Advanced Topics in Macroeconomics: Quantitative Macroeconomics

Problem Set 2 (Computation) Due Date: Optional

Question 1: (Discrete Markov Chains) Consider an AR1 process:

$$y_{t+1} = \rho y_t + \epsilon_{t+1}, \quad \text{s.t.} \quad \epsilon \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$$

We will discretize this process using Tauchen's (1986) method, as we discussed it in class an in lecture notes. You may use any software you prefer. You must provide your source code along with your written answers.

(a) Suppose $\rho = 0.6$ and $\sigma = 0.1$. Let the number of nodes on the grid for y be $n = 9$. Compute the grid points $\{y_i\}_{i=1}^n$ and conditional probabilities $\pi_{ij} \equiv \pi(z_j|z_i)$. Report the results.

(b) Pick some large T and draw $\{z_t\}_{t=0}^T$ from your π_{ij} that you constructed in part (a). Compute the serial correlation and standard deviation of innovations from the simulated data. How do these compare with ρ and σ?

(c) Now suppose $\rho = 0.95$, with $\sigma = 0.1$ and $n = 9$ as before. Recompute the gridpoints and conditional probabilities. Then redo the simulation in part (b) and report the simulated serial correlation and standard deviation of innovations. Does the fit of the computed Markov chain improve?

(d) [Bonus] Read Kopecky and Suen (2010) and redo parts (a), (b) and (c) using the Rouwenhorst method. Does the fit improve?

Question 2: (Comparative Statics) Consider the consumption-savings problem from Chapter 3.3 of the lecture notes, represented in equation (16). This is a form of the buffer-stock model (c.f. Carroll (1992, 1997, 2009)). In this problem, I will ask you to use the **Heterogeneous Agent Computational toolKit (HACK)** to compute policy functions under a particular parameterization and then recompute policy functions under a parameter perturbation. You must provide your source code along with your written answers.

(a) You should have already installed Anaconda and run the Python notebook "Exploring-Estimation-Module.ipynb" as per previous requests. Report the estimated parameters.

(b) Along with your Anaconda distribution, you have installed an IDE (interactive development environment) called Spyder (Scientific PYthon Development EnviRonment). Locate and open Spyder. From Spyder open the HACK file "SolutionLibrary.py" and locate the routine "solve_consumption_problem()." This routine computes policy functions using the **Endogenous Gridpoint Method**.

(c) Import the necessary libraries (see the preamble of "SolutionLibrary.py" and other HACK files).

(d) You will need to provide values for each input of "solve_consumption_problem()." For parameters, let the coefficient of relative risk aversion be given by $\rho = 2$; let the discount factor be $\beta = 0.95$; and let the gross interest rate be $R \equiv 1 + r = 1.03$. Let other parameter inputs be: "constrained = true", "spline_k = 1", and "tol=1e-5". The remaining inputs are constructed within the routine "init_consumer_problem()" and use input values that can be found in the HACK file "params_json_replication.json". The routine "init_consumer_problem()" constructs the asset grid (using the "grid_type = exp_mult" option) and the income shock grids and probabilities ("income_distrib" and "Gamma"). The file "params_json_replication.json" contains values for the effective discount factor via "timevary_discount_factors" and "survival_probs".

(e) Run "solve_consumption_problem()" using the appropriate inputs given in part (d). Plot policy functions in a visually useful way (e.g. as a function of wealth for some shock values and ages).

(f) Numerically compute the MPC out of transitory shocks and permanent shocks for each level of wealth. Report the results in a series of useful Plots (again, it is up to you to develop intuition from visualizing the results). Are the results as you would expect?

(g) Repeat parts (e) and (f) with a new coefficient of risk aversion $\rho = 5$. How do the new results compare to the case in which $\rho = 2$? Are the differences as you would expect?

(h) Let $\rho = 2$ again. Now repeat parts (e) and (f) with a new discount factor $\beta = 0.9$. How do the new results compare to the case in which $\beta = 0.95$? Are the differences as you would expect?

Question 3: (Policy Functions) Consider the General Equilibrium problem from Chapter 4.1 of the lecture notes. For now consider a partial equilibrium formulation with $K = K_{ss}$ where K_{ss} is the value obtained from a deterministic steady state value (e.g. a representative agent economy: when $\varepsilon = 1$ for all agents and $a = K$ for each agent).

(a) Derive K_{ss} analytically (Hint: K_{ss} should correspond with the steady state capital stock of a deterministic Neoclassical Growth Model).

(b) Assume that $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$; $\beta = 0.94$; $\alpha = 0.33$; $\underline{a} = 0$; $\delta = 0.06$; $\rho = 0.95$; $\sigma_\varepsilon = 0.2$; and $r = 0$. Compute the policy functions $c(a, \varepsilon)$ and $a'(a, \varepsilon)$ using Fixed Point Iteration. (Hint: Try a dampening parameter of 0.80 to start, if unstable increase the value). Plot policy functions with assets on the x-axis, for different levels of idiosyncratic shocks.

(c) Assume that $\rho = 0.8$ now. Compute and plot policy functions for consumption and savings. What differences do you observe between the decision rules with high and low persistence? Are these differences as you would expect?

Question 4: (Equilibrium) Consider the General Equilibrium problem from Chapter 4.1 of the lecture notes. Use the same parameter values as in Question 3 part (b).

(a) Using the policy function in Question 3 part (b), compute the stationary distribution. (Hint: Initial guesses for the distribution are important for convergence; see the Heer and Maussner textbook for guidance.) Plot this stationary distribution associated with K_{ss} and compute the implied aggregate capital. Does the new value of aggregate capital equal K_{ss}? Why? (Hint: what are individuals incentives to save when aggregate capital is K_{ss}?)

(b) Use the bisection algorithm discussed in class to compute the equilibrium interest rate. What is the interest rate? Plot the equilibrium stationary distribution.

(c) Now assume that $\sigma = 3$. Compute the equilibrium interest rate. What is the new interest rate? Is the difference between this interest rate and the answer to part (b) as you would expect?