.

Parrotta, P., D. Pozzoli, and D. Sala (2016): "Ethnic Diversity and Firms' Export Behavior," *European Economic Review*, 89, 248–263.

Rinne, H. (2014): *The Hazard rate : Theory and inference (with supplementary MATLAB-Programs)*, Justus-Liebig-Universitat.

Sager, E. and O. A. Timoshenko (2019): "The Double EMG Distribution and Trade Elasticities," *Canadian Journal of Economics*, 52, 1523–1557.

Simonovska, I. and M. E. Waugh (2014): "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 92, 34–50.

Soderbery, A. (2015): "Estimating Import Supply and Demand Elasticities: Analysis and Implications," *Journal of International Economics*, 96, 1–17.

Timoshenko, O. A. (2015a): "Learning versus Sunk Costs Explanations of Export Persistence," *European Economic Review*, 79, 113–128.

——— (2015b): "Product Switching in a Model of Learning," *Journal of International Economics*, 95, 233–249.

Figures and Tables

Table 1: Properties of the log-export quantity and log-export sales distributions across destination-year-industry observations over 1997-2000.

Statistic	Mean	Standard Deviation	Min	Max
Panel A: Properties of log-quantity				
Standard Deviation	2.46	0.48	1.24	3.38
Skewness	0.09	0.40	-1.00	0.96
Interquartile Range	3.48	0.82	1.88	5.50
Kelly Skew	0.02	0.13	-0.36	0.27
Panel B: Properties of log-sales				
Standard Deviation	2.28	0.41	1.30	3.19
Skewness	-0.13	0.27	-0.88	0.57
Interquartile Range	3.09	0.62	1.74	4.47
Kelly Skew	-0.04	0.11	-0.32	0.28

Note: the statistics are reported across 190 destination-year-industry observations where at least 100 firms export. An industry is defined as a 6-digit HS code. Export quantity is measured as export weight in kilograms.

Table 2: Trade elasticity estimates.

Measure	Extensive Margin Elasticity, γ_{ijk} Mean	Std. Dev.	Partial Trade Elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$ Mean	Std. Dev.
Panel A: Estimates of trade elasticity				
Quantity based[a]	0.07	0.34	3.38	3.64
Sales based	0.003	0.02	3.54	3.76
Panel B: Amplification effect				
Amplification effect[b]	$2.8 \cdot 10^5$	$1.6 \cdot 10^6$	1.07	0.34

[a] The quanity based measure of trade elasticity is based on a model with demand uncertainty. The summary statistics are reported across 175 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > (\epsilon_k - 1)/\epsilon_k$. The elasticities are not defined for $\lambda_R \leq (\epsilon_k - 1)/\epsilon_k$.

[b] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 175 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures.

Table 3: Sample properties of diversity measures.

Diversity Measure	No. Obs.	Mean	St. Dev.	min	25th pct	50th pct	75th pct	max
Ethnic	185	0.44	0.26	South Korea (0.002)	El Salvadore (0.20)	Dominican republic (0.43)	Nepal (0.66)	Uganda (0.93)
Linguistic	185	0.39	0.28	South Korea (0.002)	Ecuador (0.13)	Mongolia (0.37)	Belize (0.63)	Uganda (0.93)
Religious	196	0.44	0.23	Yemen (0.002)	Andorra (0.23)	Haiti (0.47)	Mauritius (0.64)	South Africa (0.86)

NOTES: Diversity scores are from Alesina et al. (2003). The number in parenthesis beneath a country's name indicates that country's respective score.

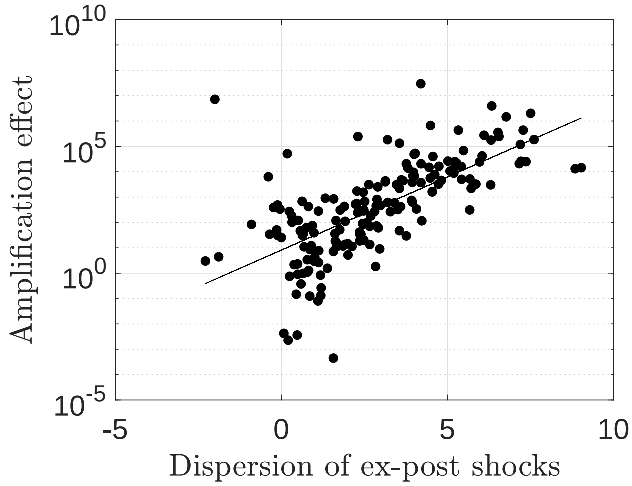
Table 4: OLS gravity regressions.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Ethnic Div.		Linguistic Div.		Religious Div.	
$\log Dist_{ij}$	-1.440^a	-1.822^a	-1.800^a	-1.723^a	-1.665^a	-1.587^a	-1.578^a
	(0.199)	(0.247)	(0.222)	(0.251)	(0.223)	(0.251)	(0.233)
$\log Dist_{ij} \cdot \log Information_j$		-0.307^b	-0.320^b	-0.181^b	-0.167^b	-0.139^c	-0.135^c
		(0.125)	(0.117)	(0.080)	(0.072)	(0.079)	(0.066)
$\log Dist_{ij} \cdot \log Information_j \cdot Dispersion_{ij}$			-0.002		-0.001		-0.014^b
			(0.002)		(0.002)		(0.007)
Amplif. Effect		17%	18%	11%	10%	9%	9%
No. obs.	2,055	2,055	2,055	2,055	2,055	2,055	2,055
R^2	0.80	0.80	0.82	0.80	0.82	0.80	0.82
orig. & dest. FE	Y	Y	Y	Y	Y	Y	Y
Addit. grav. controls	Y	Y	Y	Y	Y	Y	Y

NOTES: cross-sectional data on bilateral trade flows for the year 2001. Distance in measured as population weighted distance from *CEPII GeoDist* database (Mayer and Zignago, 2011). The dependent variable is the logarithm of a bilateral trade flows the World Bank Exporter Dynamics Database (Fernandes et al., 2016). $Information_j$ is measured as ethnic diversity in columns (2) and (3); linguistic diversity in columns (4) and (5); religious diversity in columns (6) and (7). Ethnic, religius, linguistic, diversity scores are from Alesina, Devleeschauwer, Easterly, Kurlat, and Wacziarg (2003). $Dispersion_{ij}$ is measured as the standard deviation of the export value per exporter from country i to country j normalized by the average standard deviation of the export value per exporter across all destination j where firms from origin country i export; source the World Bank Exporter Dynamics Database (Fernandes et al., 2016). The amplification effect is comuted by dividing the coefficent of the interraction term by the coeffient on log-distance, and is expressed in percent terms. Columns (3), (5), and (7) include $Dispersion_{ij}$ as an additional control.

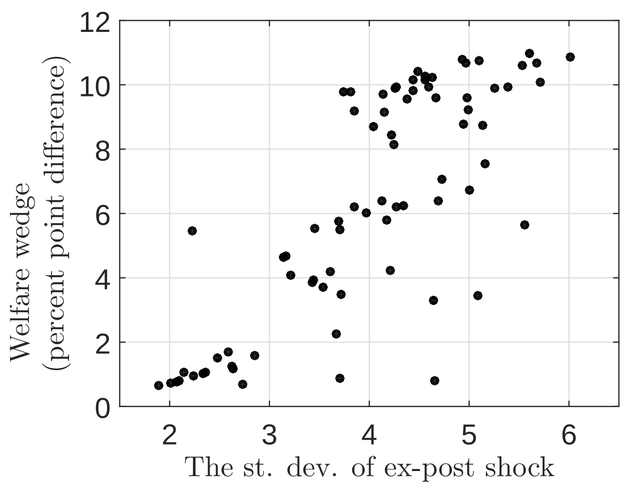
[a, b, c] statitsically significant at 1%, 5%, 10% level. Standard errors are clusters by origin and by destination using the estimator described in Correia (2017).

Figure 1: Amplification effect and uncertainty.



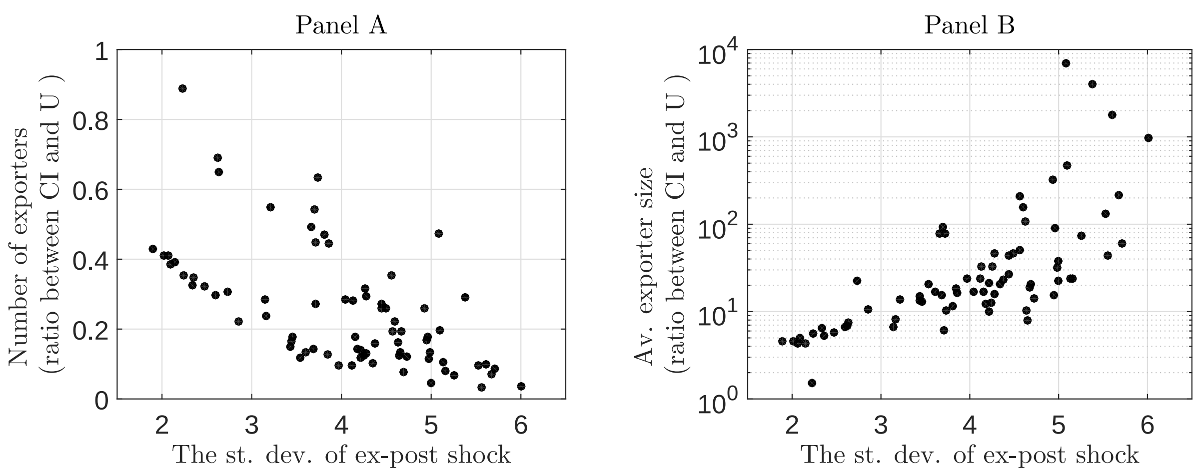
Notes: The figure depicts a scatter plot of the amplification effect and uncertainty. The amplification effect is defined as the ratio of the extensive margin elasticity estimates between the quantity based and the sales based measures. The dispersion of the ex post shocks is defined as the variance of the ex post shocks estimated using equation (31). Each dot corresponds to a destination-year-industry observation. The solid line is an OLS best fit line.

Figure 2: The welfare wedge from a 10% decline in variable trade costs.



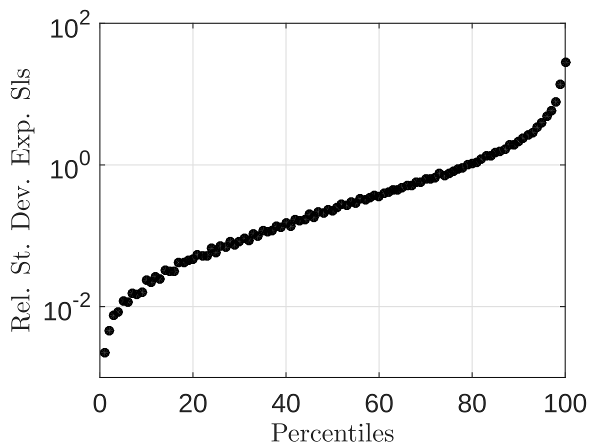
Notes: The figure depicts the percentage point difference in the estimates of the welfare gains from trade from a 10% decline in variable trade costs between a model with complete information and a model with uncertainty. Each dot is a separate observation and represents a simulation result for that observation.

Figure 3: Conterfactual number for exporters and average exporter size.



Notes: Each dot is a separate observation and represents a counterfactual result for that observation. 'CI' stands for 'Complete Information'; 'U' stands for 'Uncertainty'.

Figure 4: Distribution of the demand dispersion measure.



Notes: The figure plots the distribution of the relative to the mean standard deviation of export sales across export destinations for an average origin country.

A Theoretical Appendix

In this section we provide derivations for the theoretical results in Section 2.

A.1 Environment with Complete Information

A firm's problem selling from country i to country j in sector k consists of maximizing profit subject to the demand equation (2):

$$\pi_{ijk}(z_{ijk}) = \max_{q_{ijk}} p_{ijk}(z^p_{ijk})q_{ijk} - \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}} q_{ijk} - w_i f_{ijk}. \tag{36}$$

The first order conditions with respect to quantity yield the optimal quantity given by

$$q_{ijk}(z^a_{ijk}, z^p_{ijk}) = \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k} e^{\epsilon_k z^a_{ijk} + z^p_{ijk}} (\tau_{ij} w_i)^{-\epsilon_k} Y_{jk} P^{\epsilon_k - 1}_{jk}. \tag{37}$$

Using equations (2) and (37), a firm's optimal revenue is further given by

$$r_{ijk}(z_{ijk}) = \underbrace{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P^{\epsilon_k - 1}_{jk} w_i^{1-\epsilon_k}}_{B_{ijk}} \underbrace{\tau_{ij}^{1-\epsilon_k}}_{f^\tau(\tau_{ij})} \underbrace{e^{(\epsilon_k - 1) z^a_{ijk} + z^p_{ijk}}}_{f^z(z_{ijk})}, \tag{38}$$

where

$$z_{ijk} = \underbrace{(\epsilon_k - 1)}_{\alpha^a} z^a_{ijk} + \underbrace{1}_{\alpha^p} \cdot z^p_{ijk}.$$

Hence the revenue can be written as being determined solely by the profitability shock z_{ijk}, a function of the variable trade cost, $f^\tau(\tau_{ij})$, and an origin-destination-industry fixed effect, B_{ijk}, as stated in equation (5).

Substituting equations (38) and (37) into equation (36) yields optimal profit given by

$$\pi_{ijk}(z_{ijk}) = \frac{1}{\epsilon_k} \underbrace{\left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} Y_{jk} P^{\epsilon_k - 1}_{jk} w_i^{1-\epsilon_k} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{ijk} + z^p_{ijk}}}_{r_{ijk}(z_{ijk})} - w_i f_{ijk}. \tag{39}$$

Hence the optimal profit also depends solely on the profitability shock, z_{ijk}, and can be written as

$$\pi_{ijk}(z_{ijk}) = \frac{1}{\epsilon_k} r_{ijk}(z_{ijk}) - w_i f_{ijk}. \tag{40}$$

Notice from equation (38) that $r_{ijk}(z_{ijk})$ is a strictly increasing function of the profitability shock, z_{ijk}. By equation (40), so is the optimal profit.

A firm exports if its profit from exporting is positive:

$$\begin{aligned} \pi_{ijk}(z_{ijk}) &\geq 0 \\ \frac{1}{\epsilon_k} r_{ijk}(z_{ijk}) - w_i f_{ijk} &\geq 0 \\ \frac{1}{\epsilon_k} r_{ijk}(z_{ijk}) &\geq w_i f_{ijk} \end{aligned} \tag{41}$$

Since revenue function is strictly increasing in profitability, z_{ijk}, when held with equality, equation (41) determines the profitability entry threshold z_{ijk}^* such that a firm exports if $z_{ijk} \geq z_{ijk}^*$, and does not export otherwise. The profitability entry threshold z_{ijk}^* is implicitly determined by

$$\begin{aligned} r_{ijk}(z_{ijk}^*)/\epsilon_k &= w_i f_{ijk} \\ B_{ijk} f^\tau(\tau_{ij}) f^z(z_{ijk}^*) &= \epsilon_k w_i f_{ijk}. \end{aligned}$$

Using the definition of function $f^z(z_{ijk})$ from equation (38), the profitability entry threshold is given by

$$e^{z_{ijk}^*} = \frac{\epsilon_k w_i f_{ijk}}{B_{ijk} f^\tau(\tau_{ij})}. \tag{42}$$

Trade Elasticity: Given the endogenous selection into exporting that is based on the realization of profitability shocks, the total trade flows from country i to country j in sector k, X_{ijk}, are defined as

$$X_{ijk} = J_i(1 - G_{ijk}^z(z_{ijk}^*)) \int_{z_{ijk}^*}^{+\infty} r_{ijk}(z) \frac{g_{ijk}^z(z)}{1 - G_{ijk}^z(z_{ijk}^*)} dz,$$

where J_i is the exogenous mass of entrants in country i, and $g_{ijk}^z(.)$ and $G_{ijk}^z(.)$ are the probability and the cumulative distribution functions of the profitability shock z_{ijk}. Substituting equation (38) for the revenue yields

$$X_{ijk} = J_i(1 - G_{ijk}^z(z_{ijk}^*)) \int_{z_{ijk}^*}^{+\infty} B_{ijk} f^\tau(\tau_{ij}) f^z(z) \frac{g_{ijk}^z(z)}{1 - G_{ijk}^z(z_{ijk}^*)} dz,$$

Using the definition of function $f^z(z_{ijk})$ from equation (38) and differentiating with respect to τ_{ij} yields:

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = \frac{\partial f^\tau(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z_{ijk}^*}^{+\infty} B_{ijk} e^z g_{ijk}^z(z) dz - J_i \frac{\partial z_{ijk}^*}{\partial \tau_{ij}} B_{ijk} f^\tau(\tau_{ij}) e^{z_{ijk}^*} g_{ijk}^z(z_{ijk}^*). \tag{43}$$

Differentiate equation (42) with respect to τ_{ij} to obtain

$$\frac{\partial z^*_{ijk}}{\partial \tau_{ij}} = -\frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}}. \tag{44}$$

Substituting equation (44) into equation (43) yields

$$\begin{aligned}
\frac{\partial X_{ijk}}{\partial \tau_{ij}} &= \frac{\partial f^\tau(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{z^*_{ijk}}^{+\infty} B_{ijk} e^z g^z_{ijk}(z) dz + J_i \frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}} B_{ijk} f^\tau(\tau_{ij}) e^{z^*_{ijk}} g^z_{ijk}(z^*_{ijk}) = \\
&= \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(\tau_{ijk} J_i \int_{z^*_{ijk}}^{+\infty} B_{ijk} e^z f^\tau(\tau_{ijk}) g^z_{ijk}(z) dz + \tau_{ijk} J_i B_{ijk} f^\tau(\tau_{ij}) e^{z^*_{ijk}} g^z_{ijk}(z^*_{ijk}) \right) = \\
&= \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(\tau^{-1}_{ijk} X_{ijk} + \tau^{-1}_{ijk} X_{ijk} \frac{e^{z^*_{ijk}} g^z_{ijk}(z^*_{ijk})}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz} \right).
\end{aligned}$$

Hence,

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} = \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(1 + \frac{e^{z^*_{ijk}} g^z_{ijk}(z^*_{ijk})}{\int_{z^*_{ijk}}^{+\infty} e^z g^z_{ijk}(z) dz} \right).$$

A.2 Environment with Uncertainty

A firm's problem selling from country i to country j in sector k consists of maximizing the *expected* profit subject to the demand equation (2):

$$E_{z^p_{ijk}}[\pi_{ijk}(z^a_{ijk}, z^p_{ijk})] = \max_{q_{ijk}} E_{z^p_{ijk}} \left(p_{ijk}(z^p_{ijk}) q_{ijk} - \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}} q_{ijk} \right) - w_i f_{ijk}. \tag{45}$$

The first order conditions with respect to quantity yield the optimal quantity given by

$$q_{ijk}(z^a_{ijk}) = \underbrace{\left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k} \left(E_{z^p_{ijk}} \left(e^{\frac{z^p_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} w_i^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}}_{B_{ijk}} \underbrace{\tau_{ij}^{-\epsilon_k}}_{f^{\tau,q}(\tau_{ij})} e^{\epsilon_k z^a_{ijk}}. \tag{46}$$

Using equations (2) and (46), a firm's realized revenue is further given by

$$r_{ijk}(z^a_{ijk}, z^p_{ijk}) = \underbrace{\left(\frac{\epsilon_k - 1}{\epsilon_k} \right)^{\epsilon_k - 1} \left(E_{z^p_{ijk}} \left(e^{\frac{z^p_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k - 1} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}}_{B_{ijk}} \underbrace{\tau_{ij}^{1-\epsilon_k}}_{f^\tau(\tau_{ij})} \underbrace{e^{\frac{z^p_{ijk}}{\epsilon_k}}}_{f^p(z^p_{ijk})} \underbrace{e^{(\epsilon_k - 1) z^a_{ijk}}}_{f^a(z^a_{ijk})}. \tag{47}$$

Substituting equations (47) and (46) into equation (45) yields optimal expected profit given by

$$E_{z^p_{ijk}}[\pi_{ijk}(z^a_{ijk}, z^p_{ijk})] = \frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}} \left(E_{z^p_{ijk}} \left(e^{\frac{z^p_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k} e^{(\epsilon_k - 1) z^a_{ijk}} - w_i f_{ijk}. \tag{48}$$

A firm exports if its expected profit from exporting is positive:

$$E_{z^p_{ijk}}[\pi_{ijk}(z^a_{ijk}, z^p_{ijk})] \geq 0. \tag{49}$$

Notice from equation (48) that expected profit is strictly increasing in the ex ante profitability shock, z^a_{ijk}. Hence, when held with equality, equation (49) determines the ex ante profitability entry threshold z^{a*}_{ijk} such that a firm exports if $z^a_{ijk} \geq z^{a*}_{ijk}$, and does not export otherwise. The profitability entry threshold z^{a*}_{ijk} is therefore given by

$$e^{(\epsilon_k - 1) z^{a*}_{ijk}} = \frac{w_i f_{ijk}}{\frac{(\epsilon_k - 1)^{\epsilon_k - 1}}{\epsilon_k^{\epsilon_k}} \left(E_{z^p_{ijk}} \left(e^{\frac{z^p_{ijk}}{\epsilon_k}} \right) \right)^{\epsilon_k} w_i^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \tau_{ij}^{1-\epsilon_k}}. \tag{50}$$

Trade Elasticity: Given the endogenous selection into exporting that is based on the realization of the *ex ante component*, z^a_{ijk} of the profitability shock, the total trade flows from country i to country j in sector k, X_{ijk}, are given by

$$X_{ijk} = J_i(1 - g^a_{ijk}(z^{a*}_{ijk})) \int_{z^{a*}_{ijk}}^{+\infty} \left[\int_{-\infty}^{+\infty} r_{ijk}(z, z^p_{ijk}) g^p_{ijk}(z^p_{ijk}) dz^p_{ijk} \right] \frac{g^a_{ijk}(z)}{1 - g^a_{ijk}(z^{a*}_{ijk})} dz,$$

where $g^a_{ijk}(.)$ and $G^a_{ijk}(.)$ are the probability and the cumulative distribution functions of the ex ante component, z^a_{ijk}, of the profitability shock, z_{ijk}; function $g^p_{ijk}(z^p_{ijk})$ is the probability distribution function of the ex post component, z^p_{ijk}. Substituting equation (47) for the revenue yields:

$$X_{ijk} = J_i(1 - g^a_{ijk}(z^{a*}_{ijk})) \int_{z^{a*}_{ijk}}^{+\infty} B_{ijk} E_{z^p_{ijk}} \left[f^p(z^p_{ijk}) \right] f^\tau(\tau_{ij}) f^a(z) \frac{g^a_{ijk}(z)}{1 - g^a_{ijk}(z^{a*}_{ijk})} dz.$$

Recall from equation (47) that $f^a(z^a_{ijk}) = e^{(\epsilon_k - 1) z^a_{ijk}}$. Hence, introduce the following change of notation: $\bar{z}^a_{ijk} = (\epsilon_k - 1) z^a_{ijk}$. The new variable $\bar{z}^a_{ijk}$ is characterized by a respective probability, denoted by $g^{\bar{z},a}(.)$, and cumulative, denoted by $G^{\bar{z},a}(.)$, distribution functions.

Hence, the total trade flows can be written as

$$X_{ijk} = J_i(1 - G^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk})) \int_{\bar{z}^{a*}_{ijk}}^{+\infty} B_{ijk} E_{z^p_{ijk}} \left[f^p(z^p_{ijk}) \right] f^\tau(\tau_{ij}) e^z \frac{g^{\bar{z},a}_{ijk}(z)}{1 - G^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk})} dz.$$

Differentiation with respect to τ_{ij} yields:

$$\begin{aligned} \frac{\partial X_{ijk}}{\partial \tau_{ij}} &= \frac{\partial f^\tau(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{\bar{z}^{a*}_{ijk}}^{+\infty} B_{ijk} E_{z^p_{ijk}} \left[f^p(z^p_{ijk}) \right] e^z g^{\bar{z},a}_{ijk}(z) dz - \\ & -J_i \frac{\partial \bar{z}^{a*}_{ijk}}{\partial \tau_{ij}} B_{ijk} E_{z^p_{ijk}} \left[f^p(z^p_{ijk}) \right] f^\tau(\tau_{ij}) e^{\bar{z}^{a*}_{ijk}} g^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk}). \end{aligned} \tag{51}$$

Differentiate equation (50) with respect to τ_{ij} to obtain

$$\frac{\partial \bar{z}^{a*}_{ijk}}{\partial \tau_{ij}} = -\frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}}. \tag{52}$$

Substituting equation (52) into equation (51) yields

$$\begin{aligned} \frac{\partial X_{ijk}}{\partial \tau_{ij}} &= \frac{\partial f^\tau(\tau_{ij})}{\partial \tau_{ij}} J_i \int_{\bar{z}^{a*}_{ijk}}^{+\infty} B_{ijk} E_{z^p_{ijk}} \left[f^p(z^p_{ijk}) \right] e^z g^{\bar{z},a}_{ijk}(z) dz - \\ & +J_i \frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}} B_{ijk} E_{z^p_{ijk}} \left[f^p(z^p_{ijk}) \right] f^\tau(\tau_{ij}) e^{\bar{z}^{a*}_{ijk}} g^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk}) = \\ &= \frac{\partial \log f^\tau(\tau_{ij})}{\partial \tau_{ij}} \left(\tau_{ij}^{-1} X_{ijk} + \tau_{ij}^{-1} X_{ijk} \frac{e^{\bar{z}^{a*}_{ijk}} g^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk})}{\int_{\bar{z}^{a*}_{ijk}}^{+\infty} e^z g^{\bar{z},a}_{ijk}(z) dz} \right) \end{aligned}$$

Hence,

$$\frac{\partial \log X_{ijk}}{\partial \log \tau_{ij}} = \frac{\partial \log f^\tau(\tau_{ij})}{\partial \log \tau_{ij}} \left(1 + \frac{e^{\bar{z}^{a*}_{ijk}} g^{\bar{z},a}_{ijk}(\bar{z}^{a*}_{ijk})}{\int_{\bar{z}^{a*}_{ijk}}^{+\infty} e^z g^{\bar{z},a}_{ijk}(z) dz} \right).$$

B Proof of Result 1

Result 1 *Under a mild set of assumptions, the extensive margin of the partial trade elasticity is larger under uncertainty compared to the complete information*

Consider function $\gamma(x)$, the extensive margin, where the superscript on function $g^x(.)$ has been omitted for notational compactness:

$$\gamma(x) = \frac{e^x g(x)}{\int_x^{+\infty} e^u g(u) du}.$$

Proposition 1 below establishes two novel properties of the extensive margin $\gamma(x)$.

Proposition 1 *Let $g(x)$ be a probability density function satisfying A1. Then the following hold.*

(i) $\gamma(x) \equiv [e^x g(x)] / \int_x^{+\infty} e^z g(z) dz$ *is an increasing function of* x.

(ii) Denote the extensive margin elasticity associated with $g(x)$ *as* $\gamma(x)$. *Let* $\tilde{g}(x)$ *be a mean preserving spread of* $g(x)$, *with extensive margin elasticity* $\tilde{\gamma}(x)$. *Then* $\gamma(x)$ *and* $\tilde{\gamma}(x)$ *satisfy the single crossing property. That is, there exists* x^* *such that* $\tilde{\gamma}(x) \leq \gamma(x)$ *for all* $x \geq x^*$, *and* $\tilde{\gamma}(x) \geq \gamma(x)$ *for all* $x \leq x^*$.

Proof of Proposition 1

Part (i) First, define $h(x) = (e^x g(x))/E$, where $E = \int_{-\infty}^{+\infty} e^x g(x) dx$. Notice that $h(x)$ is positive for all x and that $\int_{-\infty}^{+\infty} h(x) dx = 1$. Hence, $h(x)$ is a probability density function. The corresponding cumulative density function is given by $H(x) = \int_{-\infty}^{x} e^z g(z) dz/E$. The corresponding survival function is given by $1 - H(x) = \int_x^{+\infty} e^z g(z) dz/E$.

Next, function $\gamma(x)$ can then be written as

$$\gamma(x) \quad = \quad \frac{e^x g(x)}{\int_x^{+\infty} e^z g(z) dz} = \frac{h(x)}{1 - H(x)}.$$

Hence, $\gamma(x)$ is a hazard rate associated with the distribution $H(x)$. By Theorem 10 in Rinne (2014), the hazard rate $\gamma(x)$ is monotonically increasing in x if and only if its logarithmic survival function, $\log(1 - H(x))$, is concave. Notice that by part (iii) of A1, $\log(1 - H(x))$ is a concave function of x. Hence, $\gamma(x)$ is increasing in x. For completeness, we reproduce the proof of this result below.

Notice that

$$\gamma(x) = -\frac{d \log(1 - H(x))}{dx}.$$

Hence,

$$\frac{d\gamma(x)}{dx} = -\frac{d^2 \log(1 - H(x))}{dx^2}.$$

Since $\log(1 - H(x))$ is a concave function of x, $d^2 \log(1 - H(x))/dx^2 < 0$. Therefore, $d\gamma(x)/dx > 0$.

Part (ii) Function $\tilde{\gamma}(x)$ is given by

$$\tilde{\gamma}(x) \quad = \quad \frac{e^x \tilde{g}(x)}{\int_x^{+\infty} e^z \tilde{g}(z) dz} = \frac{\tilde{h}(x)}{1 - \tilde{H}(x)},$$

where $\tilde{g}(.)$ is a mean preserving spread of $g(.)$, $\tilde{h}(x) = [e^x \tilde{g}(x)] / \int_{-\infty}^{+\infty} e^x \tilde{g}(x) dx$, and $\tilde{H}(x)$ is the corresponding cumulative distribution function.

$\gamma(x) > \tilde{\gamma}(x)$ if and only if $H(x) > \tilde{H}(x)$ as follows for the following set of equivalent inequalities:

$$\begin{aligned}
\gamma(x) = -\frac{d\log(1-H(x))}{dx} &> -\frac{d\log(1-\tilde{H}(x))}{dx} = \tilde{\gamma}(x) \\
d\log(1-H(x)) &< d\log(1-\tilde{H}(x)) \\
\int d\log(1-H(x)) &< \int d\log(1-\tilde{H}(x)) \\
\log(1-H(x)) &< \log(1-\tilde{H}(x)) \\
H(x) &> \tilde{H}(x).
\end{aligned}$$

We will now show in three steps that $H(x)$ crosses $\tilde{H}(x)$ once from below, and therefore there exists x^* such that $H(x) > \tilde{H}(x)$ holds for $x > x^*$, and therefore (ii) holds.

Step 1: Denote by X and $\tilde{X}$ random variables distributed according to $g(x)$ and $\tilde{g}(x)$ respectively. Since $\tilde{g}(x)$ is a mean preserving spread of $g(x)$, it holds that $\tilde{X} = X + \hat{X}$, where $\hat{X}$ is distributed according to $\hat{g}(x)$ with mean zero, and $\hat{X}$ is independent from X. Hence, $\tilde{g}(.)$ is a convolution of $g(.)$ and $\hat{g}(.)$ and can be written as

$$\tilde{g}(x) = \int_{-\infty}^{+\infty} g(x-u)\hat{g}(u)du.$$

Step 2: Denote by X^h, $\tilde{X}^h$, $\hat{X}^h$ random variables distributed according to $h(x)$, $\tilde{h}(x)$, and $\hat{h}(x)$ respectively, where $\hat{h}(x) = [e^x\hat{g}(x)]/\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx$. Similarly, it can be show that $\tilde{h}(.)$ is a convolution of $h(.)$ and $\hat{h}(.)$:

$$\begin{aligned}
\int_{-\infty}^{+\infty} h(x-u)\hat{h}(u)du &= \frac{\int_{-\infty}^{+\infty} e^{x-u}g(x-u)e^u\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{\int_{-\infty}^{+\infty} e^x g(x-u)\hat{g}(u)du}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \\
&= \frac{e^x\tilde{g}(x)}{\left[\int_{-\infty}^{+\infty} e^x g(x)dx\right]\cdot\left[\int_{-\infty}^{+\infty} e^x\hat{g}(x)dx\right]} = \tilde{h}(x).
\end{aligned}$$

Thus, it hold that $\tilde{X}^h = X^h + \hat{X}^h$, where $\tilde{X}^h$ and $\hat{X}^h$ are independent.

Step 3: Consider a random variable $\bar{X} = X^h + \hat{X}^h - E(\hat{X}^h)$ with the cumulative distribution function denoted by $\bar{H}(x)$. $\bar{X}$ is a mean preserving spread of X^h and therefore the two corresponding cumulative distribution functions satisfy the single-crossing property whereby $H(x) = \bar{H}(x)$ if $x = E(X^h)$; $H(x) < \bar{H}(x)$ for $x < E(X^h)$, and $H(x) > \bar{H}(x)$ for $x > E(X^h)$.

Next, notice that $\tilde{X}^h = \bar{X} + E(\hat{X}^h)$. Therefore the cumulative distribution function of $\tilde{X}^h$ is a shift of the cumulative distribution function of $\bar{X}$ along the x-axis, namely $\tilde{H}(x) = \bar{H}(x - E(\hat{X}^h))$. Hence $\tilde{H}(x)$ preserves the same single-crossing property with respect to $H(x)$. Namely $\exists x^*$ that that $H(x) = \tilde{H}(x)$ if $x = x^*$; $H(x) < \tilde{H}(x)$ for $x < x^*$, and $H(x) > \tilde{H}(x)$ for $x > x^*$. ■

Part (ii) of Proposition 1 shows that the extensive margin elasticity as a function of threshold values $x \in \mathbb{R}$ exhibits a single crossing property. The single crossing property establishes that the extensive margin elasticity function associated with the cumulative distribution function of the mean preserving spread, $\tilde{\gamma}(x)$, only crosses the extensive margin elasticity function associated with the less dispersed cumulative distribution function, $\gamma(x)$, once from above.

To provide further intuition for the single-crossing property of function $\gamma(.)$, recall that, by part (ii) of Proposition 1, $\tilde{g}(x)$ is a mean preserving spread of $g(x)$. Hence $G(x)$ crosses $\tilde{G}(x)$ once from below. As shown in the proof of Proposition 1 above, this single-crossing property is also preserved when defining a distribution according to the transformation in equation (17), and is also preserved by $\gamma(x)$. Therefore, $\gamma(x)$ also crosses $\tilde{\gamma}(x)$ from below once, and for all values of x sufficiently large we know $\gamma(x) > \tilde{\gamma}(x)$.

A corollary of part (ii) of Proposition 1 is that the single crossing property of the extensive margin holds for any affine transformation of the abscissa for either of the functions.

Corollary 1 *Let $g(x)$ be a probability density function satisfying A1. For all $a \in \mathbb{R}$ there exists $x^*(a)$ such that $\gamma(x) \geq \tilde{\gamma}(x + a)$ if $x \geq x^*(a)$, and $\gamma(x) \leq \tilde{\gamma}(x + a)$ if $x \leq x^*(a)$.*

Proof of Corollary 1

Notice that part (ii) of Proposition 1 implies a single crossing property of $\gamma(.)$ and $\tilde{\gamma}(.)$. This property is preserved under an affine transformation of the abscissa for either of the functions. Therefore, $\gamma(x)$ also crosses $\tilde{\gamma}(x + a)$ from above for some $x^*(a)$.■

Together, Proposition 1 and Corollary 1 imply that the extensive margin of the partial trade elasticity is larger under incomplete information.

Observe from equations (8) and (12) that the difference in the partial trade elasticity between the two information environments arises from the extensive margin component. Further, the difference in the extensive margin arises from two separate channels: the entry threshold, z^*_{ijk} versus $\bar{z}^{a*}_{ijk}$, and the distribution of the corresponding shock, $g^z_{ijk}(.)$ versus $g^{\bar{z},a}_{ijk}(.)$. Assume that both distributions satisfy Assumption 1. It is worth noting that distributions that are commonly used in the trade literature, such as Normal and Double EMG distributions, satisfy A1.

Recall that profitability shock z_{ijk} is defined as $z_{ijk} = \alpha^a z^a_{ijk} + \alpha^p z^p_{ijk} = \bar{z}^a_{ijk} + \alpha^p z^p_{ijk}$, where $\bar{z}^a_{ijk}$ and z^p_{ijk} are independent and are drawn from the probability density functions $g^{\bar{z},a}_{ijk}(.)$ and $g^p_{ijk}(.)$ respectively. Without loss of generality we can assume that the mean of $g^{\bar{z},a}_{ijk}(.)$ equals zero. In this case, $g^z_{ijk}(.)$ is a mean-preserving spread of $g^p_{ijk}(.)$. Hence, by part (ii) of Proposition 1 we know that $\gamma^U(x) > \gamma^{CI}(x)$, where 'U' stands for Uncertainty and 'CI' stands for Complete Information, for a sufficiently high entry threshold x, and therefore trade is more elastic under incomplete information given the same threshold value.

We, next, show that $\gamma^U(x) > \gamma^{CI}(x)$ holds even if the entry thresholds are different. From equations (42) and (50), the entry threshold $\bar{z}^{a*}_{ijk}$ under incomplete information versus z^*_{ijk} under complete information are related as follows: $\bar{z}^{a*}_{ijk} = z^*_{ijk} - \log\left(E_{z^p_{ijk}}\left(\exp(z^p_{ijk}/\epsilon_k)\right)\right)^{\epsilon_k}$. Therefore, the proper comparison of elasticities involves comparing $\gamma^U(x)$ and $\gamma^{CI}(x+a)$, where constant $a \equiv \log\left(E_{z^p_{ijk}}\left(\exp(z^p_{ijk}/\epsilon_k)\right)\right)^{\epsilon_k}$. By Corollary 1, for $\bar{z}^{a*}_{ijk}$ high enough, $\gamma^U(\bar{z}^{a*}_{ijk}) > \gamma^{CI}(\bar{z}^{a*}_{ijk} + a) = \gamma^{CI}(z^*_{ijk})$, i.e. trade is less elastic under complete information.

C A model with endogenous quality choice

In this section we introduce endogenous quality choice along the lines of Kugler and Verhoogen (2012) into our baseline model of trade, and demonstrate that the identification of the partial trade elasticity relies on using the sales data alone under complete information, and relies on using the quantity data under uncertainty, as suggested in the baseline model.

C.1 Demand

Consider preferences described in equation (1) where consumption $c_{ijk}(\omega)$ of variety ω is augmented by the quality of the corresponding variety, $\lambda_{ijk}(\omega)$:

$$U_j = \prod_{k=1}^{K}\left[\left(\sum_{i=1}^{N}\int_{\omega\in\Omega_{ijk}}\left(e^{z^p_{ijk}(\omega)}\right)^{\frac{1}{\epsilon_k}}\left(\lambda_{ijk}(\omega)c_{ijk}(\omega)\right)^{\frac{\epsilon_k-1}{\epsilon_k}}d\omega\right)^{\frac{\epsilon_k}{\epsilon_k-1}}\right]^{\mu_k}. \quad (53)$$

Cost minimization problem by consumers yields the optimal demand equation given by

$$c_{ijk}(\omega) = e^{z^p_{ijk}(\omega)}\lambda_{ijk}(\omega)^{\epsilon_k-1}p_{ijk}(\omega)^{-\epsilon_k}Y_{jk}P^{\epsilon_k-1}_{jk}. \quad (54)$$

C.2 Supply

The supply side of the economy is the same as that described in Section 2.2 with the amendment that firms face quality production costs as in Kugler and Verhoogen (2012). In particular, producing a good of quality λ in sector k requires incurring quality-upgrading cost given by $\lambda^{\beta_k}/\beta_k$ denominated in the units of labor, where $\beta_k > \epsilon_k - 1$. Additionally, for a given labor productivity of a firm, z^a_{ijk}, producing a good of higher quality requires more labor: The unit labor requirement into producing a good of quality λ is given by $\lambda^{\gamma_k}/z^a_{ijk}$, where $0 < \gamma_k < 1$.

C.3 Information Structure

We follow the information structure described in Section 2.3. We continue to distinguish between an ex ante productivity, z^a_{ijk}, and an ex post shocks demand, z^p_{ijk}, shocks, and define the cumulative profitability shock as $z_{ijk} = \alpha^a z^a_{ijk} + \alpha^p z^p_{ijk}$.

C.4 Environment with Complete Information

The problem of the firm is given by

$$\pi_{ijk}(z_{ijk}) = \max_{q_{ijk},\lambda_{ijk}} p_{ijk}(z^p_{ijk})q_{ijk} - \frac{w_i\tau_{ij}}{e^{z^a_{ijk}}}\lambda^{\gamma_k}_{ijk}q_{ijk} - w_i\frac{1}{\beta_k}\lambda^{\beta_k}_{ijk} - w_i f_{ijk}, \tag{55}$$

subject to the demand equation (54). The resulting optimal quality, quantity, and revenue of the firm are given by

$$\begin{aligned}
\lambda_{ijk} &= \left((1-\gamma_k)\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} P^{\epsilon_k-1}_{jk}Y_{jk}w_i^{-\epsilon_k}\right)^{\frac{1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}} \times \\
&\quad \times \tau_{ij}^{\frac{1-\epsilon_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}} e^{\frac{\epsilon_k-1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}z^a_{ijk}+\frac{1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}z^p_{ijk}} \\
q_{ijk} &= \left((1-\gamma_k)^{\frac{(\beta_k-\gamma_k)-(\beta-(1-\gamma_k)(\epsilon_k-1))}{\beta_k-\gamma_k}}\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} P^{\epsilon_k-1}_{jk}Y_{jk}w_i^{-\epsilon_k}\right)^{\frac{\beta_k-\gamma_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}} \times \\
&\quad \times \tau_{ij}^{\frac{(1-\epsilon_k)(\beta_k-\gamma_k)}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}-1} e^{\left(\frac{(\epsilon_k-1)(\beta_k-\gamma_k)}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}+1\right)z^a_{ijk}+\frac{1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}z^p_{ijk}}
\end{aligned}$$

$$\begin{aligned}
r_{ijk}(z_{ijk}) &= \underbrace{\left((1-\gamma_k)^{1-\gamma_k}\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\beta_k-\gamma_k+1}\left(P^{\epsilon_k-1}_{jk}Y_{jk}\right)^{\frac{\beta_k}{\epsilon_k-1}} w_i^{-(\beta_k-\gamma_k+1)}\right)^{\frac{\epsilon_k-1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}}}_{B_{ijk}} \times \\
&\quad \times \underbrace{\tau_{ij}^{\frac{(1-\epsilon_k)\beta_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}}}_{f^\tau(\tau_{ij})} \underbrace{e^{z_{ijk}}}_{f^z(z_{ijk})}
\end{aligned} \tag{56}$$

where z_{ijk} is given by

$$z_{ijk} = \underbrace{\frac{\beta_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}(\epsilon_k-1)}_{\alpha^a} z^a_{ijk} + \underbrace{\frac{\beta_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}}_{\alpha^p} z^p_{ijk}.$$

Notice that the revenue function admits the same general form as in the baseline model where function $f^{\tau}(.)$ is augmented with parameters of the quality production function, γ_k and β_k, yet preserves the monotonically decreasing property. The profitability shock is also defined as the weighted sum of the productivity and demand shocks with weights being augmented by the parameters of the quality production function. Selection into exporting is determined by the zero profit condition

$$\pi_{ijk}(z^*_{ijk}) = 0,$$

or equivalently

$$r_{ijk}(z^*_{ijk})\frac{\beta_k - (1-\gamma_k)(\epsilon_k - 1)}{\beta_k \epsilon_k} = w_i f_{ijk} \tag{57}$$

Given that the revenue function admits the same general form as in the baseline model (see equation (5)), the calculations of trade flows, X_{ijk}, and the partial trade elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$, are identical to those in the baseline model with complete information.

C.5 Environment with Uncertainty

The problem of the firm is given by

$$E_{z^p_{ijk}}\pi_{ijk}(z^a_{ijk}, z^p_{ijk}) = \max_{q_{ijk},\lambda_{ijk}} E_{z^p_{ijk}}\left(p_{ijk}(z^p_{ijk})q_{ijk} - \frac{w_i\tau_{ij}}{e^{z^a_{ijk}}}\lambda^{\gamma_k}_{ijk}q_{ijk} - w_i\frac{1}{\beta_k}\lambda^{\beta_k}_{ijk}\right) - w_i f_{ijk}, \tag{58}$$

subject to the demand equation (54). The resulting optimal quality, quantity, and revenue of the firm are given by

$$\begin{aligned}
\lambda_{ijk} &= \left((1-\gamma_k)\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} E_{z^p_{ijk}}\left[e^{\frac{z^p_{ijk}}{\epsilon_k}}\right]^{\epsilon_k} P^{\epsilon_k-1}_{jk}Y_{jk}w_i^{-\epsilon_k}\right)^{\frac{1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}} \times \\
&\quad \times \tau_{ij}^{\frac{1-\epsilon_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}} e^{\frac{\epsilon_k-1}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}z^a_{ijk}} \\
q_{ijk} &= \left((1-\gamma_k)^{\frac{(\beta_k-\gamma_k)-(\beta-(1-\gamma_k)(\epsilon_k-1))}{\beta_k-\gamma_k}}\left(\frac{\epsilon_k-1}{\epsilon_k}\right)^{\epsilon_k} E_{z^p_{ijk}}\left[e^{\frac{z^p_{ijk}}{\epsilon_k}}\right]^{\epsilon_k} P^{\epsilon_k-1}_{jk}Y_{jk}w_i^{-\epsilon_k}\right)^{\frac{\beta_k-\gamma_k}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}} \times \\
&\quad \times \tau_{ij}^{\frac{(1-\epsilon_k)(\beta_k-\gamma_k)}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}-1} e^{\left(\frac{(\epsilon_k-1)(\beta_k-\gamma_k)}{\beta_k-(1-\gamma_k)(\epsilon_k-1)}+1\right)z^a_{ijk}}
\end{aligned}$$

$$r_{ijk}(z^a_{ijk}, z^p_{ijk}) =$$

$$= \underbrace{\left((1-\gamma_k)^{1-\gamma_k} \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\beta_k - \gamma_k + 1} E_{z^p_{ijk}} \left[e^{\frac{z^p_{ijk}}{\epsilon_k}} \right]^{\beta_k - \gamma_k + 1} \left(P^{\epsilon_k - 1}_{jk} Y_{jk}\right)^{\frac{\beta_k}{\epsilon_k - 1}} w_i^{-(\beta_k - \gamma_k + 1)} \right)^{\frac{\epsilon_k - 1}{\beta_k - (1-\gamma_k)(\epsilon_k - 1)}}}_{B_{ijk}} \times$$

$$\times \underbrace{\tau_{ij}^{\frac{(1-\epsilon_k)\beta_k}{\beta_k - (1-\gamma_k)(\epsilon_k - 1)}}}_{f^\tau(\tau_{ij})} \underbrace{e^{\frac{1}{\epsilon_k} z^p_{ijk}}}_{f^p(z^p_{ijk})} \underbrace{e^{\frac{\beta_k}{\beta_k - (1-\gamma_k)(\epsilon_k - 1)}(\epsilon_k - 1) z^a_{ijk}}}_{f^a(z^a_{ijk})} \tag{59}$$

Notice that the revenue function admits the same general form as in the baseline model with the same caveats as discussed in the complete information case. Selection into exporting is determined by zero expected profit condition

$$E_{z^p_{ijk}} \pi_{ijk}(z^{a*}_{ijk}, z^p_{ijk}) = 0, \tag{60}$$

which determines the entry threshold z^{a*}_{ijk}.

Given that the revenue function admits the same general form as in the baseline model (see equation (10)), the calculations of trade flows, X_{ijk}, and the partial trade elasticity, $\partial \log X_{ijk} / \partial \log \tau_{ij}$, are identical to those in the baseline model with uncertainty.

C.6 Characterization of Trade Elasticites

As argued in Sections C.4 and C.5 , revenue function, r_{ijk} and the partial trade elasticity, $\partial \log X_{ijk} / \partial \log \tau_{ij}$, in a model with endogenous quality choice admit the same general form and are identical to those in the baseline model. Hence, comparison and the identification of the trade elasticities are identical to the baseline model.

Under complete information, the distribution of log-export sales identifies the distribution and the entry threshold value of the profitability shock. With endogenous quality choice, the weights on ex ante and ex post shocks in the definition of the profitability shock are augmented with the quality production function parameters. These weights however do not impact the relation between the profitability shocks and the log-export sales.

Under uncertainty, the log-export sales data embeds both the ex ante and ex post shocks, while only weighted ex ante shocks determine selection in exporting and the extensive margin of the partial trade elasticity. Hence, similarly to the baseline model, one needs the log-quantity data to recover the underlying distribution of the weighted ex ante shock. With endogenous quality choice, the weight on the ex ante shock is augmented with the quality production function parameters. This weight however does not impact the relation between the weighted ex ante shocks and the log-export quantity.

D A Model with Variable Markups

In this section we examine the implications for identification of trade elasticities in a model with variable markups along the lines of Melitz and Ottaviano (2008), and demonstrate that the identification of the partial trade elasticity relies on using the sales (and, in this case, quantity) data under complete information, and relies on using the quantity data alone under uncertainty, as suggested in the baseline model. To ease explication, we abstract from multi-sector environment and consider a model with one differentiated goods sector.

D.1 Demand

Preferences of a representative consumer c in country j are represented by the following utility function

$$U_j^c = q_{j0}^c + \sum_{i=1}^{N} \left(\int_{\Omega_{ij}} z_{ij}^p(\omega) q_{ij}^c(\omega) d\omega - \frac{\gamma}{2} \int_{\Omega_{ij}} q_{ij}^c(\omega)^2 d\omega - \frac{\eta}{2} \left(\int_{\Omega_{ij}} q_{ij}^c(\omega) \omega \right)^2 \right),$$

where N is the number of countries, q_{j0}^c is the consumption of the homogeneous good zero, Ω_{ij} is the set of varieties sold from country i to country j, $q_{ij}^c(\omega)$ is the consumption of variety ω by consumer c, $z_{ij}^p(\omega)$ is the demand shock for variety ω. Utility maximization problem yields the inverse demand equation for variety ω by consumer c given by

$$p_{ij}(\omega) = z_{ij}^p(\omega) - \gamma q_{ij}^c(\omega) - \eta Q_{ij}^c, \tag{61}$$

where $Q_{ij}^c \equiv \int_{\Omega_{ij}} q_{ij}^c(\omega) d\omega$. The total inverse demand for variety ω can be written as

$$p_{ij}(\omega) = p_{ij}^{\max} + z_{ij}^p(\omega) - \frac{\gamma}{L_j} q_{ij}(\omega) \tag{62}$$

where $q_{ij}(\omega) \equiv L_j q_{ij}^c(\omega)$ and $p_{ij}^{\max} = \eta N_{ij} \left(\frac{\bar{p}_{ij} - \bar{\theta}_{ij}}{\gamma + \eta N_{ij}} \right)$, $N_{ij} \equiv \int_{\Omega_{ij}} d\omega$, $\bar{z}_{ij}^p \equiv \int_{\Omega_{ij}} z_{ij}^p(\omega) d\omega / N_{ij}$, $\bar{p}_{ij} \equiv \int_{\Omega_{ij}} p_{ij}(\omega) d\omega / N_{ij}$.

D.2 Supply

The supply side assumptions follow those of Melitz and Ottaviano (2008). The homogeneous good zero is the numeraire good that is produced under perfect competition using the constant returns to scale production function with unit labor productivity. Each variety in the differentiated good sector is produced by a firm. Each firm draws its labor productivity (demand) level, z_{ij}^a (z_{ij}^p), from a probability distribution denoted by $g_{ij}^a(.)$ ($g_{ij}^p(.)$). Firms face variable 'iceberg' trade costs denoted by τ_{ij}. There are no fixed costs of production.

D.3 Information Structure

We follow the information structure described in Section 2.3. We distinguish between an ex ante productivity shock, z_{ij}^a, and an ex post demand shock, z_{ij}^p.

D.4 Environment with Complete Information

The problem of the firm is given by

$$\pi_{ij}(z_{ij}^a, z_{ij}^p) = \max_{q_{ij} \geq 0} p_{ij}(z_{ij}^p) q_{ij} - \frac{w_i \tau_{ij}}{e^{z_{ij}^a}} q_{ij}$$

subject to the demand equation (62). The resulting optimal quantity and revenue of the firm is given by

$$q_{ij}(z_{ij}^a, z_{ij}^p) = \frac{L_j}{2\gamma}\left(p_{ij}^{\max} + z_{ij}^p - \frac{w_i \tau_{ij}}{e^{z_{ij}^a}}\right) \quad (63)$$

$$r_{ij}(z_{ij}^a, z_{ij}^p) = \frac{L_j}{4\gamma}\left((p_{ij}^{\max} + z_{ij}^p)^2 - \left(\frac{w_i \tau_{ij}}{e^{z_{ij}^a}}\right)^2\right) \quad (64)$$

for $q_{ij} \geq 0$. If $q_{ij} < 0$ the firm does not export. Hence, selection into exporting is determined by

$$p_{ij}^{\max} + z_{ij}^p - \frac{w_i \tau_{ij}}{e^{z_{ij}^a}} \geq 0 \quad (65)$$

Inequality (65) implicitly defines a boundary in the (z_{ij}^a, z_{ij}^p) space such that, given a draw of the two shocks, a firm exports if inequality (65) holds and does not export otherwise.

Renormalization of shocks: Notice from equations (63) and (64), and inequality (65), that in a model with variable markups, the two shocks do not aggregate to a single decision relevant statistic that we referred to as the profitability shock in Section 2.3.

To proceed forward, we need to redefine shocks as follows. Let $u_{ij}^p \equiv p_{ij}^{\max} + z_{ij}^p$, and let $u_{ij}^a \equiv \frac{w_i}{e^{z_{ij}^a}}$. Notice that u_{ij}^p and u_{ij}^a are monotonic transformations of z_{ij}^p and z_{ij}^a. The sales, quantity, and selection equations can be written as

$$r_{ij}(u_{ij}^p, u_{ij}^a) = \frac{L_j}{4\gamma}\left((u_{ij}^p)^2 - (\tau_{ij} u_{ij}^a)^2\right) \quad (66)$$

$$q_{ij}(u_{ij}^p, u_{ij}^a) = \frac{L_j}{2\gamma}\left(u_{ij}^p - \tau_{ij} u_{ij}^a\right) \quad (67)$$

$$\text{if } u_{ij}^p - \tau_{ij} u_{ij}^a \geq 0. \quad (68)$$

Trade Elasticity: The total trade flows are defined as

$$X_{ij} = J_i Prob(u^p_{ij} > \tau_{ij} u^a_{ij}) \int_{u^p_{ij} > \tau_{ij} u^a_{ij}} r_{ij}(u^p_{ij}, u^a_{ij}) \frac{g^p_{ij}(u^p_{ij}) g^a_{ij}(u^a_{ij})}{Prob(u^p_{ij} > \tau_{ij} u^a_{ij})} du^p_{ij} du^a_{ij}, \tag{69}$$

where J_i is the mass of potential entrants. Functions $g^p_{ij}(.)$ and $g^a_{ij}(.)$ are the probability density functions of u^p_{ij} and u^a_{ij} respectively. The partial trade elasticity is given by

$$\frac{\partial \log X_{ij}}{\partial \log \tau_{ij}} = -2 \left(\frac{\int_{-\infty}^{+\infty} (u^p_{ij})^2 G^a_{ij}(u^p_{ij}/\tau_{ij}) g^p_{ij}(u^p_{ij}) du^p_{ij}}{\int_{-\infty}^{+\infty} (\tau_{ij} u^a_{ij})^2 \left(1 - G^p_{ij}(\tau_{ij} u^a_{ij})\right) g^a_{ij}(u^a_{ij}) du^a_{ij}} - 1 \right)^{-1}. \tag{70}$$

D.5 Environment with Uncertainty

The problem of the firm is given by

$$E_{z^p_{ij}} \pi_{ij}(z^a_{ij}, z^p_{ij}) = \max_{q_{ij} \geq 0} E_{z^p_{ij}} \left(p_{ij}(z^p_{ij}) q_{ij} - \frac{w_i \tau_{ij}}{e^{z^a_{ij}}} q_{ij} \right)$$

subject to the demand equation (62). The resulting optimal quantity and revenue of the firm is given by

$$q_{ij}(z^a_{ij}) = \frac{L_j}{2\gamma} \left(p^{\max}_{ij} + E(z^p_{ij}) - \frac{w_i \tau_{ij}}{e^{z^a_{ij}}} \right) \tag{71}$$

$$r_{ij}(z^a_{ij}, z^p_{ij}) = \frac{L_j}{4\gamma} \left(p^{\max}_{ij} + E(z^p_{ij}) - \frac{w_i \tau_{ij}}{e^{z^a_{ij}}} \right) \left(p^{\max}_{ij} + 2 z^p_{ij} - E(z^p_{ij}) + \frac{w_i \tau_{ij}}{e^{z^a_{ij}}} \right) \tag{72}$$

for $q_{ij} \geq 0$. If $q_{ij} < 0$ the firm does not export. Hence, selection into exporting is determined by

$$p^{\max}_{ij} + E(z^p_{ij}) - \frac{w_i \tau_{ij}}{e^{z^{a*}_{ij}}} = 0 \tag{73}$$

Equation (73) pins down a unique entry threshold associated with a productivity draw, i.e. an ex ante component of the profitability shock. Hence, as is the case in the baseline model with uncertainty, it is only the ex ante shocks that determine selection into exporting. Further, notice from equation (72) that export sales contain information regarding the realization of both ex ante and ex post shocks. As a result, sales data must be complemented with the quantity data to isolate the ex ante component of a firm's profitability that determines selection into exporting.

Renormalization of shocks: Following the notation in Section D.4, let $u^a_{ij} \equiv \frac{w_i}{e^{z^a_{ij}}}$ and

$u_{ij}^p \equiv p_{ij}^{\max} + z_{ij}^p$. The sales, quantity, and selection equations can then be written as

$$r_{ij}(u_{ij}^p, u_{ij}^a) = \frac{L_j}{4\gamma}\left(E(u_{ij}^p) - \tau_{ij}u_{ij}^a\right)\left(2u_{ij}^p - E(u_{ij}^p) + \tau_{ij}u_{ij}^a\right) \quad (74)$$

$$q_{ij}(u_{ij}^a) = \frac{L_j}{2\gamma}\left(E(u_{ij}^p) - \tau_{ij}u_{ij}^a\right) \quad (75)$$

$$\text{if } E(u_{ij}^p) - \tau_{ij}u_{ij}^a \geq 0. \quad (76)$$

Trade Elasticity: The total trade flows are defined as

$$X_{ij} = J_i(1 - G_{ij}^a(u_{ij}^{a*}))\int_{u_{ij}^{a*}}^{+\infty}\int_{-\infty}^{+\infty} r_{ij}(u_{ij}^a, u_{ij}^p)\frac{g_{ij}^a(u_{ij}^a)g_{ij}^p(u_{ij}^p)}{1 - G_{ij}^a(u_{ij}^{a*})}du_{ij}^p du_{ij}^a,$$

where J_i is the mass of potential entrants, functions $g_{ij}^a(.)$ and $g_{ij}^p(.)$ are the probability density functions of u_{ij}^a and u_{ij}^p respectively, and $G_{ij}^a(.)$ is the cumulative distribution function of u_{ij}^a. Notice that in contrast to the trade flows under the complete information environment as stated in equation (69), the selection into exporting under uncertainty is based on a single entry threshold associated with an ex ante profitability shock, u_{ij}^{a*}. The partial elasticity of trade flows is given by

$$\frac{\partial \log X_{ij}}{\partial \log \tau_{ij}} = -2\left(\frac{\int_{u_{ij}^{a*}}^{+\infty}\left(\frac{u_{ij}^a}{u_{ij}^{a*}}\right)^2 g_{ij}^a(u_{ij}^a)du_{ij}^a}{1 - G_{ij}^a(u_{ij}^{a*})} - 1\right)^{-1}. \quad (77)$$

D.6 Characterization of Trade Elasticites

Under complete information, selection into exporting is given by the boundary in the (u_{ij}^a, u_{ij}^p) space defined in equation (68). Hence, the partial trade elasticity in this case depends on the distribution of both of the corresponding shocks. Both of these distributions can be jointly recovered using the sales and quantity data using equations (66) and (67). By assuming a parametric distribution for u_{ij}^a and for u_{ij}^p, and observing the data on quantity and sales, one can estimate the parameters of these distributions using the maximum likelihood method.

Under uncertainty, equation (77) shows that, in contrast to the complete information environment, the partial trade elasticity is governed solely by the distribution of an ex ante profitability shock, u_{ij}^a. The distribution of this shock can be recovered from the quantity data as can be seen in equation (71).

In sum, the main message about identifying elasticities developed in the baseline model in Section 3 also holds in the environment with variable markups: different data identify trade elasticities under complete versus incomplete information environments. In the Melitz and Ottaviano (2008) framework considered in this section, one requires quantity and sales data to identify trade elasticities under complete information environment, and one requires quantity data to identify trade elasticities under uncertainty.

E Robustness

In this section we demonstrate the robustness of our theoretical and quantitative results to the way we choose to model a firm's decision under uncertainty. In the main text we assume that in a model with uncertainty, firms choose export quantities before demand shocks are realized. This assumption is consistent with the majority of the literature on learning such as Timoshenko (2015b), Arkolakis et al. (2018), Berman et al. (2019). In contrast to this literature, in this section we assume that firms choose prices before demand shocks are realized. Below, we present an alternative representation of the model with uncertainty in line with this assumption. In this model, given that firms choose prices, the price data contain information necessary to identify the partial elasticity of trade flows with respect to variable trade costs. We subsequently quantify trade elasticities according to this insight. Our quantitative results are unaffected by the change in the firm's choice variable.

The intuition for this equivalence lies in the fact that the price and quality are inversely related through sales. In a model with uncertainty where firms choose quantities, the empirical distribution of quantity identifies the underlying theoretical distribution of productivities. In a model with uncertainty where firm choose prices, the optimal price equals to the inverse of productivity. Therefore, theoretical productivity distribution is identified by the empirical distribution of the inverse of the prices, which is proportional to the empirical distribution of quantities through the equation quantity = sales/price.

E.1 Alternative Model of Demand Uncertainty

The economic environment and demand are the same as in Section 2.

$$c_{ijk}(\omega) = e^{z^p_{ijk}(\omega)} p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1}, \tag{78}$$

E.1.1 Supply

For each destination and industry firms maximize expected profits given by

$$E_{z^p_{ijk}}[\pi_{ijk}(z^a_{ijk}, z^p_{ijk})] = \max_{p_{ijk}} E_{z^p_{ijk}} \left(p_{ijk}(z^p_{ijk}) q_{ijk} - \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}} q_{ijk} \right) - w_i f_{ijk},$$

subject to the demand equation (78). The expectation over the demand draw, z^p_{ijk}, is given by the distribution from which the demand parameter is drawn, $g^p_{ijk}(.)$. Substituting equation equation (78) into the objective function and applying the expectation operator yields the problem of the firm,

$$\max_{p_{ijk}(z^a_{ijk})} p_{ijk}(z^a_{ijk})^{1-\epsilon_k} E_{z^p_{ijk}}\left(e^{z^p_{ijk}}\right) Y_{jk} P_{jk}^{\epsilon_k-1} - \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}} E_{z^p_{ijk}}\left(e^{z^p_{ijk}}\right) p_{ijk}(\omega)^{-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1} - w_i f_{ijk}.$$

The first order conditions with respect to price yield the optimal price,

$$p_{ijk}(z^a_{ijk}) = \left(\frac{\epsilon_k}{\epsilon_k - 1}\right) \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}}. \tag{79}$$

A firm's realized revenue is then given by

$$r_{ijk}(z^a_{ijk}, z^p_{ijk}) = e^{z^p_{ijk}(\omega)} \left(\frac{\epsilon_k}{\epsilon_k - 1} \frac{w_i \tau_{ij}}{e^{z^a_{ijk}}}\right)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k-1}.$$

E.1.2 Entry

Firms enter the market as long as expected profit is positive. Hence, the optimal productivity entry threshold, z^{a*}_{ijk}, is a solution to the zero-expected profit condition given by

$$E_{z^p_{ijk}}[\pi(z^p_{ijk}, z^{a*}_{ijk})] = 0,$$

and is given by

$$e^{(\epsilon_k-1) z^{a*}_{ijk}} = \frac{\epsilon_k^{\epsilon_k} w_i f_{ijk} (w_i \tau_{ij})^{\epsilon_k - 1}}{(\epsilon_k - 1)^{\epsilon_k - 1} Y_{jk} P_{jk}^{\epsilon_k - 1} E_{z^p_{ijk}}\left(e^{z^p_{ijk}}\right)}. \tag{80}$$

E.1.3 Trade Elasticity

The aggregate trade flow from country i to country j in industry k is defined as

$$X_{ijk} = M_{ijk} \int_{z^{a*}_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(z^a_{ijk}, z^p_{ijk}) g^p_{ijk}(z^p_{ijk}) \frac{g^a_{ijk}(z^a_{ijk})}{Prob^a_{ijk}(z^a_{ijk} > z^{a*}_{ijk})} dz^p_{ijk} dz^a_{ijk}, \tag{81}$$

where M_{ijk} is the mass of firms exporting from country i to country j in industry k. Given the exogenous entry assumption, the mass of firms is given by

$$M_{ijk} = J_i \times Prob^a_{ijk}(z^a_{ijk} > z^{a*}_{ijk}),$$

where J_i is the exogenous mass of entrants. Equation (81) can then be simplified as follows:

$$\begin{aligned} X_{ijk} &= J_i \int_{z^{a*}_{ijk}}^{+\infty} \int_{-\infty}^{+\infty} r_{ijk}(z^a_{ijk}) g^p_{ijk}(z^p_{ijk}) g^a_{ijk}(z^a_{ijk}) dz^p_{ijk} dz^a_{ijk} = \\ &= J_i \left(\frac{\epsilon_k - 1}{\epsilon_k}\right)^{\epsilon_k - 1} E_{z^p_{ijk}}\left(e^{z^p_{ijk}}\right) (\tau_{ij} w_i)^{1-\epsilon_k} Y_{jk} P_{jk}^{\epsilon_k - 1} \int_{z^{a*}_{ijk}}^{+\infty} e^{(\epsilon_k - 1) z^a_{ijk}} g^a_{ijk}(z^a_{ijk}) dz^a_{ijk}. \end{aligned} \tag{82}$$

Differentiate equation (82) with respect to τ_{ij} to obtain

$$\frac{\partial X_{ijk}}{\partial \tau_{ij}} = (1-\epsilon_k)\frac{X_{ijk}}{\tau_{ijk}} - \frac{X_{ijk}}{\int_{z_{ijk}^{a*}}^{+\infty} e^{(\epsilon_k-1)z_{ijk}^a} g_{ijk}^a(z_{ijk}^a) dz_{ijk}^a} e^{(\epsilon_k-1)z_{ijk}^{a*}} g_{ijk}^a(z_{ijk}^{a*}) \frac{\partial z_{ijk}^{a*}}{\partial \tau_{ij}}. \quad (83)$$

Differentiate equation (80) with respect to τ_{ij} to obtain

$$\frac{\partial z_{ijk}^{a*}}{\partial \tau_{ij}} = \frac{1}{\tau_{ij}}. \quad (84)$$

Combine equations (83) and (84) to obtain the partial elasticity of trade flows with respect to the variable trade costs being given by

$$\eta_{ijk} \equiv \frac{\partial \ln X_{ijk}}{\partial \ln \tau_{ij}} = (1-\epsilon_k)\left(1 + \frac{e^{(\epsilon_k-1)z_{ijk}^{a*}} g_{ijk}^a(z_{ijk}^{a*})}{(\epsilon_k-1)\int_{z_{ijk}^{a*}}^{+\infty} e^{(\epsilon_k-1)z_{ijk}^a} g_{ijk}^a(z_{ijk}^a) dz_{ijk}^a}\right). \quad (85)$$

E.1.4 Estimation Approach

From equation (79), the distribution $g_{ijk}(.)$ can be directly recovered from the empirical distribution of the logarithm of the inverse of export price as follows:

$$\log\left(1/p_{ijk}(\varphi_{ijk})\right) = B_{ijk} + z_{ijk}^a. \quad (86)$$

Hence, the distribution of the logarithm of the inverse of export price is given by the distribution of z_{ijk}^a. Given the distribution of $g_{ijk}^a(.)$, the productivity entry threshold, z_{ijk}^{a*}, can be recovered from matching the empirical to the theoretical average-to-minimum ratio of the inverse of export prices. From equation (79) the average of the inverse of export price, $\widetilde{1/p}_{ijk}$, and the minimum of the inverse of export price , $(1/p)_{ijk}^{\min}$, are given by

$$\widetilde{1/p}_{ijk} = \frac{\epsilon_k-1}{\epsilon_k}(\tau_{ij}w_i)^{-1}\int_{z_{ijk}^{a*}}^{+\infty} \frac{e^{z_{ijk}^a} g_{ijk}^a(z_{ijk}^a)}{Prob_{ijk}^a(z_{ijk}^a > z_{ijk}^{a*})} dz_{ijk}^a$$

$$(1/p)_{ijk}^{\min} = \frac{\epsilon_k-1}{\epsilon_k}(\tau_{ij}w_i)^{-1} e^{z_{ijk}^{a*}}.$$

Hence, the average-to-minimum ratio, $\widetilde{1/p}_{ijk}/(1/p)_{ijk}^{\min}$, is given by

$$\text{Average-to-Minimum Ratio} = e^{-z_{ijk}^{a*}}\int_{z_{ijk}^{a*}}^{+\infty} \frac{e^{z_{ijk}^a} g_{ijk}^a(z_{ijk}^a)}{Prob_{ijk}^a(z_{ijk}^a > z_{ijk}^{a*})} dz_{ijk}^a. \quad (87)$$

E.2 Trade Elasticity Estimates

Table E1 replicates results in Table 2 and shows that the quantitative magnitude of the trade elasticities and the amplification effect remains robust to the alternative firm-level choice variable under uncertainty.

Table E1: Trade elasticity estimates, the log of the inverse of prices.

	Extensive Margin Elasticity, γ_{ijk}		Partial Trade Elasticity, $\partial \log X_{ijk}/\partial \log \tau_{ij}$	
Measure	Mean	Std. Dev.	Mean	Std. Dev.
Panel A: Estimates of trade elasticity				
Price based [a]	0.03	0.13	2.88	3.51
Sales based[b]	$1.7 \cdot 10^{-4}$	$8.8 \cdot 10^{-4}$	3.82	3.86
Panel B: Amplification effect				
Amplification effect [c]	$1.1 \cdot 10^{4}$	$5.1 \cdot 10^{4}$	1.03	0.15

[a] The summary statistics are reported across 109 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[b] The sales based measure of the trade elasticity is based on a model with complete information. The summary statistics are reported across 124 destination-year-industry observations for which an estimates of the Double EMG tail parameter $\lambda_R > 1$. The elasticities are not defined for $\lambda_R \leq 1$.

[c] The amplification effect is computed as the ratio of the quantity based relative to the sales based estimate of trade elasticity. The summary statistics are reported across 77 destination-year-industry observations for which the elasticity is defined in terms of both quantity and sales based measures.

F Parameter and Threshold Estimates

In this section we present the distribution parameter and threshold estimates.

F.1 Distribution Parameters

Table F1 summarizes distribution parameter estimates across 190 observations for the log-export quantity distributions (Panel A) and log-export sales distributions (Panel B). As can be seen from Table F1 Panels A and B, the average sample value of σ for the log-quantity is 1.64 and for the log-sales is 1.66, which means that we can reject the common assumption of Exponentially (or Double Exponentially) distributed data that have $\sigma = 0$, and consequently consider an alternative distribution to model underlying shocks. Furthermore, as can be inferred from the values of the left and right tail parameters, λ_L and λ_R, distributions exhibit substantial heterogeneity in the fatness of both tails. For the log-export quantity distributions, the value of the right tail parameter, λ_R varies between 0.37 and 33.57, with about 44 percent of observations exhibiting a fat right tail, i.e. $\lambda_R < 2$. Similarly, for the log-export sales distributions, the value of the right tail parameter, λ_R varies between 0.47 and 23.46, with about 45 percent of observations exhibiting a fat right tail, i.e. $\lambda_R < 2$. These estimates are consistent with the previous empirical research documenting fatness in the right tail of sales or employment distributions across firms.[40] Furthermore, we also find that distributions exhibit fatness in the left tail ($\lambda_L < 2$) in approximately 38 percent of log-export quantity observations and 85 percent of log-export sales observations.

F.2 Entry Thresholds

Figure F1 provides a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of log-export quantity and log-export sales in Panel A and B respectively. Each dot in the Figure corresponds to a destination-year-industry observation. Figure F1 demonstrates a negative relationship between the average-to-minimum ratio and the entry threshold. The larger is the average-to-minimum ratio, the smaller is the marginal exporter relative an average exporter. Hence, the respective entry threshold must be lower.

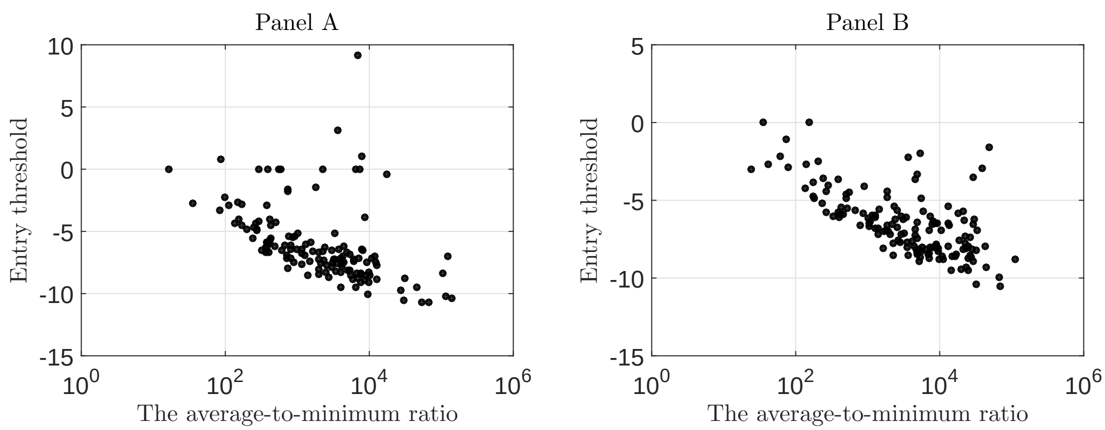
Table F1: Double EMG distribution parameter estimates.

Parameter	σ	λ_L	λ_R
Panel A: Log-quantity [a]			
Mean	1.64	1.85	8.13
Std. Dev.	0.86	4.39	7.35
Panel B: Log-sales [a]			
Mean	1.66	2.48	7.55
Std. Dev.	0.61	4.72	6.78

[a] The summary statistics are reported across 190 destination-year-industry observations. An industry is defined as a 6-digit HS code.

[40] See Axtell (2001) and di Giovanni et al. (2011).

Figure F1: The entry threshold and average-to-minimum ratio.



Notes: The figure depicts a scatter plot of the entry threshold estimates and the corresponding average-to-minimum ratios of export quantity (Panel A) or export sales (Panel B) for observation with an estimate of the Double EMG tail parameter $\lambda_R > 1$. The threshold is not defined for $\lambda_R \leq 1$. Each dot corresponds to a destination-year-industry observation. Values of the thresholds are demeaned by a corresponding estimate of μ of the Double EMG distribution.